

## MATEMATIKA

Pedagogika institutlari maxsus sirtqi bo'limi uchun o'quv- uslubiy qo'llanma

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Yaratilgan: Angren 2005 yil

Kategoriya: Matematika

Bo'lim: Maxsus sirtqi

Institut: Toshkent viloyat davlat pedagogika instituti

Fakultet: Matematika va fizika

Kafedra: Matematika

Electron fayl turi : RAR

Electron fayl xajmi:

## ANNOTATSIYA

Mazkur o'quv-uslubiy qo'llanma pedagogika institutlarining maxsus sirtqi bo'limida tahsil olayotgan talabalar uchun muljallangan.

Qo'llanma uz ichiga ma'ruzalar matni va SOIT uchun individual vazifalarni olgan.

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1 – mavzu. Matritsa va determinant.

2 – mavzu. Chiziqli tenglamalar sistemasi.

3 -mavzu. Differentsial Hisob elementlari.

4 –mavzu. Integral hisob elementlari.

5-mavzu. Ehtimollar nazariyasining matematik asoslari.

6 -mavzu. Matematik statistika elementlari.

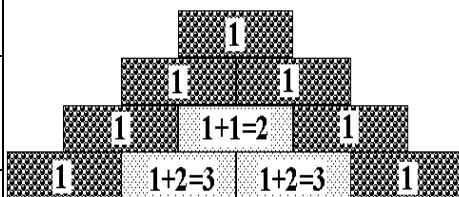
### ADABIYOTLAR

Individual vazifalar.

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Toshkent viloyat Davlat Pedagogika instituti

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|   |   |   |
|---|---|---|
| 4 | 9 | 2 |
| 3 | 5 | 7 |
| 8 | 1 | 6 |



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| ۴ | ۹ | ۲ |
| ۳ | ۵ | ۷ |
| ۸ | ۱ | ۶ |

# Matematika

*Pedagogika institutlari maxsus sirtqi bo'limi  
uchun o'quv- uslubiy qo'llanma*

Angren-2005 y.

UDK 51(075)  
BBK 22.11я73

M 367

*Taqrizchi: TVDPI Boshlang'ich ta'lim metodikasi kafedrası  
(kafedra mudiri TVDPI professori Abdiyev U.N.)*

TVDPI O'quv Metodik Kengashi qaroriga binoan nashr qilindi.

***Ismailov Sh.N., Atakulov A.M., Eshonchayeva G.N.***

M 367      Математика. О'quv-uslubiy qo'llanma. / Angren, 2005.  
ISBN 5-88874-227-9.

Mazkur o'quv-uslubiy qo'llanma pedagogika institutlarining maxsus sirtqi bo'limida tahsil olayotgan talabalar uchun muljallangan.  
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**ISBN 5-88874-227-9**

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## SO'Z BOSHI

Oliy ta'limning fundamentalligi real dunyodagi jarayonlar, ob'ektlar va xodisalar orasidagi munosabatlarni ajratib bila olishga yo'naltirilgan bo'lib, yuqori malakali yoshlarni tarbiyalashda asos sifatida qabul qilingan. Davlat ta'lim standartlari (DTS) tayyorlanayotgan kadrlarning malaka va kunikmalariga yuqori talablarni qo'ymoqda. Shu talablar zamonaviy sharoitlardan bevosita kelib chiqqan bo'lib, ularning maqsadi – ta'lim olayotgan yoshlardan milliy intellektual elitani shakllantirish, vaholanki matematik madaniyat — zamonaviy madaniyatning tarkibiy qismidir.

**Matematika** — yetarlicha umumiy nuqtai nazardan barcha miqdoriy munosabatlarni va fazoviy shakllarni klassifikatsiya qiladigan va o'rganadigan fan sifatida e'tirof e'tilib, uni o'qitishning asosiy maqsadi - talabalarni zamonaviy matematika fani insonning kundalik xayotida alohida o'rin tutishini isbotlash.

Albatta, o'quv soatlarining miqdori, tayanch tushunchalar majmuasining xajmi hamda talabalarining psixologik hususiyatlari xar bir professor-o'qituvchidan mavzuni bayon etishda yuksak e'tiborni va sabr-toqatni talab qilib, yangi axborot va ta'lim texnologiyalarning zamonaviy yutuqlardan foydalanishga chorlaydi.

Shuni ta'kidlashimiz lozimki, o'quv materialni bayon etishda tayanch tushunchalarni etarli darajada aniq va sodda berish maqsadga muvofiqdir. Albatta bunda asosiy g'oyalar va natijalarni ma'lum darajada erkin berishga majburiy. Agar teoremlarning isboti murakkab bo'lsa, uni to'la bermasdan, balki mantiqiy tomonlariga e'tiborni kuchaytirish kerak.

Amaliy mashg'ulotlarda tadbirlarda keng qo'llanilayotgan asosiy matematik usullarni o'rgatishga e'tiborni kuchaytirish lozim. Ma'lum sababalariga ko'ra ma'ruzaga kirmagan va murakkab bo'lgan tushunchalarni amaliy mashg'ulotlarda ko'rib o'tish maqsadga muvofiqdir. 2- ilovadagi vazifalar uyda bajariladi.

Tayanch tushunchalarni bayon etishda ShEHM, proektor kabi texnik vositalardan keng foydalanishi ko'zda tutilgan. Ma'ruzalarni slaydlarini yaratishda va ekranda namoyish qilishda Microsoft Power Point va Flash texnologiyalardan, ayrim sonli va analitik hisob-kitoblarda Maple kompyuter dasturidan foydalanish maqsadga muvofiq.

1- ilovadagi material asosida davra suxbatlarni utkazilishini tavsiya etamiz. Bunda talabalar Internet va boshqa manbaalardan foydalanib uzlari matematikaning inson turmushidagi o'rni, matematika metodlarining gumanitar va tabiiy fanlarga tatbiqi, matematikaning asosiy bo'limlari haqida tarixiy va boshqa ma'lumotlar, buyuk allomalar hayoti va ijodi, matematikaga qo'shgan hissalar kabi mavzular bo'yicha qisqacha ma'lumotlar bayon qiladilar. Tabiiyki, bunday tadbirlar talabalarda matematika faniga qiziqishni kuchaytiradi va yuqorida qayd etilgan maqsadlarga erishishga za'min yaratadi.

*Mualliflar.*

## *Mundarija*

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## 1 – MAVZU. CHIZIQLI ALGEBRA ELEMENTLARI.

1. Maqsad: Chiziqli algebraning asosiy usullarini bayon qilish.

2. Reja:

1. Matritsa tushunchasi.
2. Matritsalar ustida asosiy amallar.
3. Determinant.
4. Teskari matritsa.
5. Chiziqli tenglama va chiziqli tenglamalar sistemasi (ChTS).
6. Chiziqli tenglamalar sistemasining elementar almashtirishlari.
7. Gauss usuli.
8. Teskari matritsa usuli.
9. Kramer qoidasi

### 3. Mavzu bayoni

Matritsa tushunchasi.

$mn$  ta  $a_{ij}$  ( $i=1, \dots, m, j=1, \dots, n$ ) sonlardan tashkil topgan

$$A = (a_{ij}) = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

$m$  satrli va  $n$  ustunli matritsa deyiladi. Shunday matritsalar to'plamini  $R^{m \times n}$  orqali belgilaymiz.

Ustunlar (yoki satrlar) soni birga teng matritsa *vektor* deyiladi.

Ustunlar soni satrlar soniga teng matritsa *kvadrat* matritsa deyiladi.

$$\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & a_{nn} \end{pmatrix} \text{ ko'rinishdagi kvadrat matritsa } \textit{diagonal} \text{ matritsa deb yuritiladi.}$$

$m$  satrli va  $n$  ustunli ikkita  $A, B$  matritsalar *nomdosh* deyiladi.

Ta'rif.  $A, B \in R^{m \times n}$ ,  $A = (a_{ik})$ ,  $B = (b_{ik})$  matritsalar uchun  $a_{ik} = b_{ik}$ ,  $i=1, \dots, m$ ,  $k=1, \dots, n$ , tengliklar bajarilsa, bu matritsalar *teng* deyiladi.

Matritsalar ustida asosiy amallar.

Ta'rif.  $A, B \in R^{m \times n}$ ,  $A = (a_{ik})$ ,  $B = (b_{ik})$  matritsalar *yig'indisi* deb elementlari  $c_{ik} = a_{ik} + b_{ik}$ ,  $i=1, \dots, m$ ,  $k=1, \dots, n$ ,

tengliklarni qanoatlantiruvchi  $C = (c_{ik}) \in R^{m \times n}$  matritsaga aytiladi va u  $A + B$  orqali belgilanadi.

Misol.  $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 2+1 \\ 0+1 & 1+1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}$ ,

Agar  $A \in R^{m \times n}$ ,  $\lambda \in R$  bo'lsa, u holda  $A$  matritsaning  $\lambda$  *skalyarga ko'paytmasi* deb  $\lambda A = (b_{ik})$ ,  $b_{ik} = \lambda a_{ik}$ ,  $i=1, \dots, m$ ,  $k=1, \dots, n$ , matritsaga aytiladi.

Ta'rif.  $A \in R^{m \times n}$ ,  $B \in R^{n \times t}$ ,  $A = (a_{ip})$ ,  $B = (b_{pk})$  matritsalar *ko'paytmasi* deb elementlari  $c_{ik} = \sum_{p=1}^n a_{ip} b_{pk}$ ,  $i=1, \dots, m$ ,  $k=1, \dots, t$ , tengliklarni qanoatlantiruvchi

$C=(c_{ik}) \in R^{m \times t}$  matritsaga aytiladi va u  $A B$  orqali belgilanadi.

Shuni ta'kidlash lozimki,  $A \in R^{m \times n}$ ,  $B \in R^{n \times t}$  matritsalar uchun  $B A$  matritsa faqat  $t=s$  bo'lganda aniqlangan. Bu holda  $A B \in R^{m \times t}$ ,  $B A \in R^{n \times n}$ , Umuman aytganda  $m=n$  bo'lganda ham (ya'ni  $A \in R^{m \times m}$ ,  $B \in R^{m \times m}$  matritsalar uchun  $A B \neq B A$  bo'ladi, ya'ni ko'paytirish amali nokommutativ amaldir. Ushbuni qo'yidagi misol ko'rsatadi.

Misollar. A)  $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 0 + 2 \cdot 1 & 1 \cdot 1 + 2 \cdot 1 \\ 0 \cdot 0 + 1 \cdot 1 & 0 \cdot 1 + 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} =$

$$= \begin{pmatrix} 0 \cdot 1 + 1 \cdot 0 & 0 \cdot 2 + 1 \cdot 1 \\ 1 \cdot 1 + 1 \cdot 0 & 1 \cdot 2 + 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 3 \end{pmatrix}.$$

b)  $\begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 2 & 1 \cdot 4 & 1 \cdot 1 \\ 4 \cdot 2 & 4 \cdot 4 & 4 \cdot 1 \\ 3 \cdot 2 & 3 \cdot 4 & 3 \cdot 1 \end{pmatrix} = \begin{pmatrix} 2 & 4 & 1 \\ 8 & 16 & 4 \\ 6 & 12 & 3 \end{pmatrix}.$

c)  $\begin{pmatrix} 2 & 4 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} = 2 \cdot 1 + 4 \cdot 4 + 1 \cdot 3 = 2 + 16 + 3 = 21.$

d)  $\begin{pmatrix} 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} = (3+10 \quad 4+12) = (13 \quad 16).$

Ta'rif.  $E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$  ko'rinishdagi  $n$ -tartibli diagonal kvadrat matritsa *birlik matritsa* deyiladi.

*matritsa* deyiladi.

$A \in R^{m \times n}$  matritsa uchun  $AE=A$  o'rinli (agar bunda  $EA$  ko'paytma ham aniqlangan bo'lsa, u holda  $EA=A$ ).

Teorema Matritsalar ko'paytirish amali assotsiativ bo'ladi, ya'ni agar  $(A B)C$  va  $A(BC)$  ko'paytmalardan birortasi mavjud bo'lsa, u holda ikkinchisi ham mavjud bo'lib, ular uchun  $A(BC) = (A B)C$  munosabat o'rinli bo'ladi.

Bu teoremani isbot qilish uchun  $A \in R^{m \times n}$ ,  $B \in R^{n \times t}$ ,  $C \in R^{t \times s}$  matritsalar uchun  $(A B)C$  va  $A(BC)$  matritsalar elementlari hisoblanib, biri biriga solishtiriladi.

Teorema . Matritsalar ko'paytirish amali qo'shishi amaliga nisbatan distributiv bo'ladi. Ya'ni  $(A+B)C$  va  $A B+AC$  ( $A(B+C)$  va  $A B+AC$ ) matritsalaridan birortasi mavjud bo'lsa, u holda ikkinchisi ham mavjud bo'lib, ular uchun  $(A+B)C = A B+AC$  ( $A(B+C) = A B+AC$ ) munosabat o'rinli bo'ladi.

Qo'yidagi xossalar o'rinli:

$$\alpha(B+C) = \alpha B + \alpha C; (\alpha+\beta)B = \alpha B + \beta B; (\alpha\beta)B = \alpha(\beta B); 1B = B; \alpha(BC) = (\alpha B)C = B(\alpha C)$$

Determinantlar.

Ta'rif. Ikkinchi tartibli  $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in R^{2 \times 2}$  kvadrat matritsaning *determinanti* (ya'ni 2 – tartibli determinant) deb

$$\det A = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \quad (1)$$

songa aytiladi.

Ta'rif. Uchinchi tartibli  $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \in R^{3 \times 3}$  kvadrat matritsaning *determinanti*

(ya'ni 3 –tartibli determinant) deb

$$\det A = |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} =$$

$$= a_{11} a_{22} a_{33} + a_{13} a_{21} a_{32} + a_{12} a_{13} a_{31} - a_{13} a_{22} a_{31} - a_{11} a_{13} a_{32} - a_{12} a_{21} a_{33} . \quad (3)$$

songa aytiladi.

3 –tartibli determinantni hisoblashga doir konkret misollarda turli mnemonik shoidalar mavjud. Shu shoidalar bilan sizlar amaliy mashg'ulotlarda tanishasizlar.

Misollar. a)  $\begin{vmatrix} 2 & -3 & -1 \\ 4 & -1 & 2 \\ 3 & 5 & 0 \end{vmatrix}$  ni hisoblaymiz. Bu uchun (3) formulaga ko'ra

$$\begin{vmatrix} 2 & -3 & -1 \\ 4 & -1 & 2 \\ 3 & 5 & 0 \end{vmatrix} = 2 \begin{vmatrix} -1 & 2 \\ 5 & 0 \end{vmatrix} - (-3) \begin{vmatrix} 4 & 2 \\ 3 & 0 \end{vmatrix} + (-1) \begin{vmatrix} 4 & -1 \\ 3 & 5 \end{vmatrix} =$$

$$= 2(-1 \cdot 0 - 2 \cdot 5) + 3(4 \cdot 0 - 2 \cdot 3) - 1(4 \cdot 5 - (-1) \cdot 3) = -61.$$

Misol.  $\begin{pmatrix} 2 & -3 & -1 \\ 4 & -1 & 2 \\ 3 & 5 & 0 \end{pmatrix}$  matritsa uchun  $M_{31}$  va  $A_{23}$  ni hisoblaymiz.

$$M_{31} = \begin{vmatrix} -3 & -1 \\ -1 & 2 \end{vmatrix} = -3 \cdot 2 - (-1) \cdot (-1) = -6 - 1 = -7. \quad A_{23} = (-1)^5 \begin{vmatrix} 2 & -3 \\ 3 & 5 \end{vmatrix} = -(2 \cdot 5 - (-3) \cdot 3) = -19.$$

Teskari matritsa.

Ta'rif.  $A \in R^{n \times n}$  kvadrat matritsa *teskarilantiruvchi* deyiladi, agar  $AB=BA=E$  shartlarni qanoatlantiruvchi  $B$  matritsa mavjud bo'lsa, bu holda  $B$  matritsa  $A$  ga nisbatan *teskari matritsa* deb yuritiladi.

Ta'rifdan ko'rinib turibdiki,  $B \in R^{n \times n}$ ,  $E \in R^{n \times n}$  bo'ladi.

Biz ma'ruzamizda faqat  $n=4$  holni qaraymiz.

$A$  ga nisbatan teskari matritsani biz  $A^{-1}$  orqali belgilaymiz.

Teorema. Agar  $A \in R^{3 \times 3}$  matritsaning determinanti noldan farqli bo'lsa, u holda formula o'rinli.

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$



Misol.  $A = \begin{pmatrix} 5 & -1 & -1 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix}$  bo'lsa  $A^{-1}$  ni toping.

Yechim.  $\det A = \begin{vmatrix} 5 & -1 & -1 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{vmatrix} = 5(4-9) + 1(2-12) - 1(3-8) = -25 - 10 + 5 = -30.$

$$M_{11} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -5; \quad M_{21} = \begin{vmatrix} -1 & -1 \\ 3 & 2 \end{vmatrix} = 1; \quad M_{31} = \begin{vmatrix} -1 & -1 \\ 2 & 3 \end{vmatrix} = -1;$$

$$M_{12} = \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} = -10; \quad M_{22} = \begin{vmatrix} 5 & -1 \\ 4 & 2 \end{vmatrix} = 14; \quad M_{32} = \begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = 16;$$

$$M_{13} = \begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = -5; \quad M_{23} = \begin{vmatrix} 5 & -1 \\ 4 & 3 \end{vmatrix} = 19; \quad M_{33} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 11;$$

$$\begin{aligned} a_{11}^{-1} &= \frac{5}{30}; & a_{12}^{-1} &= \frac{1}{30}; & a_{13}^{-1} &= \frac{1}{30}; \\ a_{21}^{-1} &= -\frac{10}{30}; & a_{22}^{-1} &= -\frac{14}{30}; & a_{23}^{-1} &= \frac{16}{30}; \\ a_{31}^{-1} &= \frac{5}{30}; & a_{32}^{-1} &= \frac{19}{30}; & a_{33}^{-1} &= -\frac{11}{30}; \end{aligned} \quad \text{Demak, } A^{-1} = \begin{pmatrix} \frac{1}{6} & \frac{1}{30} & \frac{1}{30} \\ -\frac{1}{3} & -\frac{7}{15} & \frac{8}{15} \\ \frac{1}{6} & \frac{19}{30} & -\frac{11}{30} \end{pmatrix}.$$

Tekshiramiz:

$$A \cdot A^{-1} = \begin{pmatrix} 5 & -1 & -1 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} \begin{pmatrix} \frac{5}{30} & \frac{1}{30} & \frac{1}{30} \\ -\frac{10}{30} & -\frac{14}{30} & \frac{16}{30} \\ \frac{5}{30} & \frac{19}{30} & -\frac{11}{30} \end{pmatrix} = \frac{1}{30} \begin{pmatrix} 25+10-5 & 5+14-19 & 5-16+11 \\ 5-20+15 & 1-28+57 & 1+32-33 \\ 20-30+10 & 4-42+38 & 4+48-22 \end{pmatrix} = E.$$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1r} & a_{1,r+1} & \dots & a_{1n} \\ & a_{22} & \dots & a_{2r} & a_{2,r+1} & \dots & a_{2n} \\ & & \dots & & & \dots & \\ \bigcirc & & & a_{rr} & a_{r,r+1} & \dots & a_{rn} \end{pmatrix},$$

ko'rinishdagi matritsa *zinasimon matritsa* deyiladi, bu yerda  $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$  - noldan farqli sonlar.

Chiziqli tenglama va chiziqli tenglamalar sistemasi (ChTS).

Ta'rif.

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b \quad (1)$$

$x_1, x_2, \dots, x_n$  noma'lumli tenglama *chiziqli tenglama* deyiladi, bu yerda  $a_1, a_2, \dots, a_n$  sonlar chiziqli tenglama *koeffitsientlari* va  $b$  son chiziqli tenglama *ozod hadi* deb yuritiladi. (1) ni qanoatlantiruvchi  $(x_1, x_2, \dots, x_n)$  vektor esa uning *yechimi* deyiladi.

Agar  $b = 0$  bo'lsa, u holda (1) ga *birjinsli chiziqli tenglama*, aks holda *birjinsli bo'lmagan chiziqli tenglama* deyiladi.

Ravshanki, har qanday bir jinsli chiziqli tenglama nol yechimga ega.

*Ta'rif.*

$x_1, x_2, \dots, x_n$  lar ChTSni noma'lumlari,  $a_{ik} \in R$  – ChTS koefitsientlari, va  $b_k \in R$  esa ozod hadlar deyiladi. (2) sistemani qanoatlantiruvchi  $(x_1, x_2, \dots, x_n)$  vektor uning yechimi deyiladi. Agar (2) da barcha  $b_k$  sonlar nolga teng бўlsa, u holda (2) ga *bir-jinsli ChTS* (BChTS), aks holda u *birjinsli bo'lmagan ChTS* deyiladi.

Chiziqli tenglamalar sistemasining elementar almashtirishlari.

(1)

*Ta'rif.* qōyidagi almashtirishlar ChTSni *elementar almashtirishlari* deyiladi:

- Teorema.** ChTS ustida chekli sondagi elementar almashtirishlar bajarilsa, hosil bōlgan ChTS yechimlar to'plami berilgan ChTS yechimlar to'plami bilan ustma –ust tushdi.

Avvalambor (1) sistemada ozod hadi va koeffitsientlari nolga teng bōlgan tenglamalar b faraz qilamiz, aks holda ularni barchasini tashlab yuboramiz .

Shuning uchun (1) sistemani yechishdan oldin, undagi tenglamalarda noldan farqli sientlar mavjud deb qabul qilamiz.

Zarur b\u00f6lsa, numeratsiyani \u00f6zgartirib,  $a_{11} \neq 0$  deb qabul qilamiz.

(1) sistemani  $k$  -nchi tenglamaga ( $k=2,3,\dots,m$ ) birinchi tenglamani  $-\frac{a_{k1}}{a_{11}}$  ga

köpaytirib qoşib chiqamiz. Natijada  $2, 3, \dots, m$  tenglamalarda  $x_1$  noma'lum yôqoladi va qôvidagi (1) ga ekvivalent sistema hosil bôladi:

(2)

(2) sistemada ozod hadi va koeffitsientlari nolga teng bōlgan tenglamalar yōq deb faraz qilamiz, aks holda ularni barchasini tashlab yuboramiz .

$$\left\{ \begin{array}{l} a_{22}'x_2 + \dots + a_{2n}'x_n = b_2' \\ \vdots \\ a_{m2}'x_2 + \dots + a_{mn}'x_n = b_m' \end{array} \right.$$

Noma'lumlar yiqotish jarayonini davom ettirib, natijada qōyidagi sistemaga kelamiz:

$$\left\{ \begin{array}{l} c_{11}x_1' + c_{12}x_2' + \dots + c_{1r}x_r' + c_{1,r+1}x_{r+1}' + \dots + c_{1n}x_n' = d_1 \\ c_{22}x_2' + \dots + c_{2r}x_r' + c_{2,r+1}x_{r+1}' + \dots + c_{2n}x_n' = d_2 \\ \vdots \\ c_{rr}x_r' + c_{r,r+1}x_{r+1}' + \dots + c_{rn}x_n' = d_r \end{array} \right. \quad (3)$$

$x_1', x_2', \dots, x_r'$  noma'lumlar *bosh noma'lumlar*,  $x_{r+1}', x_{r+2}', \dots, x_n'$  noma'lumlar esa *ozod noma'lumlar* deb yuritiladi.

natijada uchburchaksimon sistemani hosil qilamiz.

(1) sistemani echishda Gauss usulidagi hisob–kitoblarni

$$\left( \begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_n \end{array} \right) \text{ matritsa ustida olib borish maqsadga muvofiq. Natijada}$$

Misol. qöyidagi ChTSni eching.

$$\begin{cases} x + y - z = 2 \\ 2x + 3y - 5z = 5 \\ 3x + 4y - 6z = 7 \end{cases}$$

$$\text{Yechim.} \left( \begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 2 & 3 & -5 & 5 \\ 3 & 4 & -6 & 7 \end{array} \right) \Leftrightarrow \left( \begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{array} \right) \Leftrightarrow \left( \begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{array} \right) \Leftrightarrow \left( \begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

Ohirgi matritsaga mos bōlgan sistema qōyidagicha yoziladi: 
$$\begin{cases} x + y - z = 2 \\ y - 3z = -1 \end{cases}.$$

Ozod noma'lumni fiksirlaymiz:  $z=t$ . Natijada

$$\begin{cases} x + y = 2 + t \\ y = -1 + 3t \end{cases} \text{ sistema hosil b\u00f3ladi. Bundan } (3-2t, -1+3t, t), t \in \mathbb{R}, \text{ yechimlar oilasini}$$

hosil qilamiz.

Teskari matritsa usuli.

Bu bandda biz

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} \quad (4)$$

ko'rinishdagi ChTS yechimini izlaymiz.

$$\text{Quyidagi belgilashlarni kiritamiz: } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}; \quad B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}; \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

Faraz qilamiz  $A$  matritsa  $\Delta$  determinanti noldan farqli.

(4) ChTS quyidagicha yozishimiz mumkin:  $A \cdot X = B$ .

Demak,  $A^{-1} \cdot A \cdot X = A^{-1} \cdot B$ . Ammo  $E \cdot X = A^{-1} \cdot B$ . U holda (4) teskari matritsa yagonaligidan ChTS  $X = A^{-1}B$  ko'rinishdagi yagona yechimga ega.

$$\text{Misol. } \begin{cases} 5x - y - z = 0 \\ x + 2y + 3z = 14 \\ 4x + 3y + 2z = 16 \end{cases}; \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 14 \\ 16 \end{pmatrix}, \quad A = \begin{pmatrix} 5 & -1 & -1 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix}$$

$$\text{Oldingi ma'ruzada biz } A^{-1} = \begin{pmatrix} \frac{1}{6} & \frac{1}{30} & \frac{1}{30} \\ -\frac{1}{3} & -\frac{7}{15} & \frac{8}{15} \\ \frac{1}{6} & \frac{19}{30} & -\frac{11}{30} \end{pmatrix} \text{ ni topganmiz.}$$

$$\text{Demak, } X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B = \begin{pmatrix} \frac{1}{6} & \frac{1}{30} & \frac{1}{30} \\ -\frac{1}{3} & -\frac{7}{15} & \frac{8}{15} \\ \frac{1}{6} & \frac{19}{30} & -\frac{11}{30} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 14 \\ 16 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} \cdot 0 + \frac{14}{30} + \frac{16}{30} \\ -\frac{1}{3} \cdot 0 - \frac{98}{15} + \frac{128}{15} \\ \frac{1}{6} \cdot 0 + \frac{266}{30} - \frac{176}{30} \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Kramer qoidasi. (Kramer Gabriel' (1704-1752) – Sveytsariya matematigi)

$$\text{Agar } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \text{ matritsa } \Delta \text{ determinanti noldan farqli bo'lsa, u holda (4)}$$

ChTSning yechimi  $x_1 = \Delta_1 / \det A$ ;  $x_2 = \Delta_2 / \det A$ ;  $x_3 = \Delta_3 / \det A$  formulalar bo'yicha topiladi, bu yerda

$$\Delta_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}; \quad \Delta_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}; \quad \Delta_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}.$$

$$\text{Misol. } \begin{cases} 5x - y - z = 0 \\ x + 2y + 3z = 14 \\ 4x + 3y + 2z = 16 \end{cases}$$

$$\Delta = \begin{vmatrix} 5 & -1 & -1 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{vmatrix} = -30; \Delta_1 = \begin{vmatrix} 0 & -1 & -1 \\ 14 & 2 & 3 \\ 16 & 3 & 2 \end{vmatrix} = -30; \Delta_2 = \begin{vmatrix} 5 & 0 & -1 \\ 1 & 14 & 3 \\ 4 & 16 & 2 \end{vmatrix} = -60; \Delta_3 = \begin{vmatrix} 5 & -1 & 0 \\ 1 & 2 & 14 \\ 4 & 3 & 16 \end{vmatrix} = -90.$$

Demak,  $x_1 = \Delta_1/\Delta = 1$ ;  $x_2 = \Delta_2/\Delta = 2$ ;  $x_3 = \Delta_3/\Delta = 3$ .

#### 4. Tayanch tushunchalar:

Matritsalar ustida asosiy amallar va ularning asosiy xossalari, determinantlar, teskari matritsa. Chiziqli tenglamalar, chiziqli tenglamalar sistemasi, yechim, elementar almashtirishlar, ekvivalent sistemalar, Gauss usuli, bosh va ozod noma'lumlar, teskari matritsa usuli, Kramer qoidasi.

#### 5. Nazorat savollari.

1. Matritsalar ustida amallarning ta'rifini va hossalari bayon qiling.
2. Qarama-qarshi matritsa qanday aniqlanadi? U qanday hossalarga ega?
3. Determinant haqida tushuncha bering.
4. Teskari matritsa deb nimaga aytiladi?
5. Teskari matritsa qanday hisoblanadi?
6. Gauss usulini bayon qiling.
7. Teskari matritsa usulini bayon qiling.
8. Kramer qoidasini bayon qiling.

## 2 -MAVZU. MATEMATIK TAHLIL ELEMENTLARI.

1. Maqsad: Matematik tahlil asoslarini takrorlash.

2. Reja:

1. Sonli funktsiyalar.
2. Funktsiya limiti va uzluksiz funktsiyalar xaqida tushuncha.
3. Funktsiya limitini hisoblash
4. Hosila tushunchasi. Hosilalar jadvali.
5. Differentsiallanuvchi funktsiyalar hossalari.
6. Hosila tadbiqlari.

3. Mavzu bayoni.

Sonli funktsiyalar.

**R** orqali xaqiqiy sonlar to'plamini belgilaymiz. Uning qism to'plamlarini sonli to'plamlar deb yuritamiz.

Ta'rif.  $D$  sonli to'plamlarga tegishli har bir  $x$  soniga biror qoidaga binoan  $x$  ga bog'liq bo'lgan  $y$  sonni taqqoslovchi moslik aniqlanish sohasi  $D$  dan iborat bo'lgan sonli funktsiya (funktsiya) deyiladi.

Funktsiyalar odatda lotin xarflari bilan belgilanadi. Ixtiyoriy  $f$  funktsiyani qaraymiz.  $x$  ga mos bo'lgan  $y$  sonni  $f$  funktsiyaning  $x$  nuqtadagi qiymati deyiladi va  $f(x)$  orqali belgilanadi.

$D$  to'plam  $f$  funktsiyaning aniqlanish sohasi deyiladi va  $D(f)$  orqali belgilanadi.

Ko'pincha sonli funktsiya formula yordamida berilgan bo'lib, uning aniqlanish sohasi argumentlarning joiz qiymatlaridan iborat (ushbu aniqlanish sohasi tabiiy deyiladi).

Ta'rif.  $f$  funktsiya berilgan bo'lsin.

a)  $\{y / (\exists x \in D(f)) y = f(x)\}$  to'plam  $f$  funktsiyaning *qiymatlar to'plami* deb yuritiladi va  $E(f)$  orqali belgilanadi.

b)  $\{(x, y) \in \mathbf{R}^2 / x \in X, y = f(x)\}$  kurinishdagi to'plam  $f$  funktsiyaning grafigi deyiladi.

Funktsiyalar ustida algebraik amallar qo'yidagicha kiritiladi:

$$(f \pm g)(x) = f(x) \pm g(x), D(f \pm g) = D(f) \cap D(g)$$

$$(f \cdot g)(x) = f(x) g(x), D(f \cdot g) = D(f) \cap D(g)$$

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)}, D\left(\frac{f}{g}\right) = \{x \in D(f) \cap D(g) / g(x) \neq 0\}$$

Ta'rif.  $f, g$  funktsiyalar uchun  $h(x) = g(f(x))$  tenglik bilan aniqlangan va aniqlanish sohasi  $D(h) = \{x \in D(f) / f(x) \in D(g)\}$  bo'lgan  $h$  funktsiya *murakkab funktsiya* deyiladi va u  $g \circ f$  yoki  $g \circ f$  orqali belgilanadi.

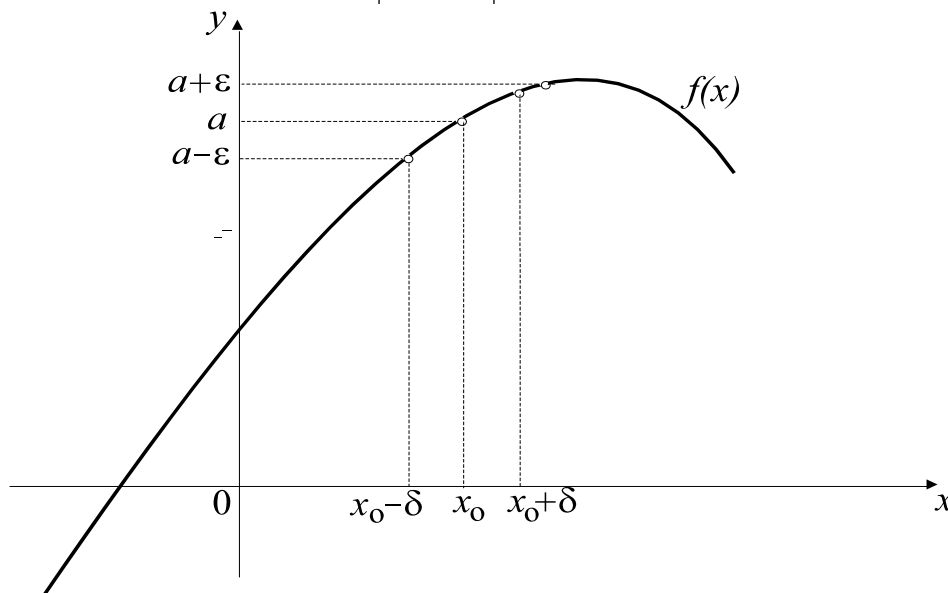
Adabiyotlarda  $g \circ f$  funktsiya  $g$  va  $f$  funktsiyalar *superpozitsiyasi* yoki *kompozi-tsiyasi* deb yuritiladi.

Ta'rif.  $f$  funktsiya uchun  $(f \circ g)(x) = x, \forall x \in D(f)$  va  $(g \circ f)(x) = x, \forall x \in D(g)$  tengliklarni ta'minlovchi  $g$  funktsiya mavjud bo'lsa, u holda  $f$  funktsiya *teskarilannuvchi funktsiya*,  $g$  funktsiya esa  $f$  funktsiyaga *teskari funktsiya* deyiladi va  $f^{-1}$  orqali belgilanadi.

Funktsiya limiti va uzluksiz funktsiyalar xaqida tushuncha.

$f(x)$  funktsiya  $x_0$  biror atrofida aniqlangan bo'lsin.

Ta'rif.  $a$  soni  $y = f(x)$  funktsiyaning  $x_0$  nuqtada limiti deyiladi  $\left[ a = \lim_{x \rightarrow x_0} f(x) \right]$ , agar ixtiyoriy  $\varepsilon > 0$  uchun shunday  $\delta(\varepsilon) > 0$  mavjudki,  $0 < |x - x_0| < \delta(\varepsilon)$  qushtengsizlikni qanoatlantiradigan barcha  $x$  lar uchun  $|f(x) - a| < \varepsilon$  tengsizlik o'rinli bo'lsa (1-rasm).



1- rasm.

### Funktsiya limitini hisoblash

Ajoyib limitlar:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1. \quad \text{- I-ajoyib limit}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = e \quad \text{- II -ajoyib limit.}$$

II - ajoyib limitdan qo'yidagi tengliklar kelib chiqadi:

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1, \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1.$$

### Misollar.

$$a) \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^3 - x} = \lim_{x \rightarrow 1} \frac{(x-1)^2}{x(x-1)(x+1)} = \lim_{x \rightarrow 1} \frac{x-1}{x(x+1)} = 0;$$

$$b) \lim_{x \rightarrow 0} \frac{\operatorname{tg} 3x}{x} = \lim_{x \rightarrow 0} \frac{3 \sin 3x}{3x \cos 3x} = 3.$$

$$c) \lim_{x \rightarrow 10} \frac{\sqrt{x-1} - 3}{x - 10} = \left[ \begin{array}{l} \sqrt{x-1} = t \\ x = t^2 + 1 \\ x \rightarrow 10 \Rightarrow t \rightarrow 3 \end{array} \right] = \lim_{t \rightarrow 3} \frac{t-3}{t^2 - 9} = \lim_{t \rightarrow 3} \frac{t-3}{(t-3)(t+3)} = \frac{1}{6}$$

d)

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} = \lim_{x \rightarrow 0} \frac{(\sqrt{x+1} - 1)(\sqrt{x+1} + 1)}{x(\sqrt{x+1} + 1)} = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+1} + 1)} = \frac{1}{2}.$$

e)

$$\lim_{x \rightarrow \infty} \left( \frac{x+3}{x-2} \right)^{2x+1} = \lim_{x \rightarrow \infty} \left( \frac{x-2+5}{x-2} \right)^{2x+1} = \lim_{x \rightarrow \infty} \left( 1 + \frac{5}{x-2} \right)^{\frac{x-2}{5} \cdot \frac{5}{x-2} (2x+1)} = e^{\lim_{x \rightarrow \infty} \frac{5(2x+1)}{x-2}} = e^{10}.$$

Agar  $f(x_0) = \lim_{x \rightarrow x_0} f(x)$  bo'lsa, u holda  $f$  funktsiya  $x_0$  nuqtada *uzluksiz* deyiladi.

Agar  $f$  funktsiya  $I$  sonli to'plamning barcha nuqtalarida uzluksiz bo'lsa, u holda u  $I$  to'plamda *uzluksiz* deyiladi.

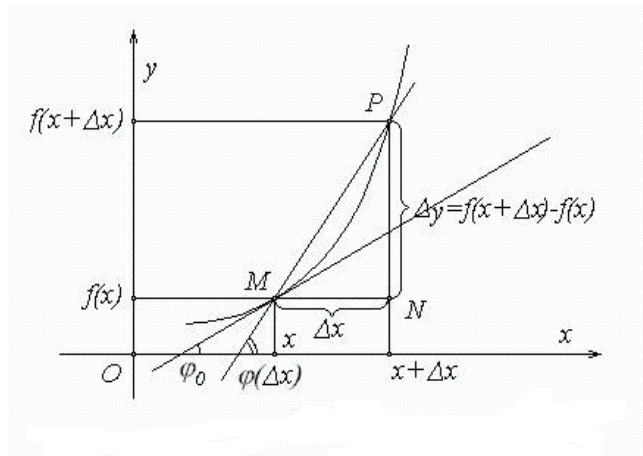
Ko'pincha  $I$  sonli to'plam sifatida oraliq qaraladi va ushbu oraliq *uzluksizlik oralig'i* deyiladi.

Shuni ta'kidlash lozimki, maktab matematikasida o'rganilgan deyarli barcha funktsiyalar o'zining tabiiy aniqlanish sohalarida uzluksiz funktsiyalar bo'ladi.

Hosila tushunchasi.

Ta'rif.  $y = f(x)$  funktsiyaning  $x_0$  nuqtadagi *hosilasi*  
 $y'(x_0) = f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{(f(x_0 + \Delta x) - f(x_0))}{\Delta x}$  formula bilan aniqlanadi.

$f$  funktsiyaning  $x_0$  nuqtadagi hosilasi ushbu funktsiyaning grafigiga  $M = (x_0, f(x_0))$  nuqta orqali o'tkazilgan o'rinmaning *burchak koeffitsientini* bildiradi, ya'ni  $f'(x) = \operatorname{tg} \varphi_0$  (2-rasm)



**2-rasm**

Agar  $f$  funktsiyaning  $x_0$  nuqtada chekli hosilaga ega bo'lsa, u holda ushbu funktsiya  $x_0$  nuqtada *differentiallanuvchi* deyiladi, biror to'plamning barcha nuqtalarda differentiallanuvchi funktsiya esa mazkur to'plamda *differentiallanuvchi* deb yuritiladi.

Shunday qilib, berilgan nuqtada differentiallanuvchi  $f$  funktsiyaga uning shu nuqtadagi  $f'$  hosilasini mos qo'yish mumkin. Ushbu moslik yordamida aniqlangan  $f'$  funktsiya  $f$  funktsiyaning hosilasi deyiladi va u qo'yidagi aniqlanish sohaga ega

$$D(f') = \{x_0 \in D(f) / \exists \lim_{\Delta x \rightarrow 0} \frac{(f(x_0 + \Delta x) - f(x_0))}{\Delta x}\}.$$

Agar  $f'$  hosila o'zi biror  $x_0 \in D(f')$  nuqtada differentiallanuvchi bo'lsa, u holda uning  $(f')'(x_0)$  hosilasi  $f$  funktsiyaning  $x_0$  nuqtadagi *ikkinchi tartibli hosilasi* deyiladi va  $f''(x_0)$  yoki  $\frac{d^2 f(x_0)}{d^2 x}$  orqali belgilanadi.

$f^{(n)}(x_0) = (f^{(n-1)})'(x_0)$  rekkurent formula yordamida  $f$  funktsiyaning  $x_0$  nuqtadagi  $n$ -*tartibli hosilasini* aniqlashimiz mumkin.



### Hosilalar jadvali.

$$1) C' = 0;$$

$$2) (x^m)' = mx^{m-1};$$

$$3) (\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$4) \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$5) (e^x)' = e^x$$

$$9) (\sin x)' = \cos x$$

$$10) (\cos x)' = -\sin x$$

$$11) (\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$$12) (\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$$

$$13) (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

Differentsiallanuvchi funktsiyalar hossalari.

Agar  $f, g$  funktsiyalar  $x_0$  nuqtada differentsiallanuvchi bo'lsa, u holda  $f \pm g$ ,

$f \cdot g, \left(\frac{f}{g}\right)$  (agar maxraji noldan farqli bo'lsa) ham shu nuqtada differentsiallanuvchi

bo'lib, qo'yidagi formulalar o'rinli:

$$a) (f \pm g)' = f' \pm g' \quad b) (f \cdot g)' = f'g + fg' \quad v) \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}.$$

Misol. a)  $y = x^3 + \ln x$ ; b)  $y = (x^3 - 5)\sin x$ ; v)  $y = \frac{x^3 - 5}{x^4 + 3}$  funktsiyalarning hosilasini

toping.

Yechish. a)  $y' = (x^3)' + (\ln x)' = 3x^2 + \frac{1}{x};$

b)  $y' = (x^3 - 5)' \sin x + (x^3 - 5) \sin' x = 3x^2 \sin x + (x^3 - 5) \cos x$

c)  $y' = \frac{(x^3 - 5)'(x^4 + 3) - (x^3 - 5)(x^4 + 3)'}{(x^4 + 3)^2} = \frac{3x^3(x^4 + 3) - (x^3 - 5) \cdot 4x^4}{(x^4 + 3)^2} = \frac{-x^6 + 20x^3 + 9x^2}{(x^4 + 3)^2}$

2) Agar  $y = f(x), u = g(x) \ (D(u) \subset D(y))$  funktsiyalar differentsiallanuvchi bo'lsa, u holda  $y = f(g(x))$  murakkab funksiya differentsiallanuvchi bo'lib, qo'yidagi formula o'rinli:

$$y' = f'(u) \cdot u'$$

Misol.  $y = \cos(x^3 - x^2 + 2)$  funktsiyaning hosilasini toping.

Yechish.  $u = x^3 - x^2 + 2, u' = 3x^2 - 2x, y = \cos u, f'(u) = -\sin u$ , demak

$$y' = (\cos(x^3 - x^2 + 2))' = -(3x^2 - 2x) \sin(x^3 - x^2 + 2)$$

### Hosila tadbiqlari.

Ta'rif. Agar ixtiyoriy turli  $x_1, x_2 \in (a, b) \subset D(f)$  nuqtalar uchun  $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$

ifoda musbat (manfiy) qiymatlarni qabul qilsa, u holda  $f$  funktsiya  $(a, b)$  oraliqda *o'suvchi* (kamayuvchi) funktsiya deyiladi,  $(a, b)$  oraliq esa *monotonlik oraliq*'i deyiladi

Ta'rif. Agar  $f$  funktsiyaning  $x_0 \in D(f)$  nuqtadagi qiymati bu nuqtaning etarli-cha kichik atrofidagi qolgan nuqtalardagi qiymatlaridan katta (kichik) bo'lsa,  $f$  funktsiya  $x_0$  nuqtada *lokal maksimumga* (lokal minimumga) erishadi deyiladi.

Ta'rif. Agar  $f$  funktsiya  $I$  to'plamda ( $I$  to'plam sifatida oraliq yoki kesma qaralishi

mumkin) aniqlangan bo'lib, uning biror  $x_0 \in I$  nuqtadagi qiymati  $I$  to'plamning qolgan nuqtalardagi qiymatlaridan kichik emas (katta emas) bo'lsa,  $f$  funktsiya  $x_0$  nuqtada *maksimumga (minimumga) erishadi* deyiladi. Funktsiyaning maksimum (minimum) nuqtaga mos bo'lgan qiymati *maksimum (minimum)* deyiladi va  $\max_I f(x)$  ( $\min_I f(x)$ ) orqali belgilanadi. Funktsiyaning minimumlari va maksimumlari funktsiyaning *ekstremumlari* deyiladi.

**Teorema** (*o'sish va kamayish sharti*). Agar  $f$  funktsiyaning ixtiyoriy  $x \in (a,b) \subset D(f)$  nuqtasi uchun  $f'(x)$  ifoda musbat (manfiy) qiymatlarni qabul qilsa, u holda  $f$  funktsiya  $(a,b)$  oraliqda *o'suvchi (kamayuvchi)* funktsiya bo'ladi.

**Misol.**  $y = x^3 - x^2 - x + 2$  funktsiyaning monotonlik oraliqlarini toping.

**Yechish.**  $y' = (x^3 - x^2 - x + 2)' = 3x^2 - 2x - 1 = 3(x-1)(x + \frac{1}{3})$ . Demak,  $(-\frac{1}{3}, 1)$  da funktsiya

kamayadi,  $(-\infty, -\frac{1}{3}) \cup (1, +\infty)$  da esa o'sadi.

**Teorema** (*ekstremumning zaruriy sharti*) Agar  $f$  funktsiya  $x_0 \in D(f)$  nuqtada lokal ekstremumga erishsa, u holda  $f$  funktsiyaning hosilasi  $x_0$  nuqtadagi qiymati nolga teng bo'ladi.

**Teorema** (*ekstremumning etarli sharti*). Agar  $f'(x_0)=0$  bo'lib,  $f''(x_0) < 0$  ( $f''(x_0) > 0$ ) bajarilsa, u holda  $f$  funktsiya  $x_0$  nuqtada lokal maksimumga (lokal minimumga) erishadi.

**Misol.**  $y = x^3 - x^2 - x$  funktsiyaning  $[0,2]$  da ekstremumlari topilsin.

**Yechish.**  $y' = (x^3 - x^2 - x)' = 3x^2 - 2x - 1 = 3(x-1)(x + \frac{1}{3}) = 0$ .  $[0,2]$  kesmaga faqat

$x=1$  «shubhali» nuqta tegishli.  $y''(1) = 4 > 0$  bo'lgani uchun  $x=1$  – lokal minimum nuqtasi bo'ladi.  $y(1)=-1$ ,  $y(0)=0$ ,  $y(2)=2$  dan  $x=1$  – minimum,  $x=2$  maksimum nuqtalari.

Demak,  $\max_{[0,2]} y = y(2)=2$ ,  $\min_{[0,2]} y = y(1)=-1$ .

**Boshlang'ich funktsiya va aniqmas integral.**

**Ta'rif.** Biror  $(a,b)$  oraliqda aniqlangan  $f$  funktsiyaning *boshlang'ich funktsiyasi* deb shu oraliqning barcha nuqtalarda  $F'(x)=f(x)$  shartni qanoatlantiradigan  $F(x)$  funktsiyaga aytiladi.

Agar  $F(x)$  va  $F(x)$  funktsiyalar  $f$  funktsiyaning boshlang'ich funktsiyalari bo'lsa, u holda  $F(x) = F(x)+C$  bo'ladi, bu yerda  $C$  - uzgarmas son, va aksincha, agar  $F(x)$  funktsiya  $f$  funktsiyaning boshlang'ich funktsiyasi bo'lsa, u holda  $F(x) = F(x)+C$  funktsiya ham  $f$  funktsiyaning boshlang'ich funktsiyasi bo'ladi.

**Ta'rif.** Agar  $F(x)$  funktsiya  $f$  funktsiyaning boshlang'ich funktsiyasi bo'lsa, u holda  $F(x) = F(x)+C$  ko'rinishdagi barcha funktsiyalar to'plami  $f$  funktsiyadan *aniqmas integrali* deyiladi va u  $\int f(x)dx$  kabi belgilanadi.

**Integrallar jadvali**

|   |                                |                         |    |                  |               |
|---|--------------------------------|-------------------------|----|------------------|---------------|
| 1 | $\int \operatorname{tg} x dx$  | $-\ln  \cos x  + C$     | 9  | $\int e^x dx$    | $e^x + C$     |
| 2 | $\int \operatorname{ctg} x dx$ | $\ln  \sin x  + C$      | 10 | $\int \cos x dx$ | $\sin x + C$  |
| 3 | $\int a^x dx$                  | $\frac{a^x}{\ln a} + C$ | 11 | $\int \sin x dx$ | $-\cos x + C$ |

|   |                                      |                                                       |    |                                    |                                         |
|---|--------------------------------------|-------------------------------------------------------|----|------------------------------------|-----------------------------------------|
| 4 | $\int \frac{dx}{a^2 + x^2}$          | $\frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$    | 12 | $\int \frac{1}{\cos^2 x} dx$       | $\operatorname{tg} x + C$               |
| 5 | $\int \frac{dx}{x^2 - a^2}$          | $\frac{1}{2a} \ln \left  \frac{x+a}{x-a} \right  + C$ | 13 | $\int \frac{1}{\sin^2 x} dx$       | $-\operatorname{ctg} x + C$             |
| 6 | $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$ | $\ln \left  x + \sqrt{x^2 \pm a^2} \right  + C$       | 14 | $\int \frac{dx}{\sqrt{a^2 - x^2}}$ | $\operatorname{arcsin} \frac{x}{a} + C$ |
| 7 | $\int x^\alpha dx$                   | $\frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$   | 15 | $\int \frac{dx}{x}$                | $\ln x  + C$                            |

### Integrallash usullari

Aniqmas integral ta'rifidan qo'yidagi integrallash qoidalarini kelib chiqadi:

a)  $\int F'(x)dx = F(x)$ ; b)  $\int c f(x)dx = c \int f(x)dx$ ; c)  $\int (f(x) \pm g(x)) dx = \int f(x)dx \pm \int g(x)dx$ ;

d)  $\int u'(x)v(x)dx = u'(x)v(x) - \int u(x)v'(x)dx$  – bo'laklab integrallash ;

e)  $x=g(t) \Rightarrow \int f(x)dx = \int f(g(t))g'(t)dt$  – almashtirish yordamida integrallash;

### Misollar.

$$\int (2x+1)^{20} dx = \{2x+1=t; \quad dt=2dx;\} = \int t^{20} \cdot \frac{1}{2} dt = \frac{1}{21} t^{21} \cdot \frac{1}{2} + C = \frac{t^{21}}{42} + C = \frac{(2x+1)^{21}}{42} + C$$

$$\int \frac{\sqrt{2-x^2} + \sqrt{2+x^2}}{\sqrt{4-x^4}} dx = \int \frac{\sqrt{2-x^2} + \sqrt{2+x^2}}{\sqrt{2-x^2} \sqrt{2+x^2}} dx = \int \frac{dx}{\sqrt{2+x^2}} + \int \frac{dx}{\sqrt{2-x^2}} = \ln|x + \sqrt{x^2+2}| +$$

$$+ \operatorname{arcsin} \frac{x}{\sqrt{2}} + C.$$

$$\int \frac{\cos x}{\sqrt{\sin^3 x}} dx = \int \sin^{-3/2} x \cos x dx = \{\sin x = t; \quad dt = \cos x dx\} = \int t^{-3/2} dt = -2t^{-1/2} + C =$$

$$= -2 \sin^{-1/2} x + C = -\frac{2}{\sqrt{\sin x}} + C.$$

Bo'laklab integrallash qoidasi yordamida qo'yidagi ko'rinishda integrallar topiladi:

$$\int Q(x) \left\{ \begin{matrix} e^x \\ \sin x \\ \cos x \end{matrix} \right\} dx = \int Q(x) d \left\{ \begin{matrix} e^x \\ -\cos x \\ \sin x \end{matrix} \right\} ; \quad \int f(x) \left\{ \begin{matrix} \ln x \\ \operatorname{arctg} \dots(x) \end{matrix} \right\} dx = \int \left\{ \begin{matrix} \ln x \\ \operatorname{arctg} \dots(x) \end{matrix} \right\} dF(x).$$

Misol :  $\int x^2 \sin x dx = \left\{ \begin{matrix} u = x^2; & dv = \sin x dx; \\ du = 2x dx; & v = -\cos x \end{matrix} \right\} = -x^2 \cos x + \int \cos x \cdot 2x dx =$

$$= \left\{ \begin{matrix} u = x; & dv = \cos x dx; \\ du = dx; & v = \sin x \end{matrix} \right\} = -x^2 \cos x + 2 \left[ x \sin x - \int \sin x dx \right] = -x^2 \cos x + 2x \sin x + 2 \cos x + C.$$

Bo'laklab integrallash qoidasini bir nexha marta takrorlash mumkin.

$$\begin{aligned}
\int x^2 e^{5x} dx &= \left\{ \begin{array}{l} u = x^2; \quad dv = e^{5x} dx; \\ du = 2x dx; \quad v = \frac{e^{5x}}{5}; \end{array} \right\} = \frac{1}{5} e^{5x} x^2 - \int \frac{1}{5} e^{5x} 2x dx = \frac{x^2 e^{5x}}{5} - \frac{2}{5} \int x e^{5x} dx = \\
&= \left\{ \begin{array}{l} u = x; \quad dv = e^{5x} dx; \\ du = dx; \quad v = \frac{1}{5} e^{5x}; \end{array} \right\} = \frac{x^2 e^{5x}}{5} - \frac{2}{5} \left[ \frac{x e^{5x}}{5} - \int \frac{1}{5} e^{5x} dx \right] = \frac{x^2 e^{5x}}{5} - \frac{2x e^{5x}}{25} + \frac{2}{25} \int e^{5x} dx = \\
&= \frac{x^2 e^{5x}}{5} - \frac{2x e^{5x}}{25} + \frac{2 e^{5x}}{125} = \frac{e^{5x}}{5} \left( x^2 - \frac{2x}{5} + \frac{2}{25} \right).
\end{aligned}$$

Trigonometrik funktsiyalarni integrallashda darajani pasaytirish va qo'yidagi formulalardan foydalanish maqsadga muvofiq:

$$\begin{aligned}
\int \cos mx \cos nxdx &= \int \frac{1}{2} [\cos(m+n)x + \cos(m-n)x] dx = \frac{1}{2} \left[ \frac{\sin(m+n)x}{m+n} + \frac{\sin(m-n)x}{m-n} \right] \\
\int \sin mx \cos nxdx &= \int \frac{1}{2} [\sin(m+n)x + \sin(m-n)x] dx = \frac{1}{2} \left[ -\frac{\cos(m+n)x}{m+n} - \frac{\cos(m-n)x}{m-n} \right] \\
\int \sin mx \sin nxdx &= \int \frac{1}{2} [-\cos(m+n)x + \cos(m-n)x] dx = \frac{1}{2} \left[ -\frac{\sin(m+n)x}{m+n} + \frac{\sin(m-n)x}{m-n} \right]
\end{aligned}$$

Misolalar. a)  $\int \sin 7x \sin 2xdx = \frac{1}{2} \int \cos 5xdx - \frac{1}{2} \int \cos 9xdx = \frac{1}{10} \sin 5x - \frac{1}{18} \sin 9x + C.$

b)

$$\begin{aligned}
\int \sin 10x \cos 7x \cos 4xdx &= \int \sin 10x [\cos 7x \cos 4x] dx = \frac{1}{2} \int \sin 10x \cos 11xdx + \frac{1}{2} \int \sin 10x \cos 3xdx = \\
&= \frac{1}{4} \int \sin 21xdx - \frac{1}{4} \int \sin xdx + \frac{1}{4} \int \sin 13xdx + \frac{1}{4} \int \sin 7xdx = -\frac{1}{84} \cos 21x - \frac{1}{4} \cos x - \frac{1}{52} \cos 13x - \\
&- \frac{1}{28} \cos 7x + C.
\end{aligned}$$

c)

$$\begin{aligned}
\int \sin^4 x dx &= \int \left( \frac{1}{2} - \frac{1}{2} \cos 2x \right)^2 dx = \frac{1}{4} \int (1 - \cos 2x)^2 dx = \frac{1}{4} \int (1 - 2 \cos 2x + \cos^2 2x) dx = \\
&= \frac{1}{4} \int dx - \frac{1}{2} \int \cos 2xdx + \frac{1}{4} \int \cos^2 2xdx = \frac{x}{4} - \frac{1}{4} \sin 2x + \frac{1}{4} \int \frac{1}{2} (1 + \cos 4x) dx = \frac{x}{4} - \frac{\sin 2x}{4} + \\
&+ \frac{1}{8} \left[ \int dx + \int \cos 4xdx \right] = \frac{x}{4} - \frac{\sin 2x}{4} + \frac{x}{8} + \frac{\sin 4x}{32} = \frac{1}{4} \left[ \frac{3x}{2} - \sin 2x + \frac{\sin 4x}{8} \right] + C.
\end{aligned}$$

$R(x) = \frac{Q(x)}{P(x)}$  to'g'ri algebraik kasrni  $(P(x) = (x-a)^\alpha \dots (x-b)^\beta (x^2+px+q)^\lambda \dots (x^2+rx+s)^\mu$

integrallash uchun u  $\frac{1}{ax+b}; \frac{Mx+N}{ax^2+bx+c}; \frac{1}{(ax+b)^m}; \frac{Mx+N}{(ax^2+bx+c)^n}$  ko'rinishdagi sodda kasrlarga ajratiladi ( $m \geq 2, n \geq 2$  - natural sonlar va  $b^2 - 4ac < 0$ ):

$$\frac{Q(x)}{P(x)} = \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \dots + \frac{A_\alpha}{(x-a)^\alpha} + \dots + \frac{B_1}{(x-b)} + \frac{B_2}{(x-b)^2} + \dots + \frac{B_\beta}{(x-b)^\beta} + \frac{M_1x + N_1}{x^2 + px + q} +$$

$$+ \frac{M_2x + N_2}{(x^2 + px + q)^2} + \dots + \frac{M_\lambda x + N_\lambda}{(x^2 + px + q)^\lambda} + \dots + \frac{R_1x + S_1}{x^2 + rx + s} + \frac{R_2x + S_2}{(x^2 + rx + s)^2} + \dots + \frac{R_\mu x + S_\mu}{(x^2 + rx + s)^\mu}$$

Bu yerda  $A_i, B_i, M_i, N_i, R_i, S_i$  – sonlar. Ularni izlash uchun aniqmas koeffitsientlar usulini qullash maqsadga muvofiq. Uning mohiyatini misolda ko'rsatib beramiz:

Misol.  $\int \frac{9x^3 - 30x^2 + 28x - 88}{(x^2 - 6x + 8)(x^2 + 4)} dx$

Yechim:  $(x^2 - 6x + 8)(x^2 + 4) = (x - 2)(x - 4)(x^2 + 4)$ , demak

$$\frac{9x^3 - 30x^2 + 28x - 88}{(x - 2)(x - 4)(x^2 + 4)} = \frac{A}{x - 2} + \frac{B}{x - 4} + \frac{Cx + D}{x^2 + 4}$$

Umumiy mahrajga keltiramiz va suratlarni tenglashtiramiz:

$$A(x - 4)(x^2 + 4) + B(x - 2)(x^2 + 4) + (Cx + D)(x^2 - 6x + 8) = 9x^3 - 30x^2 + 28x - 88$$

$$(A + B + C)x^3 + (-4A - 2B - 6C + D)x^2 + (4A + 4B + 8C - 6D)x + (-16A - 8B + 8D) = 9x^3 - 30x^2 + 28x - 88.$$

Demak,

$$\begin{cases} A + B + C = 9 \\ -4A - 2B - 6C + D = -30 \\ 4A + 4B + 8C - 6D = 28 \\ -16A - 8B + 8D = -88 \end{cases} \quad \begin{cases} C = 9 - A - B \\ D = -30 + 4A + 2B + 54 - 6A - 6B \\ 2A + 2B + 4C - 3D = 14 \\ 2A + B - D = 11 \end{cases}$$

$$\begin{cases} C = 9 - A - B \\ D = 24 - 2A - 4B \\ 2A + 2B + 36 - 4A - 4B - 72 + 6A + 12B = 14 \\ 2A + B - 24 + 2A + 4B = 11 \end{cases} \quad \begin{cases} C = 9 - A - B \\ D = 24 - 2A - 4B \\ 4A + 10B = 50 \\ 4A + 5B = 35 \end{cases}$$

$$\begin{cases} C = 9 - A - B \\ D = 24 - 2A - 4B \\ 4A + 10B = 50 \\ 50 - 10B + 5B = 35 \end{cases} \quad \begin{cases} C = 9 - A - B \\ D = 24 - 2A - 4B \\ 4A + 10B = 50 \\ B = 3 \end{cases} \quad \begin{cases} A = 5 \\ B = 3 \\ C = 1 \\ D = 2 \end{cases}$$

Natijada:

$$\int \frac{5}{x-2} dx + \int \frac{3}{x-4} dx + \int \frac{x+2}{x^2+4} dx = 5 \ln|x-2| + 3 \ln|x-4| + \int \frac{x}{x^2+4} dx + \int \frac{2}{x^2+4} dx =$$

$$= 5 \ln|x-2| + 3 \ln|x-4| + \frac{1}{2} \ln(x^2+4) + \arctg \frac{x}{2} + C.$$

formula hosil bo'ladi.

Noto'g'ri ratsional algebraik kasrlarni integrallashdan oldin butun qismi ajratiladi:

Misol:

$$\int \frac{6x^5 - 8x^4 - 25x^3 + 20x^2 - 76x - 7}{3x^3 - 4x^2 - 17x + 6} dx$$

Yechim.

$$\begin{array}{r|l} -\frac{6x^5 - 8x^4 - 25x^3 + 20x^2 - 76x - 7}{6x^5 - 8x^4 - 34x^3 + 12x^2} & \frac{3x^3 - 4x^2 - 17x + 6}{2x^2 + 3} \\ \hline & -\frac{9x^3 + 8x^2 - 76x - 7}{9x^3 - 12x^2 - 51x + 18} \\ & \hline & \frac{20x^2 - 25x - 25}{20x^2 - 25x - 25} \end{array}$$

$$\int \left[ 2x^2 + 3 + \frac{20x^2 - 25x - 25}{3x^3 - 4x^2 - 17x + 6} \right] dx = \int 2x^2 dx + \int 3 dx + 5 \int \frac{4x^2 - 5x - 5}{3x^3 - 4x^2 - 17x + 6} dx = \frac{2}{3}x^3 + 3x +$$

$$+ 5 \int \frac{4x^2 - 5x - 5}{3x^3 - 4x^2 - 17x + 6} dx$$

Maxrajni yoyamiz:  $x = 3$  uning ildizi bo'lgani uchun:

$$\begin{array}{r|l} \frac{3x^3 - 4x^2 - 17x + 6}{3x^3 - 9x^2} & \frac{x - 3}{3x^2 + 5x - 2} \\ \hline & -\frac{5x^2 - 17x}{5x^2 - 15x} \\ & \hline & -\frac{2x + 6}{-2x + 6} \\ & \hline & 0 \end{array}$$

Demak,  $3x^3 - 4x^2 - 17x + 6 = (x - 3)(3x^2 + 5x - 2) = (x - 3)(x + 2)(3x - 1)$ .

$$\frac{4x^2 - 5x - 5}{(x - 3)(x + 2)(3x - 1)} = \frac{A}{x - 3} + \frac{B}{x + 2} + \frac{C}{3x - 1}$$

$$A(x + 2)(3x - 1) + B(x - 3)(3x - 1) + C(x - 3)(x + 2) = 4x^2 - 5x - 5$$

Biz  $x$  o'rniga  $-3, -2, 1/3$  qiymatlarni quyamiz:

$$\begin{cases} 40A = 16 \\ 35B = 21 \\ C = 1 \end{cases} \quad \begin{cases} A = 2/5 \\ B = 3/5 \\ C = 1 \end{cases}$$

Demak.

$$\int \frac{6x^5 - 8x^4 - 25x^3 + 20x^2 - 76x - 7}{3x^3 - 4x^2 - 17x + 6} dx = \frac{2}{3}x^3 + 3x + 3 \int \frac{dx}{x + 2} + 2 \int \frac{dx}{x - 3} + 5 \int \frac{dx}{3x - 1} =$$

$$= \frac{2}{3}x^3 + 3x + 3 \ln|x + 2| + 2 \ln|x - 3| + \frac{5}{3} \ln|3x - 1| + C.$$

Misol.

$$\int \frac{3x^4 + 14x^2 + 7x + 15}{(x + 3)(x^2 + 2)^2} dx = \int \frac{A}{x + 3} dx + \int \frac{Bx + C}{(x^2 + 2)^2} dx + \int \frac{Dx + E}{x^2 + 2} dx$$

$$A(x^2 + 2)^2 + (Bx + C)(x + 3) + (Dx + E)(x + 3)(x^2 + 2) = 3x^4 + 14x^2 + 7x + 15$$

$$Ax^4 + 4Ax^2 + 4A + Bx^2 + 3Bx + Cx + 3C + Dx^4 + 2Dx^2 + 3Dx^3 + 6Dx + Ex^3 + 2Ex + 3Ex^2 + 6E = (D+A)x^4 + (3D+E)x^3 + (A+B+2D+3E+4A)x^2 + (3B+C+6D+2E)x + (2A+3C+6E+4A)$$

$$\begin{cases} D+A=3 \\ 3D+E=0 \\ B+2D+3E+4A=14 \\ 3B+C+6D+2E=7 \\ 3C+6E+4A=15 \end{cases} \quad \begin{cases} D=3-A \\ E=-9+3A \\ B+6-2A-27+9A+4A=14 \\ 3B+C+18-6A-18+6A=7 \\ 3C-54+18A+4A=15 \end{cases}$$

$$\begin{cases} D=3-A \\ E=-9+3A \\ B+11A=35 \\ 3B+C=7 \\ 3C+22A=69 \end{cases} \quad \begin{cases} D=3-A \\ E=-9+3A \\ 11A=35-B \\ C=7-3B \\ 21-9B+70-2B=69 \end{cases} \quad \begin{cases} A=3 \\ B=2 \\ C=1 \\ D=0 \\ E=0 \end{cases}$$

Demak:

$$3 \int \frac{dx}{x+3} + \int \frac{2x+1}{(x^2+2)^2} dx = 3 \int \frac{dx}{x+3} + 2 \int \frac{x}{(x^2+2)^2} dx + \int \frac{dx}{(x^2+2)^2} = 3 \ln|x+3| - \frac{1}{x^2+2} + \frac{x}{4(x^2+2)} + \frac{1}{4\sqrt{2}} \operatorname{arctg} \frac{x}{\sqrt{2}} + C.$$

$\int R(\sin x, \cos x) dx$  – ko'rinishdagi integralda qo'yidagi almashtirish qullaniladi:  $\operatorname{tg} \frac{x}{2} = t$ ,

$$\text{bunda } x = 2 \operatorname{arctg} t, \sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}, dx = \frac{2dt}{1+t^2}$$

Misollar:

$$\begin{aligned} \int \frac{dx}{4 \sin x + 3 \cos x + 5} &= \int \frac{\frac{2dt}{1+t^2}}{4 \frac{2t}{1+t^2} + 3 \frac{1-t^2}{1+t^2} + 5} = 2 \int \frac{dt}{8t + 3 - 3t^2 + 5 + 5t^2} = 2 \int \frac{dt}{2t^2 + 8t + 8} = \\ \text{a) } &= \int \frac{dt}{t^2 + 4t + 4} = \int \frac{dt}{(t+2)^2} = -\frac{1}{t+2} + C = -\frac{1}{\operatorname{tg} \frac{x}{2} + 2} + C. \end{aligned}$$

$$\int \frac{dx}{9+8\cos x + \sin x} = \int \frac{2dt}{(1+t^2) \left[ 9 + \frac{8(1-t^2)}{1+t^2} + \frac{2t}{1+t^2} \right]} = 2 \int \frac{dt}{t^2 + 2t + 17} = 2 \int \frac{dt}{(t+1)^2 + 16} =$$

b)

$$= \frac{1}{2} \operatorname{arctg} \frac{t+1}{4} + C = \frac{1}{2} \operatorname{arctg} \frac{\operatorname{tg} \frac{x}{2} + 1}{4} + C.$$

### Aniq integral.

Ta'rif. Agar  $F$  funksiya  $f$  funksiyaning  $[a, b]$  dagi boshlang'ich funksiyasi bo'lsa, u holda  $F(b) - F(a)$  soni  $f$  funksiya  $a$  dan  $b$  gacha olingan *aniq integrali* deyiladi va bunday belgilanadi:

$$\int_a^b f(x) dx$$

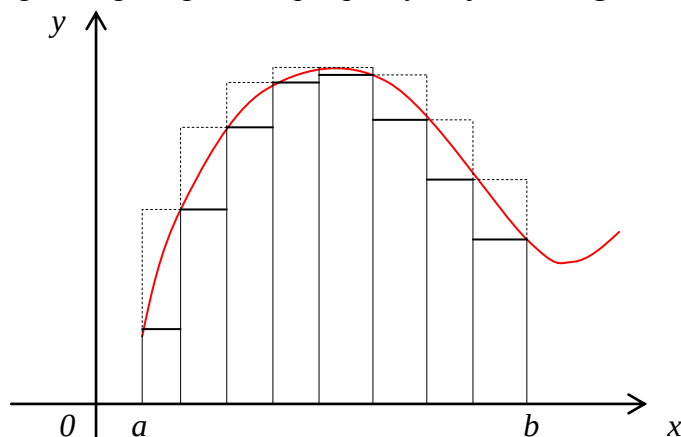
(« $f(x)$  dan  $x$  bo'yicha  $a$  dan  $b$  gacha olingan aniq integral» deb o'qiladi)

Mazkur ta'rifda  $f(x)$  – *integral ostidagi funksiya*,  $[a, b]$  kesma – *integrallash oralig'i*,  $x$  – *integrallash o'zgaruvchisi*,  $a$  va  $b$  sonlar *mos ravishda integrallash qo'yi va yuqori chegarasi* deyiladi.

$$\int_a^b f(x) dx = F(b) - F(a)$$

formula N'yuton-Leybnits formulasi deb yuritiladi (Isaak N'yuton (1643—1727) – ingliz matematigi, Gotfrid Vilgel'm Leybnits (1646—1716) nemis matematigi).

Aniq integralning absolyut qiymati integral ostidagi funksiya grafigi hamda  $x=a$ ,  $x=b$ ,  $y=0$  to'g'ri chiziqlar bilan chegaralangan egri chiziq trapetsiya  $S$  yuzini anglatadi (3-rasm).



**3-rasm.**

Aniq integral qo'yidagi xossalarga ega :

a) Agar  $a=b$  bo'lsa, u holda  $\int_a^b f(x) dx = 0$ ; b)  $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ ;

c)  $\int_a^b c f(x) dx = c \int_a^b f(x) dx$ ; g)  $\int_a^b (f(x) \pm g(x)) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$ ;



Misol.

$$\int_0^1 \sqrt{1-x^2} dx = \left\{ \begin{array}{l} x = \sin t; \\ \alpha = 0; \beta = \pi/2 \end{array} \right\} = \int_0^{\pi/2} \sqrt{1-\sin^2 t} \cos t dt = \int_0^{\pi/2} \cos^2 t dt = \frac{1}{2} \int_0^{\pi/2} (1 + \cos 2t) dt = \\ = \frac{1}{2} \left( t + \frac{1}{2} \sin 2t \right) \Big|_0^{\pi/2} = \frac{\pi}{4} + \frac{1}{4} \sin \pi = \frac{\pi}{4}.$$

Eslatma. Aniq integralni hisoblashda maxsus komp'yuter dasturlaridan foydalanish maqsadga muvofiq (Maple, MathCad, Mathematica, Mathlab).

4. Tayanch tushunchalar:

Sonli funktsiyalar, funktsiya limiti, uzluksiz funktsiyalar, ajoyib limitlar, hosila, hosilaning geometrik ma'nosi, differentsiallanuvchi funktsiya, yuqori tartibli hosila, hosilalar jadvali, differentsiallanuvchi funktsiyalar hossalari, o'suvchi va kamayuvchi funktsiya, monotonlik oralig'i, lokal maksimum va lokal minimum, maksimum va minimum, o'sish va kamayish sharti, lokal ekstremumning zaruriy sharti. Boshlang'ich funktsiya, aniqmas integral, integrallar jadvali, integrallash usullari, aniq integral.

5. Nazorat savollari.

1. Sonli funktsiyalar xaqida tushuncha bering.
2. Funktsiya limiti xaqida tushuncha bering.
3. Uzluksiz funktsiyalar xaqida tushuncha bering.
4. Ajoyib limitlar xaqida tushuncha bering.
5. Funktsiya limitini hisoblashga misollar keltiring.
6. Hosila tushunchasini bayon qiling. Hosilalar jadvalini keltiring.
7. Funktsiyani tekshirishda hosila qanday qullaniladi?
8. Boshlang'ich funktsiya ta'rifini bayon qiling.
9. Aniqmas integral ta'rifini bayon qiling.
10. Integrallar jadvalini keltiring
11. Integrallash usullari keltiring
12. Aniq integral ta'rifini va xossalari bayon qiling.

### 3-MAVZU. EHTIMOLLAR NAZARIYASINING MATEMATIK ASOSLARI. MATEMATIK STATISTIKA ELEMENTLARI.

#### Reja:

1. Ehtimollik haqida tushuncha.
2. Ehtimollarni qo'shish va ko'paytirish.
3. Binomial taqsimot
4. Tanlanma. Tajriba natijalarini statistik ishlanmasi.

#### 3. Mavzu bayoni.

##### Ehtimollik haqida tushuncha.

Tasodifiy hodisa – bu berilgan sharoitda ruy beradigan yoki ro'y bermaydigan hodisa. Tanga tashlanganda raqam tomoni bilan tushishi, lotereya bo'yicha yutuq chiqishi, otilgan o'qning nishonga tegishi tasodifiy hodisalarga misol sifatida qaralishi mumkin. Shu bilan birga amaliyot nuqtai-nazardan alohida olingan hodisalar bilan emas, balki etarlicha ko'p sonli, ommaviy xarakterga ega hodisalarning qonuniyatlarini o'rganish maqsadga muvofiq. Masalan, korhona uchun alohida maxsulot emas, balki tayorlangan maxsulotlardan qanchasi sifatli yoki yaroqsiz ekanini bilish ahamiyatliroq.

Shu kabi masalalarni echish uchun alohida tajriba ya'ni sinash o'tkaziladi va ularning oqibatlari o'rganiladi. Har bir tajriba ma'lum shartlar va sharoitlar asosida bir necha marotaba o'tkazish mumkinligi bilan xarakterlanadi. Bunda bir-birini rad etuvchi va ro'y berish imkoniyatlari bir hil bo'lgan joiz oqibatlar (elementar hodisalar) to'plami alohida o'rin tutadi. Shu to'plamni biz  $\Omega$  orqali belgilaymiz.

1-Misol. o'yin kubchasi bir marta tashlansin. Bunda elementar hodisalar to'plami  $\Omega = \{E_1, E_2, E_3, E_4, E_5, E_6\}$  ko'rinishga ega, bu yerda  $E_i$  – «i – raqamli tomoni bilan tushdi» elementar hodisasi.

2-Misol. Elektr asbobni ishdan chiqmasdan hizmat qilish. Bunda har bir elementar hodisa musbat haqiqiy son bilan ustma-ust tushadi, ya'ni elementar hodisalar to'plami  $\Omega = (0, +\infty)$  ko'rinishga ega.

Amaliyotda elementar hodisalardan tashqari, murakkabroq hodisalar qiziqtiradi. Masalan, o'yin kubchasi bir marta tashlanganda «juft raqamli tomoni bilan tushdi» hodisasi, yoki «elektr asbob 3000 soat mobaynida ishdan chiqmasdan hizmat qildi» kabi hodisa.

$\emptyset$  - hodisa xech qanday elementar hodisalarni o'z ichiga olmaydi (ya'ni, xech qanday xollarda ro'y bermaydi), shuning uchun *ro'y bermasligi aniq* hodisa,  $\Omega$  hodisa esa doimo ro'y berib *mukarrar hodisa* deb yuritiladi.

Ehtimollar nazariyasi tasodifiy hodisalarni ro'y berish qonuniyatlarini o'rganadi. Shuning uchun tasodifiy xodisa ruy berishi imkoniyatlarini kattaligini bildirish uchun qiymatlari  $[0,1]$  segmentda qabul qiladigan maxsus funktsiya – ehtimollik kiritilishi lozim.

Tabiiyki, bunda mukarrar hodisaning ehtimolini 1 ga, ro'y bermasligi aniq xodisaning ehtimolini esa 0 ga teng deb qabul qilishimiz lozim. Bundan tashqari elementar hodisalarning ro'y berish imkoniyatlari bir hil deb hisoblaymiz.

Mashhur matematik Kolmogorov A.N. ehtimol tushunchasini qo'yidagi aksiomalar orqali kiritgan:

Tasodifiy xodisaning ehtimolligi deb qo'yidagi aksiomalarga bo'ysinadigan  $R$  funktsiyaga aytiladi:

- 1)  $0 \leq P(A) \leq 1 \quad \forall A$ ; 2)  $P(\Omega) = 1$ ; 3) Agar  $A_1, A_2, \dots, A_n, \dots$  hodisalar juft-jufti bilan birgalikda ro'y bermasa (ya'ni  $A_i A_j = \emptyset \quad \forall i \neq j$  bo'lsa), u holda  $P(\sum_i A_i) = \sum_i P(A_i)$ .

Berilgan  $A$  va  $B$  hodisalar uchun  $A \cup B$  birlashma  $A$  va  $B$  hodisalardan kamida bittasi ro'y berishidan iborat bo'lgan hodisadir. Unga  $A$  va  $B$  hodisalar *yig'indisi* aytiladi va u ko'pincha  $A + B$  orqali belgilanadi.

Xuddi shunday  $A \cap B$  kesishma  $A$  va  $B$  hodisalardan bir vaqtda ruy berishidan iborat bo'lgan hodisadir. Unga  $A$  va  $B$  hodisalar *ko'paytmasi* aytiladi va u ko'pincha  $AB$  orqali belgilanadi.

Hodisalar yig'indisi va ko'paytmasi tabiiy ravishda cheksiz sondagi  $A_1, A_2, \dots, A_n, \dots$  hodisalar uchun ham kiritilishi mumkin, bunda ular uchun mos ravishda  $\sum_i A_i$  va  $\prod_i A_i$  belgilashlar qabul qilingan.

$A$  hodisaga *qarama-qarshi hodisa* sifatida  $\bar{A} = \Omega/A$  hodisa qaraladi va u  $A$  hodisa ro'y bermasligidan iborat bo'lgan hodisadir.

O'zaro kesishmaydigan hodisalar ya'ni *birgalikda ro'y bermaydigan hodisalar* deyiladi. Shuning uchun o'zaro qarama-qarshi hodisalar birgalikda ro'y bermaydi.

#### Binomial taqsimot.

$n$  ta sinashdan iborat bo'lgan tajriba Bernulli sinash sistemasi deyiladi agar a) ixtiyoriy sinashda  $A$  hodisaning ro'y berishi  $p$  ehtimoli uning boshqa sinashlarda ro'y berish-bermasligiga bog'liq emas;

b) istalgan sinash faqat  $A$  va  $\bar{A}$  o'zaro qarama-qarshi oqibatga ega, bunda  $\bar{A}$  hodisaning ehtimoli  $q=1-p$  ga teng.

$P(n, m)$  orqali  $n$  ta sinashdan iborat bo'lgan tajriba mobaynida  $A$  hodisa  $t$  marta ro'y berishi ehtimolini belgilaymiz.

$A$  hodisa  $C_n^m = \frac{n!}{m!(n-m)!}$  ta har biri  $p^m \cdot q^{n-m}$  ehtimolga ega bo'lgan elementlar

hodisalardan tashkil topgan bo'lib,  $P(n, t)$  ehtimol qo'yidagicha topiladi:

$$P(n, m) = C_n^m p^m q^{n-m} \quad (1)$$

1-Misol. o'g'il bola tug'ilishining ehtimoli 0,515 ga teng. Tavaqqal tanlangan 10 ta chaqaloqdan 6 tasi o'g'ilbola bo'lishining ehtimoli taxminan

$$P(10, 6) = C_{10}^6 (0,515)^6 (0,485)^4 \approx 0,2167 \text{ ga teng.}$$

2-Misol. Mahsulotning nosoz bo'lishining ehtimoli 0,01 ga teng. Tavaqqal tanlangan 100 maxsulotdan 3 ta dan ortiq nosoz maxsulot chiqishining ehtimoli

$$P(A) = C_{100}^0 (0,01)^0 (0,99)^{100} + C_{100}^1 (0,01)^1 (0,99)^{99} + C_{100}^2 (0,01)^2 (0,99)^{98} \approx 0,9816 \text{ ga teng.}$$

#### Tanlanma. Tajriba natijalarini statistik ishlash.

Inson o'z turmushida  $A$  to'plamni tashkil qilgan va xossalari noma'lum bo'lgan ob'ektlar bilan tezda uchratib turadi. Ushbu to'plamni o'rganish maqsadida uning biror chekli  $V$  qism to'plamining hossalari o'rganishga doir tajriba o'tkaziladi va ushbu tajriba natijalariga  $A$  to'plam xaqida biror umumiy xulosaga ega bo'lish masalasi dolzarb hisoblanadi. Mazkur masala matematik statistikaning asosiy masalasi deyiladi.

$A$  to'plam *bosh majmua*,  $V$  qism to'plam esa *tanlanma* deyiladi. Tanlanmadagi elementlar soni uning *hajmi* deb yuritiladi.

Umumiylikka putur etkazmasdan, bosh to'plamning elementlari qandaydir taqsimot funksiyasiga ega bo'lgan tasodifiy miqdorning qiymatlar to'plami deb faraz qilishimiz mumkin.

Ko'pincha tanlanmaning elementlari o'sish tartibida joylashtiriladi va natijada *variatsion qator* deb yuritiluvchi ketma-ketlikka ega bo'lamiz.

Masalan, 0, 5, 3, 6, 3, 4, 1, 3, 4, 6 sonlar hajmi 10 ga teng bo'lgan tanlanmani tashkil qilib, uning variatsion qatori qo'yidagi ko'rinishga ega: 0,1,3,3,3,4,4,5,6,6.

Tanlanmaning elementlari takroran o'chrashishi mumkin.

U holda hajmi  $N$  bo'lgan tanlanma uchun qo'yidagi jadval tuzish maqsadga muvofiq

|       |         |         |         |         |         |         |
|-------|---------|---------|---------|---------|---------|---------|
| $x$   | $x_1$   | $x_2$   | $\dots$ | $x_i$   | $\dots$ | $x_n$   |
| $\nu$ | $\nu_1$ | $\nu_2$ | $\dots$ | $\nu_i$ | $\dots$ | $\nu_n$ |

(1)

Bu yerda  $\nu_i - x_i$  ning absolyut takrorligi.

Agar biz  $W_i = \nu_i / N - x_i$  ning nisbiy takrorligini kiritsak, u holda tanlanma uchun qo'yidagi jadval tuzsa bo'ladi

|     |       |       |         |       |         |       |
|-----|-------|-------|---------|-------|---------|-------|
| $x$ | $x_1$ | $x_2$ | $\dots$ | $x_i$ | $\dots$ | $x_n$ |
| $W$ | $W_1$ | $W_2$ | $\dots$ | $W_i$ | $\dots$ | $W_n$ |

(2)

Ravshanki,  $\nu_1 + \nu_2 + \dots + \nu_n = N$ ,  $W_1 + W_2 + \dots + W_n = 1$ .

(1) va (2) jadvallar tanlanmaning taqsimot qonunlari deb yuritiladi.

1-Misol. 0, 5, 3, 6, 3, 4, 1, 3, 4, 6 tanlanmaning taqsimot qonunlari qo'yidagicha bo'ladi:

|       |     |     |     |     |     |     |
|-------|-----|-----|-----|-----|-----|-----|
| $x$   | 0   | 1   | 3   | 4   | 5   | 6   |
| $\nu$ | 1   | 1   | 3   | 2   | 1   | 2   |
| $W$   | 0,1 | 0,1 | 0,3 | 0,2 | 0,1 | 0,2 |

Bundan buyon biz tanlanma o'zining taqsimot qonuni yordamida berilgan deb faraz qilamiz.

$F^*(x) = \frac{1}{N} \sum_{i: x_i < x} \nu_i$  funktsiya tanlanmaning empirik taqsimot funktsiyasi (ETF) deyiladi.

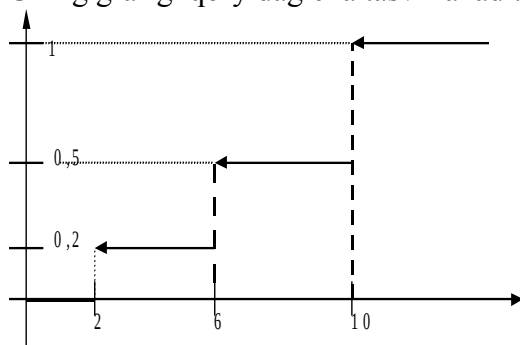
2-Misol. Hajmi 60 bo'lgan

|       |      |      |     |
|-------|------|------|-----|
| $x$   | 2    | 6    | 10  |
| $\nu$ | 12   | 18   | 30  |
| $W$   | 2/10 | 3/10 | 1/6 |

tanlanmaning ETF i qo'yidagi ko'rinishga ega:

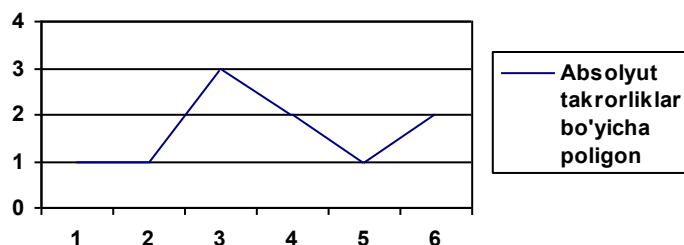
$$F^*(x) = \begin{cases} 0 & x \leq 2 \\ 0,2 & 2 < x \leq 6 \\ 0,5 & 6 < x \leq 10 \\ 1 & 10 < x \end{cases}$$

Uning grafigi qo'yidagicha tasvirlanadi:



Uchlari  $(x_i, v_i)$  yoki  $(x_i, W_i)$  nuqtalarda bo'lgan siniq chiziqlar (*tanlanma poligonlari*) tanlanma xaqida qo'shimcha tushuncha hosil qilishga imkoniyat beradi.

3-Misol. 1-misoldagi tanlanmaning  $v$  bo'yicha poligoni qo'yidagi ko'rinishga ega:



$OX$  o'qini chekli sondagi o'zaro kesishmaydigan  $\Delta_i$ ,  $i=1,2,\dots,k$ , oraliqlarga ajratib, tanlanmaning  $\Delta_i$  ga tegishli elementlar sonini  $t_i$  hisoblaymiz.

$f(x) = t_i / N$ ,  $x \in \Delta_i$ ,  $i=1,2,\dots,k$ , funktsiyaning grafigi tanlanma *gistogrammasi* deyiladi. Ayrim hollarda gistogrammada  $t_i / N$  kattalik o'rniga  $t_i$  olinadi.

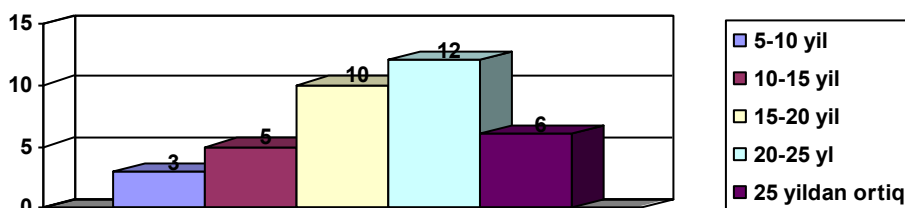
Shuni ta'kidlash lozimki, gistogrammalarni hozirgi kunda keng tarqalgan Microsoft Office dasturlar oilasi yordamida nisbatan tez va sifatli yasash imkoniyati mavjud.

4-Misol. Maktab o'qituvchilari ish staji xaqida ma'lumot

| Ish staji (yillar) | O'qituvchilar soni |
|--------------------|--------------------|
| 5–10               | 3                  |
| 10–15              | 5                  |
| 15–20              | 10                 |
| 20–25              | 12                 |
| 25 da ortiq        | 6                  |
| Jami               | 36                 |

Jadvalda o'z aksini topgan bo'lsa, u holda ShEHM gistogrammani qo'yidagicha ko'rinishda yasaydi.

O'qituvchilarning ish staji bo'yicha gistogrammasi



Q'yidagi kattaliklar tanlanmaning sonli xarakteristikallari sifatida ahamiyatlidir:

$$1) \quad Mx = \frac{1}{N} \sum_{i=1}^n v_i x_i = \sum_{i=1}^n W_i x_i - \text{tanlanma o'rta qiymati};$$

$$2) \quad Dx = \frac{1}{N} \sum_{i=1}^n v_i (x_i - Mx)^2 = \sum_{i=1}^n W_i (x_i - Mx)^2 - \text{tanlanma dispersiyasi};$$

$$3) \quad s = \sqrt{Dx} - \text{o'rta kvadratik chetlanish};$$

5-Misol. 1-misoldagi tanlanma uchun o'rta qiymat, dispersiya va o'rta kvadratik chetlanish topilsin.

Yechish.

$$Mx=0,1 \cdot 0+0,1 \cdot 1+0,3 \cdot 3+0,2 \cdot 4+0,1 \cdot 5+0,2 \cdot 6=3,5;$$

$$Dx=0,1 \cdot (0-3.5)^2 + 0,1 \cdot (1-3.5)^2 + 0,3 \cdot (3-3.5)^2 + 0,2 \cdot (4-3.5)^2 + 0,1 \cdot (5-3.5)^2 + 0,2 \cdot (6-3.5)^2 = 4;$$
$$s = 2.$$

4. Tayanch tushunchalar: Hodisalar, ehtimollik, ehtimollarni qo'shish va ko'paytirish. ro'y bermasligi aniq hodisa, qarama-qarshi hodisa, mukarrar hodisa, hodisalar yig'indisi, hodisalar ko'paytmasi, birgalikda ro'y bermaydigan yoki noo'rindosh hodisalar binomial taqsimot, bosh majmua, tanlanma, variatsion qator, tanlanmaning taqsimot qonunlari, empirik taqsimot funktsiyasi, tanlanma poligonlari va gistogrammasi, tanlanma o'rta qiymati, tanlanma dispersiyasi, o'rta kvadratik chetlanish.

#### 5. Nazorat savollari.

1. Tasodifiy hodisaga tushuncha bering.
2. Ehtimollik haqida tushuncha bering.
3. Ehtimollarni qo'shish va ko'paytirish formulalarini keltiring.
4. Binomial taqsimot haqida tushuncha bering.
5. Bosh majmua va tanlanma xaqida tushuncha bering va misollar keltiring.
6. Variatsion qator deb nimaga aytiladi?
7. Tanlanmaning taqsimot qonunlari va empirik taqsimot funktsiyasiga tushuncha bering.
8. Tanlanma poligonlari va gistogrammasi deb nimaga aytiladi?
9. Tanlanma o'rta qiymati, dispersiyasi va o'rta kvadratik chetlanishni topish qanday formulalar yordamida amalga oshiriladi?

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## ILOVA. INDIVIDUAL VAZIFALAR.

**1-vazifa.  $A^{-1}$  ni toping. Natijani tekshiring.**

$$1) A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 2 & -3 & 3 \end{pmatrix}$$

$$6) A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -5 & 0 \\ 1 & 3 & 1 \end{pmatrix}$$

$$2) A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 3 & -1 & -1 \end{pmatrix}$$

$$7) A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ -1 & 2 & 1 \end{pmatrix}$$

$$3) A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$$

$$8) A = \begin{pmatrix} 1 & -1 & 3 \\ 4 & 3 & 2 \\ 1 & -2 & 5 \end{pmatrix}$$

$$4) A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$$

$$9) A = \begin{pmatrix} 2 & -3 & 5 \\ -1 & 4 & -2 \\ 3 & -1 & 1 \end{pmatrix}$$

$$5) A = \begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}$$

$$10) A = \begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}$$

**2-vazifa. ChTS ni Gauss, teskari matritsa usullari va Kramer qoidasi bilan yeching va javoblarni solishtiring.**

$$1) \begin{cases} x_1 - x_2 + x_3 = 6 \\ x_1 - 2x_2 + x_3 = 9 \\ x_1 - 4x_2 - 2x_3 = 3 \end{cases}$$

$$2) \begin{cases} x_1 + 2x_2 + x_3 = 3 \\ 2x_1 + x_2 + 4x_3 = 13 \\ x_1 + 2x_2 + 3x_3 = 7 \end{cases}$$

$$3) \begin{cases} x_1 + x_2 + x_3 = 5 \\ x_1 + 6x_2 + x_3 = 10 \\ x_1 + x_2 + 6x_3 = 0 \end{cases}$$

$$4) \begin{cases} 3x_1 + x_2 - x_3 = -1 \\ 2x_1 - x_2 + x_3 = 1 \\ x_1 + 2x_2 - 3x_3 = -3 \end{cases}$$

$$4) \begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 11 \end{cases}$$

$$6) \begin{cases} 3x_1 + x_2 - x_3 = -1 \\ 2x_1 - x_2 + x_3 = 11 \\ x_1 + 2x_2 - 3x_3 = -3 \end{cases}$$

$$7) \begin{cases} 2x_1 - x_3 = 1 \\ 2x_1 + 4x_2 - x_3 = 1 \\ -x_1 + 8x_2 + 3x_3 = 2 \end{cases}$$

$$8) \begin{cases} x_1 + x_2 + x_3 = 2 \\ x_1 + 3x_2 + x_3 = 4 \\ x_1 + x_2 + 3x_3 = 0 \end{cases}$$

$$9) \begin{cases} x_1 + 2x_2 + 3x_3 = 2 \\ 2x_1 - x_2 - 2x_3 = 2 \\ 3x_1 + 2x_2 - x_3 = 8 \end{cases}$$

$$10) \begin{cases} 2x_1 + 2x_2 + 3x_3 = -2 \\ x_1 - x_2 = 4 \\ -x_1 + 2x_2 + x_3 = -7 \end{cases}$$

**3-vazifa. ChTS ni Gauss usuli bilan yeching va ikkita hususiy yechmini toping.**

$$\begin{array}{ll}
 1) \begin{cases} x_1 - 3x_2 + x_3 - x_4 = -2 \\ x_1 - 4x_2 + x_3 + x_4 = -3 \\ 2x_1 + x_2 + 3x_3 - 3x_4 = 3 \end{cases} & 6) \begin{cases} x_1 - 4x_2 + 4x_3 + x_4 = -3 \\ x_1 - 7x_2 + 6x_3 - 2x_4 = 2 \\ 3x_1 - 10x_2 + 14x_3 + 4x_4 = 1 \end{cases} \\
 2) \begin{cases} x_1 + x_2 + 3x_3 + 2x_4 = 1 \\ 2x_1 - x_2 + x_3 + x_4 = 4 \\ 5x_1 + x_2 + 5x_3 + 3x_4 = -4 \end{cases} & 7) \begin{cases} 3x_1 - x_2 - x_3 = 1 \\ 2x_1 + 2x_2 + 2x_3 + 3x_4 = 3 \\ x_1 + 3x_2 + 5x_3 + 6x_4 = 9 \end{cases} \\
 3) \begin{cases} x_1 - 2x_2 + x_3 - x_4 = -2 \\ x_1 + 3x_2 + 2x_3 - 2x_4 = 5 \\ -x_1 + 4x_2 + 3x_3 - x_4 = 8 \end{cases} & 8) \begin{cases} x_1 + 3x_2 - 2x_3 + 5x_4 = -2 \\ 3x_1 + 8x_2 - 7x_3 + 10x_4 = 4 \\ -2x_1 + x_2 + 5x_3 - 6x_4 = 2 \end{cases} \\
 4) \begin{cases} x_1 - 2x_2 + x_3 - x_4 = 0 \\ 2x_1 + x_2 - x_3 + 2x_4 = 1 \\ 3x_1 - 2x_2 - x_3 + x_4 = -2 \end{cases} & 9) \begin{cases} x_1 - x_2 + x_3 + x_4 = 0 \\ 2x_1 + x_2 - x_3 - x_4 = 6 \\ 3x_1 + 3x_2 - 4x_3 - 5x_4 = 2 \end{cases} \\
 5) \begin{cases} x_1 + x_2 - 3x_3 - x_4 = -2 \\ x_1 - x_2 + 2x_3 = 1 \\ 2x_1 - x_2 + 3x_3 - 2x_4 = 5 \end{cases} & 10) \begin{cases} x_1 + 2x_2 - x_3 - x_4 = 1 \\ x_1 - x_2 + x_3 - 2x_4 = -1 \\ -x_1 - 2x_2 + 3x_3 + 2x_4 = 5 \end{cases}
 \end{array}$$

**4-vazifa.  $f'(x)$  ni toping.**

$$\begin{array}{ll}
 1) f(x) = 3\sin x - \cos 9x & 6) f(x) = 5\cos x - \sin 5x \\
 2) f(x) = \sin 2x - 2\cos x & 7) f(x) = 2\sin x + \cos 4x \\
 3) f(x) = \cos 5x - 5\sin x & 8) f(x) = \operatorname{tg} 3x - 3\cos x \\
 4) f(x) = 3\cos x + \sin 6x & 9) f(x) = \cos^2 x - \sin 2x \\
 5) f(x) = 3\cos x + \sin 3x & 10) f(x) = \sin^2 x + \cos 2x
 \end{array}$$

**5-vazifa. Berilgan oraliqda funktsiyaning eng katta va eng kichik qiymatlarini toping.**

$$\begin{array}{ll}
 1) f(x) = x^3 - 6x^2 + 9x, [2;4] & 2) f(x) = x^3 - 2x^2 + x - 1, [1/2;2] \\
 3) f(x) = 2x^3 + 9x^2 - 24x + 1, [0;1] & 4) f(x) = x^3 + 3x^2 - 24x - 1, [0;3] \\
 5) f(x) = x^3 - 27x + 13, [0;4] & 6) f(x) = 2x^3 + 9x^2 - 36x, [0;3] \\
 7) f(x) = x^3 + 3x^2 - 24x - 1, [-5;0] & 8) f(x) = x^3 - 6x^2 + 9x, [2;4] \\
 9) f(x) = x^3 + 6x^2 - 36x, [-7;0] & 10) f(x) = x^3 - 27x + 13, [-4;0]
 \end{array}$$

**6-vazifa.  $\int f(x)dx$  ni toping.**

$$\begin{array}{ll}
 1) f(x) = 3\sin x - \cos 9x & 6) f(x) = 5\cos x - \sin 5x \\
 2) f(x) = \sin 2x - 2\cos x & 7) f(x) = 2\sin x + \cos 4x \\
 3) f(x) = \cos 5x - 5\sin x & 8) f(x) = \operatorname{tg} 3x - 3\cos x \\
 4) f(x) = 3\cos x + \sin 6x & 9) f(x) = \cos^2 x - \sin 2x \\
 5) f(x) = 3\cos x + \sin 3x & 10) f(x) = \sin^2 x + \cos 2x
 \end{array}$$



**7-vazifa. Hisoblang.**

- a)  $\int e^{\cos^2 x} \sin 2x dx;$
  - b)  $\int x \arctg x dx;$
1.
  - c)  $\int \frac{dx}{x^3 + 27};$
  - d)  $\int \sin^2 x \cos^3 x dx$
- a)  $\int \frac{x^2 dx}{(x^3 + 4)^6};$
  - b)  $\int e^x \ln(1 + e^x) dx;$
2.
  - c)  $\int \frac{x dx}{x^3 + 8};$
  - d)  $\int \cos^2 x \cdot \sin^3 x dx$
- a)  $\int \frac{x^2 dx}{\sqrt{1 - x^6}};$
  - b)  $\int x 2^x dx;$
- 3
  - c)  $\int \frac{(5x + 6) dx}{x^3 + x^2 + x + 1};$
  - d)  $\int \sin^3 x \cos^3 x dx$
4.
  - a)  $\int \frac{dx}{\sin^2 x (2 \ctg x + 1)};$
  - b)  $\int \frac{x \arccos x}{\sqrt{1 - x^2}} dx;$
  - c)  $\int \frac{dx}{x^3 - x^2 + 2x - 2};$
  - d)  $\int \sin^2 x \cos^2 x dx.$
5. a)  $\int \frac{\sin 2x dx}{5 - \cos 2x};$  b)  $\int x^2 e^{5x} dx;$  c)  $\int \frac{(x + 1) dx}{x^3 - 2x^2 + x};$  d)  $\int \cos^4 x dx.$
6. a)  $\int \frac{\cos x dx}{\sqrt{\sin^3 x}};$  b)  $\int x \arccos \frac{1}{x} dx;$  c)  $\int \frac{(2x + 1) dx}{x^3 + 3x^2 - 4x};$  d)  $\int \sin^4 x dx.$
7. a)  $\int \frac{\arcsin x dx}{\sqrt{1 - x^2}};$  b)  $\int x \ln(x^2 + 1) dx;$  c)  $\int \frac{x dx}{x^4 + 5x^2 + 6};$  d)  $\int \sin^5 x \cos^2 x dx.$
8. a)  $\int \frac{\arctg x}{x^2 + 1} dx;$  b)  $\int x \cos 2x dx;$  c)  $\int \frac{x dx}{x^4 - 81};$  d)  $\int \sin^2 x \cos^5 x dx.$
9. a)  $\int \frac{\cos x dx}{\sqrt[3]{8 + 3 \sin x}};$  b)  $\int x \ln^2 x dx;$  c)  $\int \frac{(x^2 + x - 1) dx}{x^4 + 3x^2 - 4};$  d)  $\int \sin^3 x \cos^4 x dx.$
10. a)  $\int \frac{\sqrt{3 + \ln x}}{x} dx;$  b)  $\int x^2 \sin 3x dx;$  c)  $\int \frac{(x^3 + x) dx}{x^4 + 5x^2 + 6};$  d)  $\int \sin^4 x \cos^3 x dx.$

**8-vazifa. Hisoblang.**

1.  $\int_0^{\frac{\pi}{2}} x \sin x dx;$
2.  $\int_0^1 x \arctg x dx.$
3.  $\int_1^2 \frac{\ln x}{x} dx.$
4.  $\int_0^1 \frac{5x+1}{x^2+2x+1} dx.$
5.  $\int_0^{\pi} \sin 2x \cos^2 x dx.$
6.  $\int_1^2 \sqrt{x} \ln x dx.$
7.  $\int_0^{\pi/2} \frac{dx}{\sin x + \cos x}.$
8.  $\int_0^1 x \ln(1+x) dx.$
9.  $\int_0^1 \frac{dx}{x^2+x+1}.$
10.  $\int_0^{\sqrt{2}/2} \frac{x dx}{\sqrt{1-x^4}}.$

**9-vazifa. Masalani yeching.**

1) Qutida 10 ta shar joylashgan. Ular ichida 6 ta yashil rangli shar bor. Tasodifan olingan 4 ta shar ichida 2 ta yashil shar chiqishining ehtimolini toping.

2) Qutida 9 ta yashil va 4 ta sariq shar joylashgan.

Tasodifan olingan 6 ta shar ichida 3 ta yashil shar chiqishining ehtimolini toping.

3) Qutida 10 ta shar joylashgan. Ular ichida 6 ta yashil rangli shar bor. Tasodifan olingan 5 ta shar ichida 3 ta yashil shar chiqishining ehtimolini toping.

4) Qutida 15 ta shar joylashgan. Ular ichida 10 ta yashil rangli shar bor. Tasodifan olingan 5 ta shar ichida 2 ta yashil shar chiqishining ehtimolini toping.

5) Qutida 25 ta shar joylashgan. Ular ichida 10 ta yashil rangli shar bor. Tasodifan olingan 4 ta shar ichida 2 ta yashil shar chiqishining ehtimolini toping.

6) Qutida 7 ta yashil va 4 ta sariq shar joylashgan. Tasodifan olingan 5 ta shar ichida 3 ta yashil shar chiqishining ehtimolini toping.

7) Qutida 30 ta shar joylashgan. Ular ichida 18 ta yashil rangli shar bor. Tasodifan olingan 7 ta shar ichida 5 ta yashil shar chiqishining ehtimolini toping.

8) Qutida 27 ta shar joylashgan. Ular ichida 7 ta yashil rangli shar bor. Tasodifan olingan 5 ta shar ichida 2 ta yashil shar chiqishining ehtimolini toping.

9) Qutida 12 ta shar joylashgan. Ular ichida 7 ta yashil rangli shar bor. Tasodifan olingan 4 ta shar ichida 2 ta yashil shar chiqishining ehtimolini toping.

10) Qutida 9 ta yashil va 6 ta sariq shar joylashgan. Tasodifan olingan 4 ta shar ichida 2 ta yashil shar chiqishining ehtimolini toping.

**10-vazifa. Taqsimot qonuni yordamida aniqlangan (jadvalga qarang) tanlanmaning empirik taqsimot funktsiyasi, poligoni, tanlanma o'rta qiymati, dispersiyasi va o'rta kvadratik chetlanishi topilsin.**

|    |       |   |     |     |     |
|----|-------|---|-----|-----|-----|
| 1. | $x$   | 1 | 3   | 6   | 7   |
|    | $\nu$ |   | 0,3 | 0,2 | 0,1 |

|    |       |     |   |     |     |
|----|-------|-----|---|-----|-----|
| 2. | $x$   | 1   | 2 | 4   | 7   |
|    | $\nu$ | 0,2 |   | 0,4 | 0,1 |

|    |       |     |   |     |     |
|----|-------|-----|---|-----|-----|
| 3. | $x$   | 1   | 2 | 3   | 4   |
|    | $\nu$ | 0,4 |   | 0,3 | 0,1 |

|    |       |     |     |     |   |
|----|-------|-----|-----|-----|---|
| 4. | $x$   | -4  | -1  | 2   | 3 |
|    | $\nu$ | 0,1 | 0,3 | 0,2 |   |

|    |       |     |     |   |     |
|----|-------|-----|-----|---|-----|
| 5. | $x$   | -2  | 0   | 2 | 4   |
|    | $\nu$ | 0,4 | 0,3 |   | 0,1 |

|    |       |     |    |     |     |
|----|-------|-----|----|-----|-----|
| 6. | $x$   | -3  | -1 | 3   | 5   |
|    | $\nu$ | 0,1 |    | 0,3 | 0,4 |

|    |       |     |     |   |     |
|----|-------|-----|-----|---|-----|
| 7. | $x$   | 2   | 3   | 4 | 5   |
|    | $\nu$ | 0,1 | 0,3 |   | 0,1 |

|    |       |     |   |     |     |
|----|-------|-----|---|-----|-----|
| 8. | $x$   | 1   | 2 | 3   | 4   |
|    | $\nu$ | 0,1 |   | 0,3 | 0,2 |

|    |       |     |     |     |   |
|----|-------|-----|-----|-----|---|
| 9. | $x$   | -1  | 0   | 1   | 2 |
|    | $\nu$ | 0,3 | 0,4 | 0,1 |   |

|     |       |   |     |     |     |
|-----|-------|---|-----|-----|-----|
| 10. | $x$   | 1 | 2   | 3   | 4   |
|     | $\nu$ |   | 0,2 | 0,2 | 0,1 |