O’ZBEKISTON RESPUBLIKASI OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI
RESPUBLIKASI TA’LIMI MARKAZI

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STEREOMETRIK MASALALARNI YECHISH

5130100-matematika bakalavr yo‘nalishi talabalari, akademik litseylar va umumiy o‘rta ta’lim maktablari va uchun o‘quv qo‘llanma

Qayta ishlangan ikkinchi nashr

TOSHKENT

X. Norjigitov, $\boxed{\text{Ch. Mirzayev}}$, O. Dushabayev.
Stereometrik masalalarni yechish. O‘quv qo‘llanma.

Mazkur o‘quv qo‘llanma 5130100-matematika bakalavr yo‘nalishi talabalariga o‘tiladigan tanlov fan “Maktab va akademik litseylarda matematika kursi” hamda, akademik litseylar va umumiy o‘rta ta’lim maktablarida o‘tiladigan “Geometriya” fani dasturi asosida yozilgan bo‘lib u oliy o‘quv yurti talabalari, akademik litsey, va umumiy o‘rta ta’lim maktab o‘quvchilari va oliy o‘quv yurtlariga kiruvchilar hamda o‘qituvchilar uchun mo‘jjallangan. Shu bilan birga undan oliy o‘quv yurtlari talabalari ham foydalanishlari mumkin.

Qo‘llanmada B.Q.Xaydarovning umumiy o‘rta ta’lim maktablari 10-11sinflari uchun va I.Isroilov, Z.Pashshaevning akademik litseylar uchun “Geometriya” darsligining “Stereometriya” qismidagi masalalaridan yechib ko‘rsatilgan.

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O‘quv qo‘llanma Guliston davlat universiteti Kengashining 2018-yil 29-sentyabrdagi 2-sonli yig‘ilish qarori hamda Respublika ta‘lim markazi huzuridagi “Matematika” fani yo‘nalishi bo‘yicha ilmiy metodik kengashning 2019-yil 27-iyuldagi 6-sonli yig‘ilish qarori bilan nashrga tavsiya etilgan.

SO‘Z BOSHI

Mamlakatimizda ta’lim sohasida ulkan o‘zgarishlar amalga oshirilmoqda.

O’zbekiston Respublikasi Prezidenti Sh. M. Mirziyoyevning 2017-yil 8-avgustdagi “O‘zbekiston Respublikasi Xalq ta’limi vazirligi faoliyatini takomillashtirish” to‘g‘risidagi 3180- qaroriga binoan 11 yillik umumiy o‘rta ta’lim joriy etildi. Hozirgi davrda maktab, litsey va kasb-hunar kolleji o‘quvchilari uchun dasturlarga mos bo‘lgan o‘quv qo‘llanma va darsliklari yaratilmoqda. Mazkur o‘quv qo‘llanma hozir darsliklardan B.Q.Xaydarovning 10-11-sinflar uchun yozilgan “Geometriya” darsligi, I.Isroilov, Z.Pashshaevning akademik litseylar uchun “Geometriya” darsligining “Stereometriya” qismilari asosida yozilgan bo‘lib, misol va masalalar yechib ko‘rsatilgan.

Respublikamizda barcha fanlardan maktab, litsey, kollejlar ta’lim tizimiga mos keladigan o‘quv qo‘llanmalari va darsliklar yaratil-moqda. Buning uchun yetakchi olimlar va mutaxassislar jalb qilinmoqda. Bu quvonarli hol, albatta. Yozilayotgan qo‘llanma va darsliklaming sifati davr talabiga qanchalik mosligini o‘qitish jarayoni ko‘rsatadi. Hozirgi vaqtda respublikamizda qabul qilingan “Ta’lim to‘g‘risida”gi qonun va “Kadrlar tayyorlashning milliy dasturi” hamda Davlat ta’lim standartlari talablariga mos keladigan o‘quv qo‘llanmalar va darsliklar tayyorlash maqsadga muvofiqdir.

Matematikaning o‘quvchiga qiziqarli, ko‘rgazmali, shu bilan birga o‘zlashtirilishi murakkab bo‘lgan qismi geometriyadir. Geometriya maktab dasturida ikki qismdan iborat.

1) Planimetriya — o‘quvchi uchun zavqli, qiziqarli, uning tajribasiga yaqin hamda tushunarli.

2) Stereometriya — o‘quvchida geometrik tasavvurni rivojlan-tiradi. Maktabda geometriyani o‘qitish tajribasi shuni ko‘rsatayaptiki, stereometriya qismi o‘quvchi oldida yechilishi lozim bo‘lgan qator muammolar keltirib chiqaradi. Bu muammolar qatoriga planimetriya kursida o‘quvchini fazoviy tasavvur qilishga tayyorlash vazifasi yetarlicha bajarilishini, nazariy va amaliy qismlari orasidagi bog‘lanish bayonini rivojlantirishdan iborat.

O‘qituvchiIarimizning bilimi va metodik saviyasini oshirish maqsadida stereometriya qismidagi amaliy mashqlarni to‘la ishlab chiqish hamda undan unumli foydalanish maqsadga muvofiqdir.

Geometriyani o‘rganishda chizmaning roli muhim. Teoremalarni isbotlashda chizma(rasm)lardan foydalaniladi.

Maktablarimizda hozirgi vaqtda stereometriya kursidagi ko‘pgina masalalami yechish (garchi ular juda foydali bo‘lsada) o‘quvchilarga og‘irlik qiladi. Geometriya o‘quv dasturi talab qilayotgan geometrik materialning mazmuni, ya’ni an’anaviy planimetriya va stereomeriya masalalarini hamda uning yangi bo‘limlari (koordinatalar sistemasi, vektorlar, almashtirishlar kabi) aksiomalar sistemasi asosida yaratilgan.

Qo‘llanma sakkiz bobdan iborat bo‘lib, ularda stereometrik aksiomalar va ulardan kelib chiqadigan natijalar, to‘g‘ri chiziq va tekisliklarning parallelligiga, perpendikularligiga doir mashqlar, fazoda dekart koordinatalari va vektorlarga doir mashqlar hamda stereometriyaning boshqa mavzulari yoritilgan.

Mazkur o‘quv qo‘llanma ikkinchi nashr bo‘lib, birinchi nashrdagi elementar xatolar to‘g‘rilangan va bitta bob qo‘shilgan.

Masalalar yechishdan asosiy maqsad:

1. Masala nazariyani yaxshi tushunish uchun yechiladi.
   Demak, masala yechishdan maqsad nazariyani egallash bilan birga uni amalda qo‘llash hamdir.
2. Nazariyani amalda qo‘llashda uning qaysi jihatlariga e’tibor berilgan yoki e’tibor kam berilganligi masala yechishdagina bilinadi. Demak, nazariyaning qay darajada qo‘llanilayotganligi masala yechish orqali ko‘rinadi.
3. Masala yechish o‘quvchini ijodiy va mustaqil ishlashga o‘rgatadi. Natijada o‘quvchining ijodiy qobiliyatini rivojlantirish uchun kerakli omillarni topish imkoni bo‘ladi.

Keltirilgan bu fikrlarimiz kelajakda geometriyadan o‘quv darsliklari va qo‘llanmalami yaratishda foydali bo‘ladi degan umiddamiz.

O‘quv qo‘llanma haqida fikr-mulohaza bildirgan hamkasblarga oldindan minnatdorchilik bildiramiz.

mualliflar

STEREOMETRIYADAGI ASOSIY TUSHUNCHALAR

1. STEREOMETRIYA AKSIOMALARI VA ULARDAN KELIB CHIQADIGAN NATIJALAR

$\boldsymbol{S_1}$. Agar uchta nuqta bir toʻgʻri chiziqda yotmasa, u holda ular orqali yagona tekislik oʻtkazish mumkin.

$\boldsymbol{S_2}$. Agar ikki tekislik umumiy nuqtaga ega boʻlsa, u holda bu tekisliklar shu nuqtadan oʻtuvchi umumiy toʻgʻri chiziqqa ham ega boʻladi.

$\boldsymbol{S_3}$. Agar toʻgʻri chiziqning ikkita nuqtasi bir tekislikda yotsa, u holda uning barcha nuqtalari shu tekislikda yotadi.

$\mathbf{P_1}$. Tekislikda qanday toʻgʻri chiziq olmaylik, unda bu toʻgʻri chiziqqa tegishli va tegishli boʻlmagan nuqtalar mavjud.

$\mathbf{P_2}$. Ikkita turli nuqtalardan bitta va faqat bitta toʻgʻri chiziq oʻtkazish mumkin.

$\mathbf{P_3}$. Tekislikda toʻgʻri chiziqdan tashqaridagi nuqtadan shu toʻgʻri chiziqqa parallel toʻgʻri chiziq oʻtkazish mumkin va faqat bitta.

$\mathbf{P_4}$. Bir toʻgʻri chiziqda olingan istalgan uchta nuqtaning faqat bittasi qolgan ikkitasining orasida yotadi.

$\mathbf{P_5}$. Har bir toʻgʻri chiziq tekislikni ikki boʻlakka: ikkita yarimtekislikka ajratadi.

$\mathbf{P_6}$. Har qanday kesma noldan farqli tayin uzunlikka ega boʻlib, u musbat son bilan ifodalanadi. Kesma uzunligi uning ixtiyoriy nuqtasi ajratgan boʻlaklari uzunliklari yigʻindisiga teng.

$\mathbf{P_7}$. Har qanday burchak tayin gradus oʻlchoviga ega boʻlib, uning qiymati musbat son bilan ifodalanadi. Yoyiq burchakning gradus oʻlchovi $180°$ ga teng. Burchakning gradus oʻlchovi burchak tomonlari orasidan oʻtuvchi ixtiyoriy nur ajratgan burchaklar gradus oʻlchovlarining yigʻindisiga teng.

$\mathbf{P_8}$. Ixtiyoriy nurga uning uchidan boshlab, berilgan kesmaga teng yagona kesmani qoʻyish mumkin.

$\mathbf{P_9}$. Ixtiyoriy nurga tayin yarimtekislikka berilgan, yoyiq boʻlmagan burchakka teng yagona burchakni qoʻyish mumkin.

$\mathbf{P_{10}}$. Har qanday uchburchak uchun unga teng uchburchak mavjud va uni nurdan tayin yarimtekislikka yagona tarzda qo‘yish mumkin.

Teoremalar

1.1. To‘g‘ri chiziq va unda yotmaydigan nuqta orqali bitta va faqat bitta tekislik o‘tkazish mumkin.

1.2. Berilgan kesishuvchi ikkita to‘g‘ri chiziq orqali yagona tekislik o‘tkazish mumkin.

2. TO‘G‘RI CHIZIQ VA TEKISLIKLARNING PARALLELLIGI

Teoremalar

2.1. Bir to‘g‘ri chiziqqa parallel ikki to‘g‘ri chiziq o‘zaro paralleldir.

2.2. Agar tekislikda yotmagan to‘g‘ri chiziq, shu tekislikdagi biror to‘g‘ri chiziqqa parallel bo‘lsa, bu to‘g‘ri chiziq tekislikning o‘ziga ham paralleldir.

2.3. Agar bir tekislikning kesishuvchi ikki to‘g‘ri chizig‘i ikkinchi tekislikdagi ikki to‘g‘ri chiziqqa mos holda parallel bo‘lsa, bu tekisliklar paralleldir.

2.4. Tekislikdan tashqaridagi nuqta orqali berilgan tekislikka parallel qilib bitta va faqat bitta tekislik o‘tkazish mumkin.

2.5. Ikki parallel tekislikni uchinchi tekislik bilan kesishishidan hosil bo‘lgan to‘g‘ri chiziqlar paralleldir.

3. TO‘G‘RI CHIZIQLAR VA TEKISLIKLARNING PERPENDIKULARLIGI

Teoremalar

3.1. Agar to‘g‘ri chiziq tekislikdagi kesishuvchi ikkita to‘g‘ri chiziqqa perpendikular bo‘lsa, bu to‘g‘ri chiziq tekislikka ham perpendikular bo‘ladi.

3.2. Agar tekislik ikkita parallel to‘g‘ri chiziqdan biriga perpendikular bo‘lsa, u holda ikkinchisiga ham perpendikulardir.

3.3. Bitta tekislikka perpendikular ikkita to‘g‘ri chiziq o‘zaro paralleldir.

3.4. Tekislikda og‘maning asosidan uning proyeksiyasiga perpendikular qilib o‘tkazilgan to‘g‘ri chiziq og‘maning o‘ziga ham perpendikular bo‘ladi va aksincha, tekislikdagi to‘g‘ri chiziq og‘maga perpendikular bo‘lsa, u og‘maning proyeksiyasiga ham perpendikular bo‘ladi.

3.5. Agar tekislik boshqa bir tekislikka perpendikular to‘g‘ri chiziq orqali o‘tsa, bu tekisliklar perpendikulardir.

4. FAZODAGI DEKART KOORDINATALAR SISTEMASI VA VEKTORLAR

Formulalar

4.1. $A_1(x_1, y_1, z_1)$ va $A_2(x_2, y_2, z_2)$ nuqtalar orasidagi masofa
$|A_1A_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2)}$.

4.2. $C(x, y, z)$ nuqta A_1 va A_2 nuqtalar o‘rtasida bo‘lsa,

$$x = \frac{x_1 - x_2}{2}, \qquad y = \frac{y_1 - y_2}{2}, \qquad z = \frac{z_1 - z_2}{2}.$$

4.3. A nuqtadan α tekislikka tushirilgan perpendikulyar $|AO|$ bo‘lsin. AO to‘g‘ri chiziqning davomida $|AO|$ ga teng $|A'O|$ kesma olamiz. U holda A' nuqta α tekislikka nisbatan A ga simmetrik nuqta deyiladi.

4.4. Ko‘pburchakning tekislikdagi ortogonal proyeksiyasining yuzi, ko‘pburchak yuzini uning tekislik bilan proyeksiyasi tekisligi orasidagi burchak kosinusi ko‘paytmasiga teng:

$$S_{pr} = S \cdot cos\varphi$$

5. KO‘PYOQLAR

Teoremalar

5.1. Muntazam piramidaning a) yon yoqlari; b) yon qirralari; c) apofemalari o‘zaro teng.

5.2. Muntazam piramidaning yon sirti uning asosining yarim perimetri va apofemasining ko‘paytmasiga teng.

6. AYLANISH JISMLARI

Teoremalar

6.1. Silindr asosi tekisligiga parallel tekislik, uning yon sirtini asos aylanasiga teng aylana bo‘yicha kesadi.

6.2. Konusning yon sirti uning asosi yuzining yarmi va yasovchisining ko‘paytmasiga teng.

7. KO‘PYOQLARNING HAJMLARI

Teoremalar

7.1. O‘xshash bo‘lgan ikkita jism hajmlarining nisbati ularning mos chiziqli o‘lchovlari kublarining nisbatiga teng.

UMUMIY METODIK KO‘RSATMALAR

Planimetriyani yaxshi o‘zlashtirgan o‘quvchilar ham stereometriya qismida biroz qiynalishadi. Bunga sabab stereometriya kursi nazariyasiga juda kam o‘rin berilganligidir.

Shuning uchun biz ushbu qo‘llanmada stereometriyaga doir berilgan masala va misollarni yechish yo‘llariga ko‘rsatmalar berdik. Stereometriya kursi asosan uchta maqsadda o‘rganiladi:

1) mantiqiy fikrlashni kengaytirish;

2) fazoviy tushunchalami rivojlantirish;

O‘quvchilarni stereometriya kursi bilan tanishtirishdan avval, hech bo‘lmasa bir soat quyidagilarni takrorlab o‘tish maqsadga muvofiqdir.

1. Stereometriya — fazodagi shakllarni o‘rganadi. Fazodagi asosiy tushunchaiar nuqta, to‘g‘ri chiziq va tekislikdir.

a) nuqtalar — lotin alfavitining katta harflari bilan belgilanadi:

$A,\ B,\ C, \dots$.

b) to‘g‘ri chiziqlar — lotin alfavitining kichik harflari bilan belgilanadi:

$a,\ b,\ c, \dots$ yoki $AB,\ BC,\ ED, \dots$.

d) tekisliklar — grek alfavitining ayrim harflari bilan belgilanadi:

$\alpha,\ \beta, \dots, \gamma, \dots$.

2. Stereometriya kursida — “yotadi” (tegishli), “yotmaydi” (tegishli emas), “kesishadi”, “kesishmaydi”, “. . . ko‘ra” so‘zlari ko‘p uchraydi. Shuning uchun quyidagi belgilashlarni kiritamiz.

$\in$ — tegishli,

$\notin$ — tegishli emas,

$\cap$ — kesishadi (kesishmasi),

... $\cap$ ... = $\emptyset$ — kesishmaydi,

$\Rightarrow$ — «... ko‘ra», (...kelib chiqadi).

1-misol. A nuqta α tekislikda yotadi, ammo β tekislikda yotmaydi:

$A \in \alpha$, $A \notin \beta$.

2- misol. A nuqta a va b to‘g‘ri chiziqning kesishish nuqtasi.

$A = a \cap b$.

3. Har bir masala yoki misolni yechish asosan uchta qismdan iborat.

A) masalaning berilishi (Ber.);

B) berilishiga ko‘ra chizmasi;

D) asosiy qism — yechish.

I. STEREOMETRIYA AKSIOMALARI VA ULARDAN KELIB CHIQADIGAN NATIJALAR

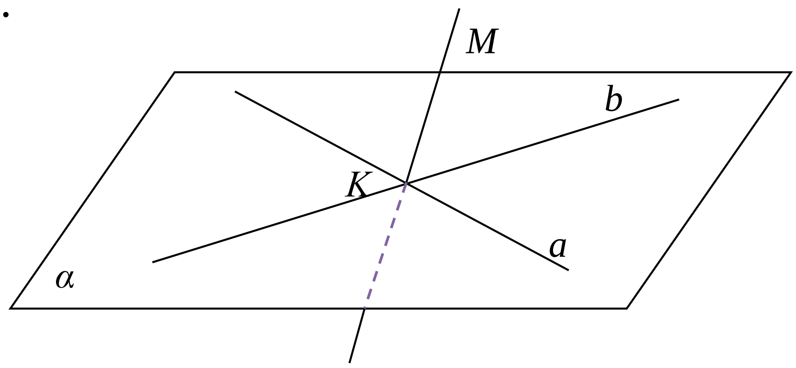
1- masala. A, B, C va D nuqtalar bir tekislikda yotmaydi. AB va CD to‘g‘ri chiziqlarning kesishmasligini isbotlang.

Berilgan: $A, B, C, D \notin \alpha$.

Ko‘rsatish kerak: $AB \cap CD = \emptyset$.

Yechish: Teskarisini, faraz qilaylik, ya’ni AB va CD ayqash bo‘lmasin. U holda AB va CD ning joylashuviga ko‘ra ikkita hol bo‘lishi mumkin, ya’ni ular yoki parallel, yoki kesishuvchi bo‘lishi mumkin. Bu ikki holda ham AB va CD lar bitta tekislikda yotadi.

Demak, $AB \in \alpha$ va $CD \in \alpha$, bundan $A, B, C, D \in \alpha$. Bu esa masala shartiga zid. Demak, farazimiz noto‘g‘ri, shunday qilib $AB \cap CD = \emptyset$.



1-rasm

2- masala. Berilgan ikki to‘g‘ri chiziqning kesishish nuqtasidan bu to‘g‘ri chiziqlar bilan bir tekislikda yotmaydigan to‘g‘ri chiziq o‘tqazish mumkinmi? Javobingizni tushuntiring.

Berilgan: a, b, $a \cap b = K$, $A \in a$, $B \in b$, $a, b \in \alpha$.

Ko‘rsatish kerak: $K \in c$, $c \notin \alpha$.

Yechish: Mumkin, haqiqatdan, $\mathbf{S_3}$ aksiomaga ko‘ra, $A \in a$, $B \in b$, $a \cap b = K$, $a \in \alpha$, $b \in \alpha$. $\mathbf{P_1}$ aksiomaga ko‘ra $M \notin \alpha$ mavjud. $\mathbf{P_2}$ aksiomaga ko‘ra, K va M nuqtalardan faqat bitta to‘g‘ri chiziq o‘tkazish mumkin.

Bu biz izlayotgan c, ya’ni $c = KM$ to‘g‘ri chiziqdir (1- rasm).

3- masala. A, B, C nuqtalar ikki turli tekisliklarning har birida yotadi. Bu nuqtalarning bir to‘g‘ri chiziqda yotishini isbotlang.

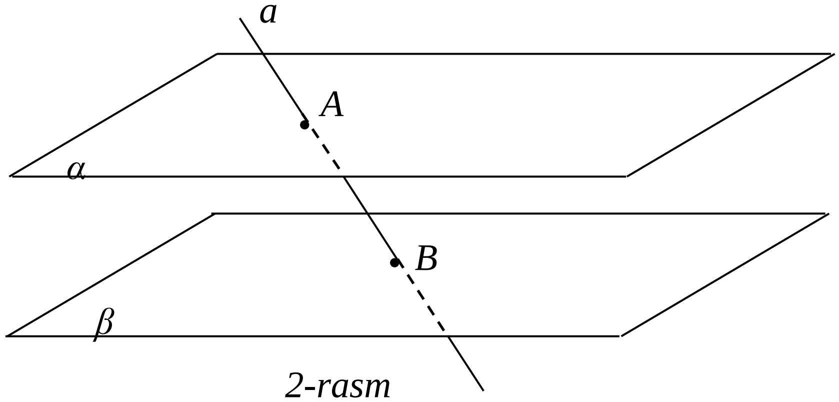
Berilgan: $A, B, C \in \alpha$, $A, B, C \in \beta$.

Ko‘rsatish kerak: $A, B, C \in \alpha$.

Yechish: $A, B, C \in \alpha$ va $A, B, C \in \beta \Rightarrow A, B, C \in \alpha \cap \beta$.

$\mathbf{S_2} \Rightarrow \alpha \cap \beta = a$, demak $A, B, C \in \alpha$.

4- masala. Kesishmaydigan ikkita tekislik berilgan (2- rasm). Bu tekisliklardan birini kesuvchi to‘g‘ri chiziq ikkinchisini ham kesishishini isbotlang.



2-rasm

Berilgan: α, β, a; $\alpha \cap \beta = \emptyset$; $\alpha \cap a = A$.

Ko‘rsatish kerak: $\beta \cap a = B$.

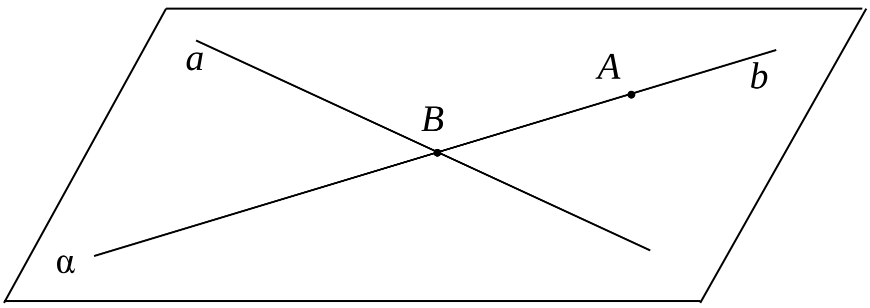
Yechish: $\alpha \cap \beta = \emptyset \Rightarrow \alpha \parallel \beta$.

Agar $\alpha \cap a = A$ va $\alpha \parallel \beta \Rightarrow \beta \cap a = B$ (2- rasm).

5- masala. Berilgan to‘g‘ri chiziqni kesib o‘tadigan va to‘g‘ri chiziqdan tashqaridagi nuqtadan o‘tadigan hamma to‘g‘ri chiziqlarning bitta tekislikda yotishini isbotlang.

Berilgan: a, A, α; $A \notin a$, $a \in \alpha$; $a \cap b = B$ (3- rasm).

Ko‘rsatish kerak: $B \in b \Rightarrow b \in \alpha$.



3-rasm

Yechish: $A \notin a$ va **1.1-** teoremadan $\Rightarrow$ $A \notin \alpha$, $a \in \alpha$, α — yagona. $a \cap b = B$, $a \in \alpha$, $B \in a \Rightarrow B \in \alpha$.

Demak, A va B dan o‘tuvchi b to‘g‘ri chiziqning ikkita nuqtasi α tekislikda yotadi. Shuning uchun, **1.2-** teoremaga ko‘ra, $b \in \alpha$.

6- masala. Agar AB va CD to‘g‘ri chiziqlar bir tekislikda yotma-sa, AC va BD to‘g‘ri chiziqlar ham shu tekislikda yotmasligini isbotlang.

Berilgan: A, B, C, D nuqtalar bir tekislikda yotmaydi. Buni qulayroq qilib quyidagicha belgilashni taklif etamiz.

$AB,\ CD \notin \alpha$

Ko'rsatish kerak: $AC,\ BD \notin \alpha$.

Yechish: Teskarisini faraz qilaylik, ya'ni $AC,\ BD \in \alpha$. U holda $A,\ B,\ C,\ D \in \alpha \Rightarrow$**1.2-** teoremaga ko'ra, $AB \in \alpha,\ CD \in \alpha$ yoki $AB,\ CD \in \alpha$. Bu hol esa masala shartiga zid. Demak, farazimiz noto'g'ri. Shuning uchun, $AC,\ BD \notin \alpha$.

7- masala. Bir tekislikda yotmaydigan to'rtta nuqta berilgan. Bu nuqtalarning uchtasidan o'tuvchi nechta turli tekislik o'tkazish mumkin? Javobingizni tushuntiring.

Berilgan: $A, B, C, D \in \alpha$.

Ko'rsatish kerak: Ixtiyoriy 3 tasi orqali nechta tekislik o'tkazish mumkin.

Yechish: $A, B, C, D \notin \alpha \Rightarrow$ ixtiyoriy uchtasi bir to'g'ri chiziqda yotmaydi, aks holda A, B, C va D nuqtalar bir tekislikda yotadi. $\mathbf{S_3}$ aksiomaga ko'ra $(A,\ B,\ C)$, $(A,\ B,\ D)$, $(B,\ C,\ D)$, $(A,\ C,\ D)$ uchlik nuqtalar orqali har xil tekislik o'tkazish mumkin.

Demak, to'rtta nuqtaning ixtiyoriy 3 tasi orqali 4 ta har xil tekislik o'tkazish mumkin ekan.

8- masala. Bir to'g'ri chiziqda yotgan uchta nuqtadan ikkita turli tekislik o'tkazish mumkinmi?

Berilgan: $A, B, C \in a$.

Ko'rsatish kerak: $A, B, C \in \alpha_1$, va $A, B, C \in \alpha_2$ mumkinmi?

Yechish: Mumkin, haqiqatan, $A,\ B,\ C \in \alpha, \beta$ bo'lsin. U holda $\mathbf{S_2}$ aksiomaga ko'ra $\alpha \cap \beta = a$. Demak, haqiqatan, $A, B, C \in \alpha, \beta$. $\mathbf{P_1}$ aksiomaga ko'ra, shunday $M \in a$ mavjud. 1.1- teoremaga ko'ra, M nuqta va a to'g'ri chiziq orqali α_1 tekislik o'tkazish mumkin. $\mathbf{P_1}$ aksiomaga ko'ra, shunday, $N \notin \alpha_1$ mavjud. 1.1- teoremaga ko'ra, N nuqta va a to'g'ri chiziq orqali α_2 tekislik o'tkazish mumkin.

9- masala. To'rtta nuqta berilgan. Bu nuqtalarning istalgan ikkitasidan o'tuvchi to'g'ri chiziqning qolgan ikkita nuqtadan o'tuvchi to'g'ri chiziq bilan kesishmasligi ma'lum. Berilgan to'rtta nuqtaning bir tekislikda yotmasligini isbotlang.

Berilgan: $A, B, C, D;\ AB \cap CD = \emptyset,\ AC \cap BD = \emptyset,$ $AD \cap BC = \emptyset$.

Ko‘rsatish kerak: $A, B, C, D \notin \alpha$.

Yechish: Teskarisini faraz qilaylik, ya’ni $A, B, C, D \in \alpha$. U holda A, B, C, D nuqtalarni shunday tanlash mumkinki, AB va CD, AC va BD to‘g‘ri chiziqlar parallel bo‘lishi mumkin. U holda A, B, C, D nuqtalar $ABCD$ parallelogrammning uchlari bo‘ladi. Bundan ko‘rinadiki, AD va BC to‘g‘ri chiziqlar parallelogrammning diagonallari bo‘ladi. Planimetriya kursidan ma’lumki, parallelogramm diagonallari bir nuqtada kesishadi, ya’ni $AD \cap BC = K$. Bu esa masala shartiga zid. Shunday qilib,

$A, B, C, D \notin \alpha$.

MUSTAQIL ISHLASH UCHUN

1. A, B, C, D nuqtalar bitta tekislikda yotmaydi. K va M nuqtalar mos ravishda AB va BC kesmalarning o‘rtalari.
1) AC va DK, 2) BD va KM, 3) AD va KM to‘g‘ri chiziqlar kesishmasligini isbotlang.

2. Fazodagi birorta shakl quyidagi xossaga ega: «Shaklning istalgan to'rtta nuqtasi bitta tekislikda yotadi». Shu figuraning to‘liqligicha tekislikda yotishini isbotlang.

II. TO‘G‘RI CH IZIQ VA TEKISLIKLARNING PARALLELLIGI

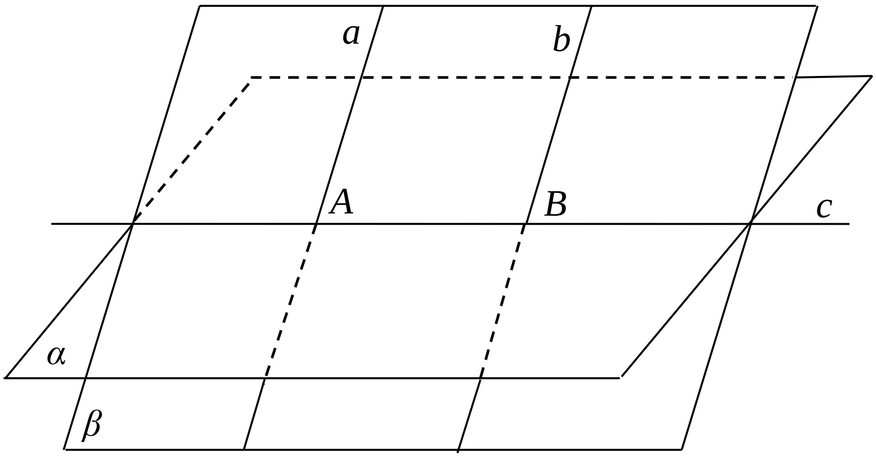
1- masala. Agar tekislik ikkita parallel to‘g‘ri chiziqdan biri kesib o‘tsa, ikkinchisini ham kesib o‘tishini isbotlang.

Berilgan: $a, b, \alpha, a \| b, \alpha \| a = A$.

Ko‘rsatish kerak: $\alpha \cap b = B$.

Yechish: $a \| b$, bundan $a, b \in \beta$, $\beta \neq \alpha$. $\mathbf{S_2}$ aksiomaga ko‘ra, $\alpha \cap \beta = c$, $\alpha \cap a = A \Rightarrow A \in c$. Shunday qilib, masala planimetrik masalaga keltirildi, ya’ni a va b bitta β tekislikda yotadi va c to‘g‘ri chiziq a to‘g‘ri chiziq bilan kesishadi, planimetriyadagi aksiomalarga ko‘ra a va b parallel bo‘lgani uchun, c to‘g‘ri chiziq b to‘g‘ri chiziq bilan ham biror B nuqtada kesishadi. $b \cap c = B$. $c = \alpha \cap \beta \Rightarrow c \in \alpha \Rightarrow B \in \alpha$.

Demak, $b \cap \alpha = B$ (4- rasm).

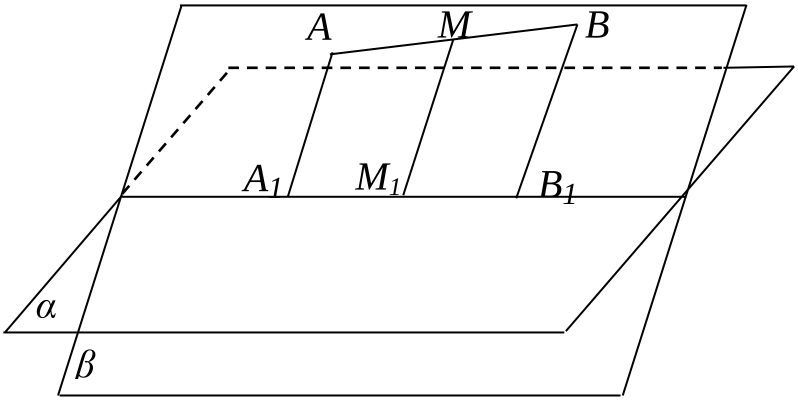


4-rasm

2-masala. AB kesmaning uchlaridan va M o‘rtasidan biror tekislikni A_1, B_1, M_1 nuqtalarda kesuvchi parallel to‘g‘ri chiziqlar o‘tkazilgan (5- rasm). Agar AB kesma tekislikni kesib o‘tmasa va $AA_1 = a$, $BB_1 = b$ bo‘lsa, MM_1 kesmaning uzunligi topilsin.

Berilgan: $AB, \alpha, M \in AB,\ MA = MB,\ AB \cap \alpha = \emptyset, \quad AA_1 = a$, $BB_1 = b$.

Topish kerak(t.k.): $MM_1 -$?

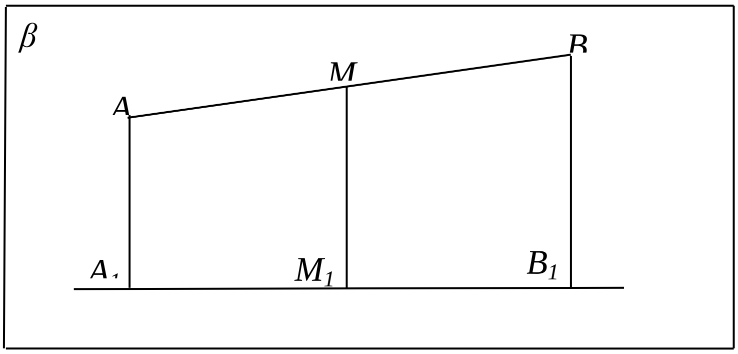


5-rasm

Yechish: $AA_1 \| BB_1 \| MM_1$ ekanligidan $AA_1, BB_1, MM_1 \in \beta$. $A_1, B_1, M_1 \in \alpha, \beta$ tekislikdagi kesimini qarasak yetarli.

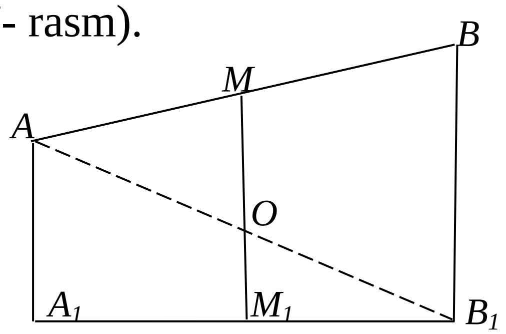
Fales teoremasiga ko‘ra, M_1 nuqta A_1B_1 kesmaning o‘rtasi.
U holda MM_1 kesma ABB_1A_1 trapetsiyaning o‘rta chizig‘i boladi
(6- rasm).

Bu masalani yechishning ikkinchi usulini ham o‘quvchilarga ko‘rsatish foydalidir.



6-rasm

A va B_1 nuqtalarni tutashtirib, ikkita uchburchak hosil qilamiz. MO va M_1O kesmalar mos ravishda ABB_1 va AB_1A_1 uchburchak-larning o‘rta chizig‘i (7- rasm).



7-rasm

Demak, $MO = \frac{BB_1}{2} = \frac{b}{2}$; $M_1O = \frac{AA_1}{2} = \frac{a}{2}$;

$$MM_1 = MO + M_1O = \frac{b}{2} + \frac{a}{2} = \frac{b+a}{2}$$

(o‘quvchilardan masalani yechishning boshqa usulini topishni talab qilish mumkin).

3- masala. Bundan oldingi masalani AB kesma tekislikni kesib o‘tgan hol uchun yeching.

Berilgan: $AB,\ \alpha,\ M \in AB, |MA| \Rightarrow |MB|, AB \cap \alpha = K$
$AA_1 = a, BB_1 = b.$

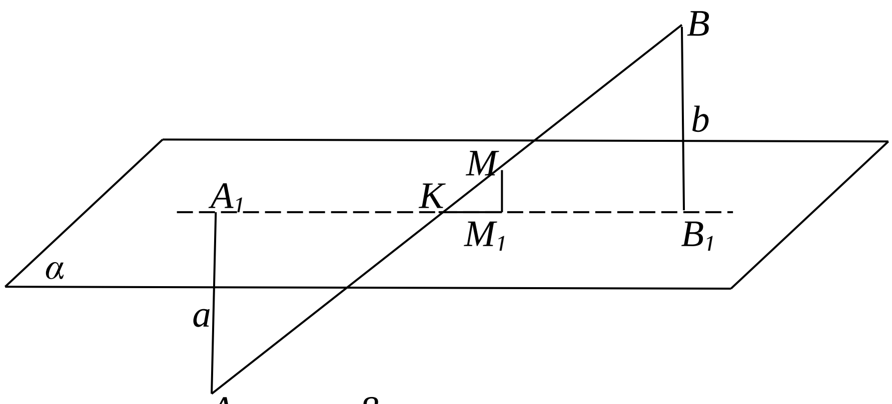
Topish kerak: $MM_1 -$?

Yechish: Faraz qilaylik, $a < b$. 2- masaladagidek mulohaza yuritib, stereometrik masaladan planimetrik masalaga kelamiz (8-rasm).

$\Delta AA_1K \sim \Delta MM_1K$ (3- alomatga ko‘ra) $\frac{AA_1}{MM_1} = \frac{AK}{KM}$.

$\Delta BB_1K \sim \Delta MM_1K$ (3- alomatga ko‘ra), $\frac{BB_1}{MM_1} = \frac{BK}{KM}$,

$\frac{BB_1}{MM_1} = \frac{BM+MK}{KM}$ yoki $\frac{BB_1}{MM_1} = \frac{BM}{KM} + 1$



8-rasm

(1) dan $\frac{AA_1}{MM_1} = \frac{AM-MK}{KM} = \frac{AM}{KM} - 1$ yoki $\frac{AM}{KM} = \frac{AA_1}{MM_1} + 1$

(2) va $MA = MB$ ni hisobga olib, $\frac{BB_1}{MM_1} = \frac{AA_1}{MM_1} + 2$, $\frac{b}{MM_1} = \frac{a}{MM_1} + 2$ yoki $\frac{b-a}{MM_1} + 2$, bundan $MM_1 = \frac{b-a}{2}$. Xuddi shuningdek, $b < a$ deb faraz qilib, MM_1 ni topamiz.

Shunday qilib, $MM_1 = \frac{|b-a|}{2}$.

4- masala. AB kesmaning A uchidan tekislik o‘tkazilgan. Shu kesmaning B uchidan va C nuqtasidan tekislikni B_1 va C_1 nuqta-larda kesuvchi parallel to‘g‘ri chiziqlar o‘tkazilgan. Agar $AC = a$, $BC = b$ va $CC_1 = c$ bo‘lsa, BB_1 kesmaning uzunligi topilsin.

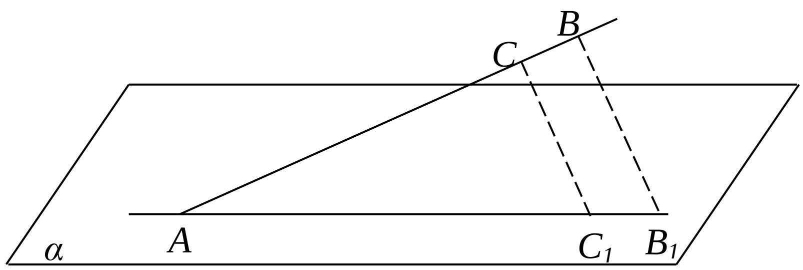
Berilgan: $AB,\ \alpha,\ A \in \alpha$, $C \in AB$, $BB_1 \| CC_1$, $AC = a$, $BC = b$, $CC_1 = c$ (10- rasm)

Topish kerak: $BB_1 - ?$

Yechish: $BB_1 \| CC_1$, b’olgani uchun ular bir tekislik (β) da yotadi. AB kesmaning ikkita (B va C) nuqtasi β tekislikda yotgani uchun A nuqta ham β tekislikda yotadi. A va B_1 nuq-talarni tutashtirib ikkita uchburchak hosil qilamiz (9- rasm).

$\Delta ACC_1 \sim \Delta ABB_1$ (3- alomatga коʻrа), $\frac{BB_1}{CC_1} = \frac{AC+CB}{AC}$.

masala shartini hisobga olib, $\frac{BB_1}{c} = \frac{a+b}{a}$, bundan $BB_1 = \frac{c(a+b)}{a}$.



9-rasm

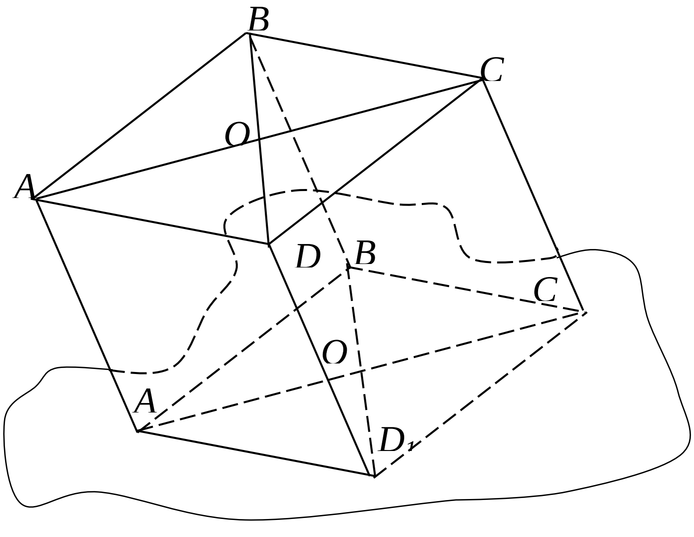
5- masala. $ABCD$ parallelogramm va uni kesmaydigan tekislik berilgan (10- rasm). Parallelogrammning uchlaridan berilgan tekislikni A_1, B_1, C_1, D_1 nuqtalarda kesib o‘tuvchi to‘g‘ri chiziqlar o‘tkazilgan. Agar $AA_1 = a$, $BB_1 = b$ va $CC_1 = c$ bo‘lsa, DD_1 kesma uzunligi topilsin.

Berilgan: $ABCD$ - parallelogramm, α - tekislik.
$ABCD \cap \alpha = \emptyset$, $AA_1 \| BB_1 \| CC_1 \| DD_1$, $AA_1 = a$, $BB_1 = b$, $CC_1 = c$.

Topish kerak: $DD_1 -?$

Yechish: AC va BD diagonallarni o‘tkazamiz. Ular O nuqtada kesishsin. Figuraning parallel kesmalari chizma tekisligida parallel kesmalar bilan tasvirlangani uchun $A_1B_1C_1D_1$ figura ham parallelogrammdir. Agar O nuqtadan AA_1 ga parallel to‘g‘ri chiziq o‘tkazsak, Fales teoremasiga ko‘ra u O_1 nuqtadan o‘tadi.

Endi ACC_1A_1 figurani qaraylik. Bu trapetsiyadir, chunki $AA_1 \| CC_1$. Parallelogramm diagonallari kesishish nuqtasida teng ikkiga bolingani uchun OO_1 kesma ACC_1A_1 trapetsiyaning o‘rta chizig‘idir. Demak, $OO_1 = \frac{AA_1 + CC_1}{2} = \frac{a+c}{2}$. (1)



10-rasm

Endi BDD_1B_1 figurani qaraymiz. Bu ham trapetsiya. ($DD_1 \| BB_1$). OO_1 kesma BDD_1B_1 trapesiyaning o‘rta chizig‘i.

Demak, (l) ni hisobga olib

$$OO_1 = \frac{BB_1 + DD_1}{2} = \frac{b + DD_1}{2}, \quad DD_1 = a + c - b.$$

6- masala. a va b to‘g‘ri chiziqlar bitta tekislikda yotmaydi. a, b to‘g‘ri chiziqlarga parallel c to‘g‘ri chiziq o‘tkazish mumkinmi?

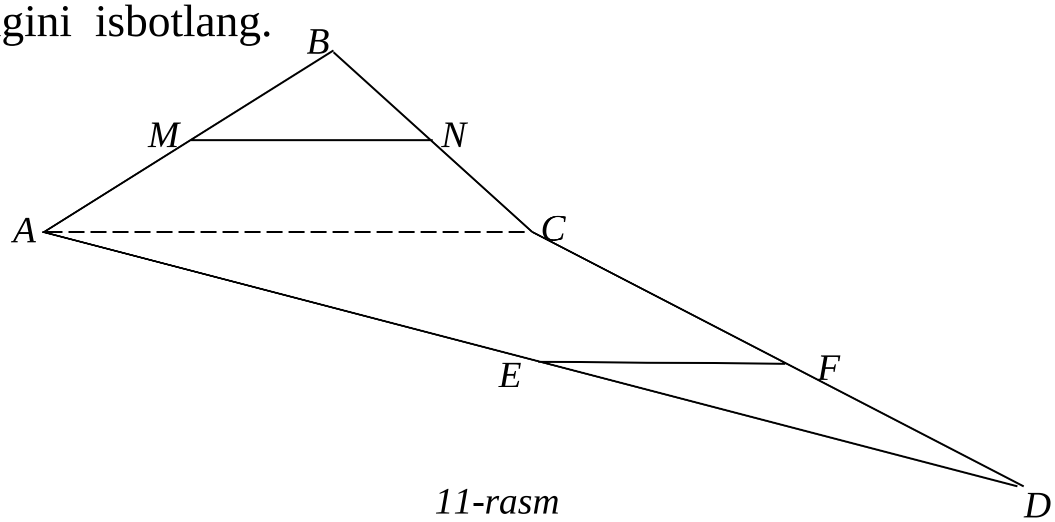
Berilgan: $a, b \notin \alpha$, a, b, α.

Ko‘rsatish kerak: $a \| c$, $b \| c$ mumkinmi?

Yechish: Faraz qilaylik, mumkin bo‘lsin. $a \| c$, $b \| c \Rightarrow$ **2.3-**

teoremaga $\Rightarrow$ $a \| b \Rightarrow a, b \in \alpha$. Bu masala shartiga zid. Demak, c to‘g‘ri chiziqni o‘tkazish mumkin emas.

7- masala. A, B, C, D nuqtalar bir tekislikda yotmaydi. AB va BC kesmalarning o‘rtalaridan o‘tuvchi to‘g‘ri chiziq AD va CD kesmalarning o‘rtalaridan o‘tuvchi to‘g‘ri chiziqqa parallel ekanligini isbotlang.



11-rasm

Berilgan: $A, B, C, D \notin \alpha$,

M nuqta — AB ning o‘rtasi bo‘lsin.

N nuqta — BC ning o‘rtasi bo‘lsin.

E nuqta — AD ning o‘rtasi bo‘lsin.

F nuqta — CD ning o‘rtasi bo‘lsin (11- rasm).

Ko‘rsatish kerak: $MN \| EF$.

Yechish: A va C nuqtalarini tutashtiramiz. U holda MN kesma ABC uchburchakning o‘rta chizig‘i bo‘ladi. Demak, $MN \| EF$, xuddi shuningdek, EF kesma ACD uchburchakning o‘rta chizig‘idir. Demak, $EF \| AC$ 2.3-teoremaga ko‘ra, $EF \| MN$.

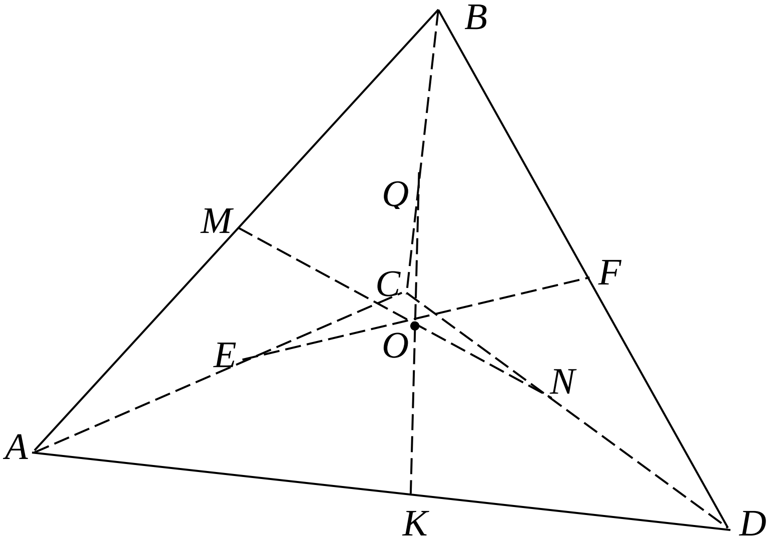
8- masala. Bitta tekislikda yotmagan to‘rtta A, B, C, D nuqtalar berilgan. AB va CD, AC va BD, AD va BC kesmalarning o‘rtalarini tutashtiruvchi to‘g‘ri chiziqlarning bitta nuqtada kesishishini isbotlang(12- rasm).

Berilgan: A, B, C, D, α; A, B, C, D, α; M, N, E, F, Q va K nuqtalar mos ravishda AB, CD, AC, BD, AD va BC kesmalarning o‘rtalari.

Ko‘rsatish kerak: $MN \cap EF \cap QK = O$

Yechish: 7- misolga ko‘ra, $MNEF$ — parallelogramm.
Demak, $MN \cap EF = O$ va kesishish nuqtasi O da teng ikkiga bo‘linadi. Xuddi shuningdek, $MQNK$ ham parallelogramm.

Demak, $QK \cap MN = O$ va kesishish nuqtasida teng ikkiga bo‘linadi. Demak, $MN \cap EF \cap QK = O$.



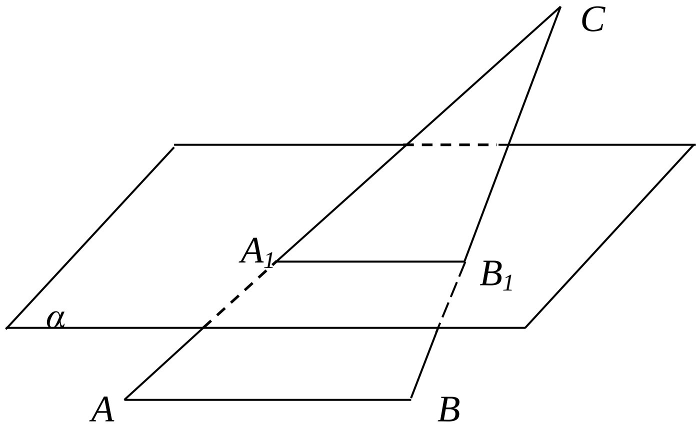
12-rasm

9-masala. ABC uchburchak berilgan. AB to‘g‘ri chiziqqa parallel tekislik bu uchburchakning AC tomonini A_1 nuqtada, BC tomonini B_1 nuqtada kesib o‘tadi. Agar $AA_1 = a$, $AB = b$, $A_1C = c$ bolsa, A_1B_1 kesmaning uzunligi topilsin (13- rasm).

Berilgan: ABC — uchburchak, α — tekislik. $AB \| \alpha$, $B_1 = BC \cap \alpha$, $A_1 = AC \cap \alpha$. $AA_1 = a$, $AB = b$, $A_1C = c$

Topish kerak: $A_1B_1 -?$

Yechish: $AB \| \alpha$ bo‘lgani uchun $AB \| A_1B_1$. Demak,



13- rasm

$\Delta ABC \sim \Delta A_1B_1C_1 \Rightarrow \frac{A_1B_1}{AB} = \frac{A_1C}{AC}$, $\frac{A_1B_1}{AB} = \frac{A_1C}{AA_1 + A_1C}$, bundan

$$A_1B_1 = \frac{AB \cdot A_1C}{A_1C + AA_1} = \frac{b \cdot c}{c + a}.$$

10- masala. Berilgan nuqtadan berilgan ikkita kesishuvchi tekislikning har biriga parallel to‘g‘ri chiziq o‘tkazing.

Berilgan: α, β, A, $A \notin \alpha$, $A \notin \beta$. $\alpha \cap \beta = a$, $A \in b$.

O‘tkazish kerak: $b \| \alpha$, $b \| \beta$.

Yechish: $\alpha \cap \beta = a \Rightarrow A \notin a$.

2.1- teoremaga $\Rightarrow$ shunday to‘g‘ri chiziq mavjudki, $b||a$. $a \in \alpha, \beta$, $\Rightarrow$ **2.2-** teoremaga ko‘ra, $b||\alpha$, $b||\beta$.

MUSTAQIL ISHLASH UCHUN

1. A, B, C, D nuqtalar bir tekislikda yotmaydi, K, M, P nuqtalar AB, BC, CD kesmalarning o‘rtalari bo‘lsa, KMP tekislik AC va BD to‘g‘ri chiziqlarga parallel ekanligini isbotlang.

2. Ikkita parallel tekislik berilgan. P nuqta bu ikki tekislikning o‘rtasida yotmaydigan, berilgan ikki tekislikdan tashqaridagi nuqta bo‘lsin. P nuqta yaqinroq bo‘lgan tekislikni A_1 va A_2 nuqtalarda, uzoqroq bo‘lgan tekislikni B_1 va B_2 nuqtalarda kesib o‘tadi. Agar $A_1B_2 = 10$ sm, $PA_1 : A_1B_1 = 2:3$ kabi nisbatda bo‘lsa, B_1B_2 kesma uzunligi topilsin.

3. A, B, C, D fazoviy nuqtalar. AB, BC, CD, DA kesmalarda mos ravishda A_1, B_1, C_1, D_1 nuqtalar olingan va ular, $AA_1 : A_1B = 1:2$, $BB_1 : B_1C = 2:1$, $CC_1 : C_1D = 3:1$, $DD_1 : D_1A = 1:3$ kabi nisbatda bo‘lsa, A_1, B_1, C_1, D_1 nuqtalar bir tekislikda yotishini isbotlang.

4. Aylana va uning diametrining parallel proyeksiyasi berilgan. Aylananing berilgan diametriga perpendikular diametrning proyeksiyasi qanday yasaladi?

5. AB va CD to‘g‘ri chiziqlar kesishadi. AC va BD to‘g‘ri chiziqlar ayqash bo‘lishi mumkinmi?

6. Agar a to‘g‘ri chiziq bo‘yicha kesishadigan ikki tekislik α tekislikni parallel to‘g‘ri chiziqlar bo‘yicha kesib o‘tsa, u holda a to‘g‘ri chiziqning α tekislikka parallel bo‘lishini isbotlang.

7. Agar to‘g‘ri chiziq ikkita parallel tekislikdan birini kesib o‘tsa, ikkinchisini ham kesib o‘tishini isbotlang.

8. Fazoda berilgan nuqtadan ikkita ayqash to‘g‘ri chiziqning har birini kesib o‘tadigan to‘g‘ri chiziq o‘tkazing. Buni hamma vaqt ham bajarish mumkinmi?

9. Uchlari ikkita ayqash to‘g‘ri chiziqda yotgan to‘g‘ri chiziqlar o‘rtalarining geometrik o‘rni tekislik bo‘lishini isbotlang

10. Berilgan nuqtadan ikkita kesishuvchi to‘g‘ri chiziqning har biriga parallel bo‘lgan tekislik o‘tkazing. Buni har doim ham bajarish mumkinmi?

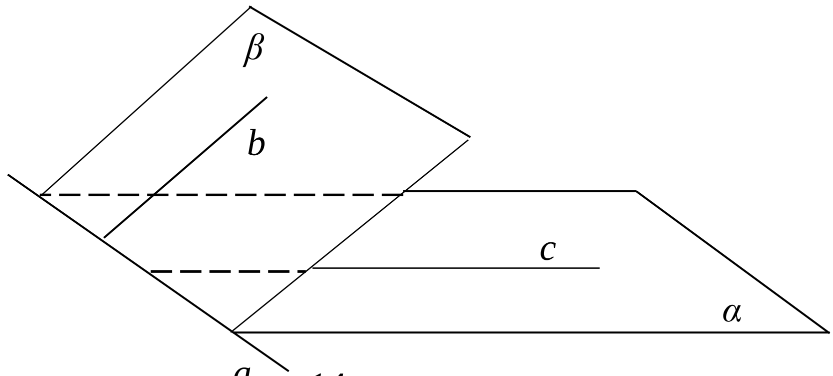
11. $ABCD$ va ABC_1D_1 parallelogrammlar turli tekisliklarda yotadi. CDD_1C_1 parallelogramm ekanligini isbotlang.

12. Ikkita parallel tekislikning birida yotuvchi $ABCD$ parallelogrammning uchlaridan ikkinchi tekislikni A_1, B_1, C_1, D_1 nuqtalarda kesib o‘tuvchi parallel to‘g‘ri chiziqlar o‘tkazilgan. $A_1B_1D_1C_1$ to‘rtburchak parallelogramm ekanini isbotlang.

III. TO‘G‘RI CHIZIQ VA TEKISLIKLARNING PERPENDIKULARLIGI

1- masala. Fazodagi istalgan to‘g‘ri chiziq orqali unga perpendikular ikkita turli to‘g‘ri chiziqlar o‘tkazish mumkinligini isbotlang.

Berilgan: a — to 'g‘ri chiziq, $b \neq c$ (14- rasm).



14- rasm

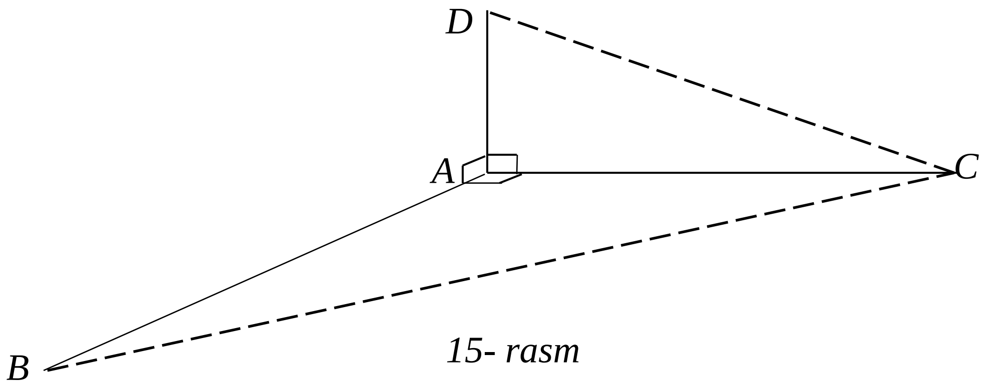
Isbot qilish kerak: $b \perp a$, $c \perp a$ bo‘lishi mumkinmi?

Yechish: a to‘g‘ri chiziq orqali ikkita turli tekislik o‘tkazish mumkin. Bu tekisliklarda berilgan to‘g‘ri chiziqqa perpendikular b va c to‘g‘ri chiziqlar o‘tkazish mumkin.

2- masala. AB, AC, AD to‘g‘ri chiziqlar juft-jufti bilan perpendikular. Agar $AB = a$, $BC = b$, $AD = d$ bol’sa, CD kesma topilsin (15- rasm).

Berilgan: $AB \perp AC$, $AB \perp AD$, $AC \perp AD$, Agar $AB = a$, $BC = b$ va $AD = d$.

Topish kerak: CD—?



15- rasm

Yechish: ACD uchburchak to‘g‘ri burchakli, Pifagor teoremasiga ko‘ra:

$$CD = \sqrt{AD^2 + AC^2} = \sqrt{d^2 + AC^2}. \qquad (1)$$

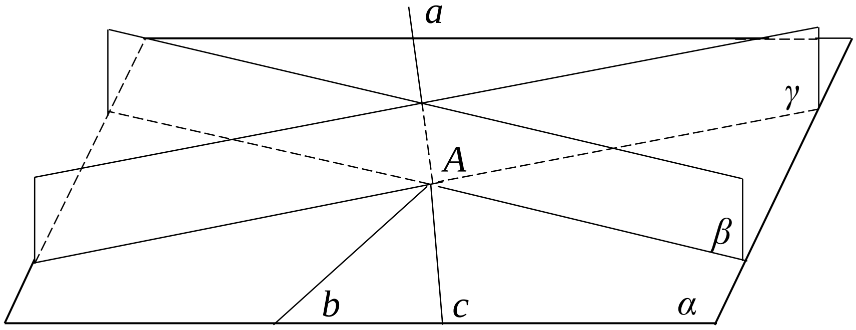
Demak, CD ni topish uchun AC ni topish kerak. ABC uchburchak to‘g‘ri burchakli: $AC = \sqrt{BC^2 - AB^2} = \sqrt{a^2 + b^2}$.

Demak, (1) ga ko‘ra, $CD = \sqrt{d^2 + a^2 - b^2}$.

3- masala. Tekislikning istalgan nuqtasidan unga perpendikular to‘g‘ri chiziq o‘tkazish mumkinligini isbotlang (16- rasm).

Berilgan: α — tekislik, $A \in \alpha$.

Ko‘rsatish kerak: A nuqta orqali α ga perpendikular a to‘g‘ri chiziq o‘tkazish mumkin.



16-rasm

Yechish: b, c — to‘g‘ri chiziqlar α tekislikda yotadi, $b \in \alpha$, $c \in \alpha$, $b \cap c = A$ bo‘lsin. A nuqtadan o‘tuvchi va b to‘g‘ri chiziqqa perpendikular β tekislik, xuddi shuningdek, A nuqtadan o‘tuvchi va c to‘g‘ri chiziqqa perpendikular γ tekislik o‘tkazish mumkin, ya’ni $A \in \beta$, $\beta \perp b$ va $A \in \gamma$, $\gamma \perp C$, β va γ — turli xil, $\beta \neq \gamma$. $\beta \cap \gamma = a$ bo‘lsin. $A \in \beta$ va $A \in \gamma \Rightarrow A \in \beta \cap \gamma = a$, $A \in \alpha$, $\beta \perp b \Rightarrow b \perp a$, $\gamma \perp c \Rightarrow c \perp a$. Demak, **3.2-** teoremaga ko‘ra $a \perp \alpha$.

4- masala. Berilgan tekislikda yotmagan nuqtadan tekislikka perpendikular bo‘lgan bittadan ortiq to‘g‘ri chiziq o‘tkazib bo‘lmasligini isbotlang.

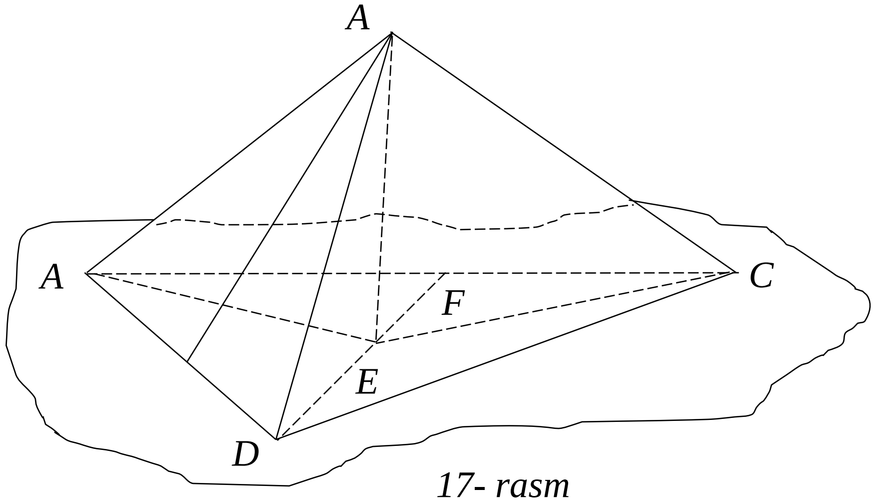
Berilgan: C — nuqta, α — tekislik, $C \notin \alpha$.

Ko‘rsatish kerak: $C \notin a$, $C \notin b$, $a \perp \alpha$, $b \perp \alpha$ mumkin emas.

Yechish: Faraz qilaylik, bitta nuqtadan α tekislikka ikkita perpendikular o‘tkazish mumkin bo‘lsin. $a \cap \alpha = A$ va $b \cap \alpha = B$ bo‘lsin. U holda ABC uchburchak hosil bo‘Iadi. Bu uchburchakning ikkita $\angle A$ va $\angle B$ burchaklari 90°, bunday bo‘lishi mumkin emas.

Demak, nuqtadan berilgan tekislikka bittadan ortiq perpendikular o‘tkazish mumkin emas.

5- masala. A nuqta tomoni a ga teng bo‘lgan teng tomonli uchburchak uchlaridan a masofada yotadi. A nuqtadan uchburchak tekisligigacha masofa topilsin (17- rasm).



17- rasm

Berilgan: $A \notin \alpha$, $\Delta BCD \in \alpha$, $BC = BD = CD = a$, $AE \perp \alpha$.

Topish kerak: AE–?

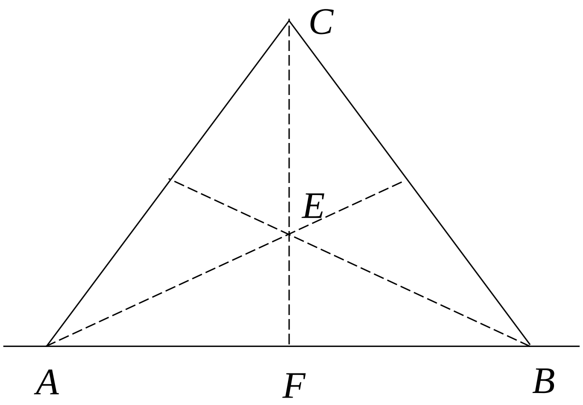
Yechish: $AB = AC = AD \Rightarrow$ uning proyeksiyalari ham teng, ya’ni $BE = DE = CE$. BAE — to‘g‘ri burchakli chburchak.

Pifagor teoremasiga ko‘ra,

$$AE = \sqrt{AB^2 - BE^2} = \sqrt{d^2 - BE^2}. \qquad (1)$$

Demak, AE ni topish uchun BE ni topish yetarli. Stereometrik masala planimetrik masalaga keltirildi. BCD uchburchakni olamiz (18- rasm).

E nuqta ACD uchburchak medianalarining kesishish nuqtasi, shuning uchun, $DE = \frac{2}{3}DF$.



18-rasm

ΔBDF dan: $BF = FC = \frac{a}{2}$. $DF = \sqrt{BD^2 - BF^2} = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}a}{2}$

Shunday qilib, $DE = BE = \frac{2}{3} \cdot \frac{\sqrt{3}a}{2} = \frac{a}{\sqrt{3}}$, buni (1) ga qo‘yib, hosil qilamiz: $AE = \sqrt{a^2 - \frac{a^2}{3}} = a\sqrt{\frac{2}{3}}$.

6- masala. a) Tekislikning hamma nuqtalaridan unga parallel tekislikkacha bo‘lgan masofalar bir xil ekanini isbotlang.

Berilgan: $\alpha \| \beta$, $\rho_{\alpha\beta}$ - tekisliklar orasidagi masofa.

Ko‘rsatish kerak: $\rho_{\alpha\beta}$ - const.

Yechish: $A, B \in \alpha$ bo‘lsin. U holda $AB \in \alpha$, $\alpha \| \beta \Rightarrow AB \| \beta$ bo‘ladi. A va B nuqtalar ixtiyoriyligidan $\rho_{\alpha\beta}$ — const.

b) Berilgan nuqtadan tekislikka tushirilib, berilgan uzunlikdagi og‘malar asoslarining geometrik o‘rni topilsin.

Berilgan: $A \notin \alpha$, $X \in \alpha$, AX — og‘ma $= const = a$.

Ko‘rsatish kerak: X nuqtalar to‘plamini?

Yechish: $O \in \alpha$, $OA \perp \alpha$ bo‘lsin. U holda $OX = \sqrt{AX^2 - AO^2} = \sqrt{a^2 - AO^2}$. X nuqta α tekislikning ixtiyoriy nuqtasi bo‘lgani bilan, OX masofa o‘zgarmasdan qolar ekan. Bundan, nuqtalar to‘plami markazi O nuqtada va radiusi $R = \sqrt{a^2 - AO^2}$ bo‘lgan aylana bo‘lar ekan.

7- masala. Berilgan nuqtadan tekislikka uzunliklari 10 sm va 17 sm bo‘lgan ikkita og‘ma o‘tkazilgan. Bu og‘malar proyeksiyalarining ayirmasi 9 sm bo‘lsa, og‘malarning proyeksiyalari topilsin (19- rasm).

Berilgan: $S \notin \alpha$, SA, SB — og‘malar, $SO \perp \alpha$, $SA = 10$ sm,

$SB = 17$ sm, $OB - OA = 9$ sm.

Topish kerak: $OB-?$, $OA-?$

Yechish: $DB = x$, $OA = y$ bo‘lsin. $x - y = 9$ (1)

ΔSOB dan: $SO^2 = BS^2 - OB^2 = 289 - x^2$

ΔSOA dan: $SO^2 = AS^2 - AO^2 = 100 - y^2$, bundan,

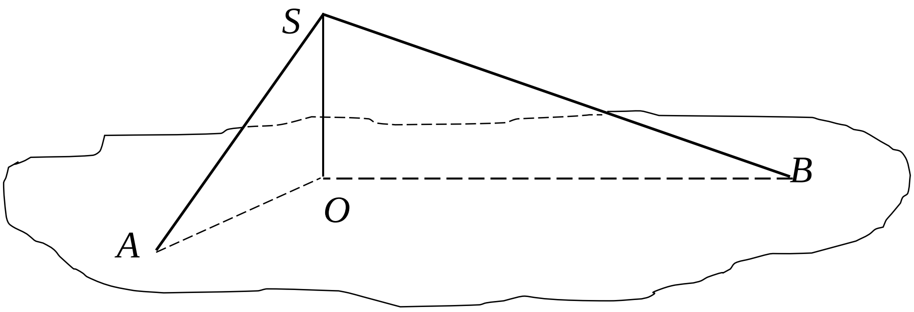
$$289 - x^2 = 100 - y^2. \quad (2)$$

(2) va (1) birgalikda yechamiz.

$$\begin{cases} x - y = 9 \\ x^2 - y^2 = 189 \end{cases} \Rightarrow \begin{cases} x - y = 9 \\ (x-y)(x+y) = 189 \end{cases} \Rightarrow \begin{cases} x - y = 9 \\ x + y = 21 \end{cases} \Rightarrow$$

$\Rightarrow 2x = 30$, $x = 15$, $y = 6$.

Shunday qilib $OB = 15$ sm, $OA = 6$ sm.



19- rasm

8- masala. Nuqtadan tekislikka ikkita og‘ma o‘tkazilgan. Agar og‘malar $1:2$ nisbatda bo‘lib, ularning proyeksiyalari 1 sm va 7 sm bo‘lsa, og‘malarning uzunliklari topilsin (20- rasm).

Berilgan: $S \notin \alpha$, SA, SB — og‘malar, $SO \perp \alpha$, $SA:SB = 1:2$, $OA = 1$ sm, $OB = 7$ sm.

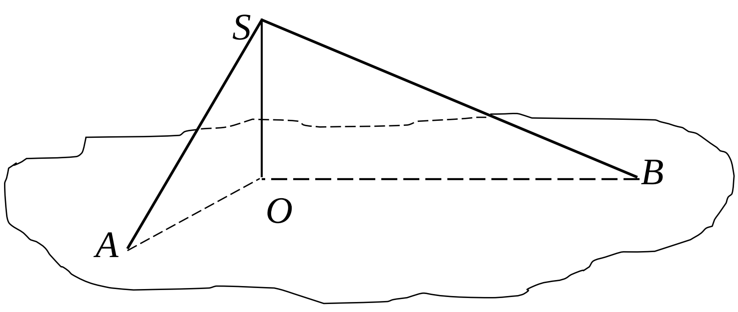
Topish kerak: $SA-?$, $SB-?$

Yechish: $BS = x$ bo‘lsin, u holda $AS = \frac{1}{2}x$.

ΔAOS va ΔBOS dan: $SO^2 = AS^2 - AO^2 = BS^2 - OB^2$.

Demak, $\left(\frac{1}{2}x\right)^2 - 1 = x^2 - 49$. $\frac{1}{4}x^2 - 1 = x^2 - 49$ Bundan,

$x = 8$ sm.



20- rasm

Shunday qilib, $SA = 4$ sm, $SB = 8$ sm.

9- masala. Nuqtadan tekislikka 23 sm va 33 sm ga teng ikkita og‘ma o‘tkazilgan. Agar og‘malarning proyeksiyalari $2:3$ nisbatda bo‘lsa, shu nuqtadan tekislikkacha masofa topilsin.

Berilgan: $SA = 23$ sm, $SB = 33$ sm, $OA:OB = 2:3$.

Topish kerak: $SO-?$

Yechish: $SO = x$ bo‘lsin. ΔAOS va ΔBOS dan:

$OA = \sqrt{23^2 - x^2}$, $OB = \sqrt{33^2 - x^2}$. Masala shartiga ko‘ra,

$OA:OB = 2:3 \Rightarrow OA^2:OB^2 = 4:9$

U holda, $\frac{23^2 - x^2}{33^2 - x^2} = \frac{4}{9}$ yoki $9 \cdot (23^2 - x^2) = 4 \cdot (33^2 - x^2)$

$5x^2 = 9 \cdot 23^2 - 4 \cdot 33^2 = 81 \cdot 5$

$$x^2 = 81,\ x = \pm 9,\ x > 0 \Rightarrow x = 9$$

Javob: $SO = 9$ sm.

10- masala. Uchburchakka tashqi chizilgan aylananing markazidan uchburchak tekisligiga perpendikular to‘g‘ri chiziq chiqarilgan. Bu to‘g‘ri chiziqning har bir nuqtasi uchburchakning uchlaridan baravar uzoqlikda yotishini isbotlang.

Berilgan: $\Delta ABC \in \alpha$, ρ — uchburchakka tashqi chizilgan aylana. O — aylana markazi, $O \in \alpha$, $a \perp \alpha$, $\forall X \notin \alpha$.

Ko‘rsatish kerak: $AX = BX = CX = const$.

Yechish: Masala shartiga ko‘ra, $OA = OB = OC$, chunki bu ΔABC ga tashqi chizilgan aylana radiuslaridir. Ikkinchi tomondan esa, $OA = pr_\alpha AX$, $OB \perp pr_\alpha BX$, $OC = pr_\alpha CX$.

Demak, proyeksiyalari teng og‘malarning o‘zlari ham tengdir:

$$AX = BX = CX.$$

11- masala. α tekislikning tashqarisida S nuqtadan unga uchta teng SA, SB, SC og‘malar va SO perpendikular o‘tkazilgan. Perpendikulaming asosi O nuqta ΔABC uchburchakka tashqi chizilgan aylana markazi bolishini isbotlang.

Berilgan: $S \notin \alpha$, $SA = SB = SC$, $SO \perp \alpha$.

Ko‘rsatish kerak: O nuqta — aylana markazi.

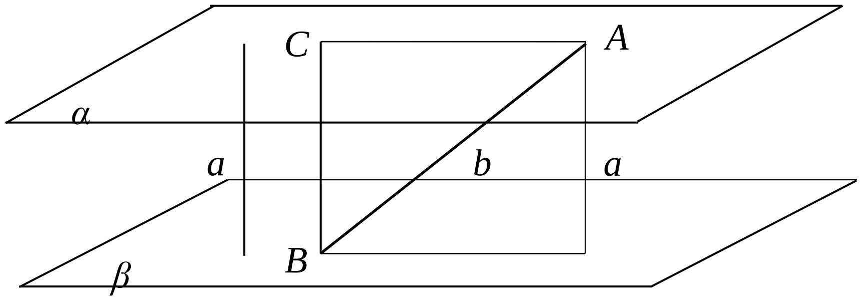
Yechish: SA, SB, SC og‘malar teng bo‘lgani uchun AO, BO, CO proyeksiyalar ham teng, demak, O nuqta ΔABC ga tashqi chizilgan aylana markazi bo‘lar ekan.

12- masala. Ikkita parallel tekislik orasidagi masofa a ga teng. b uzunlikdagi kesmaning uchlari bu tekislikka tiralib turibdi. Kesmaning har bir tekislikdagi proyeksiyasi topilsin (21- rasm).

Berilgan: $\alpha \| \beta$, $\rho_{\alpha\beta} = a$, $AB = b$ — og‘ma, $A \in \alpha$, $B \in \beta$.

Topish kerak: $pr_\alpha b - ?$, $pr_\beta b - ?$

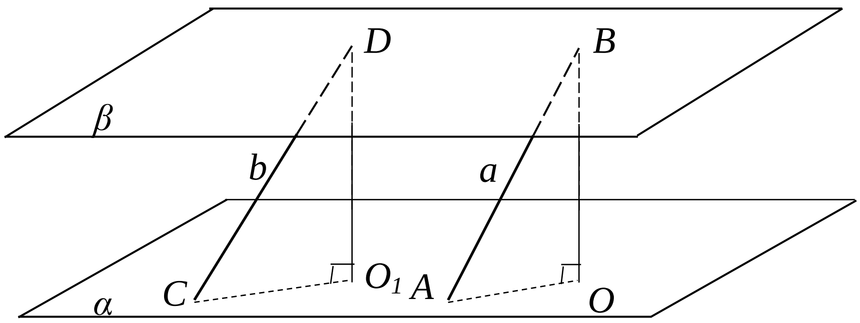
Yechish: Ikkala proyeksiya ham teng: $pr_\alpha b = pr_\beta b = \sqrt{b^2 - a^2}$



21- rasm

13- masala. a va b uzunlikdagi ikkita og‘ma α va β parallel tekislikka tiralib turadi. a kesmaning tekislikdagi proyeksiyasi c bo‘lsa, b kesmaning proyeksiyasi topilsin (22- rasm).

Berilgan: $\alpha \| \beta$, $AB = a$, $A \in a$, $B \in b$, $CD = b$, $c \in \alpha$, $D \in b$, $pr_\beta a = c$.



22- rasm

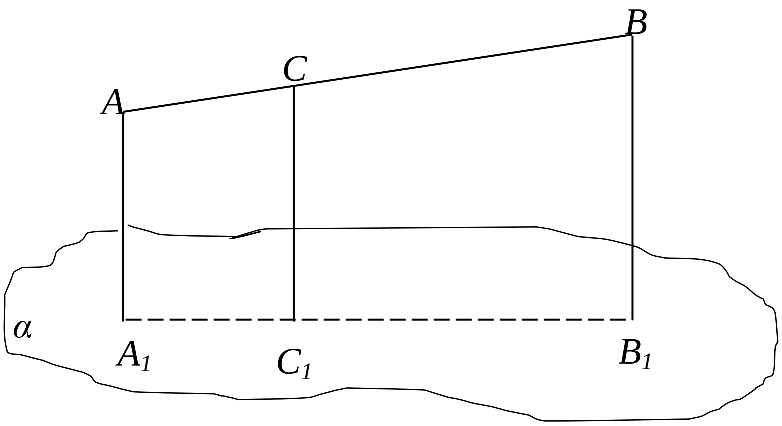
Topish kerak: $pr_\beta b - ?$

Yechish: ΔABO dan: $BO = \sqrt{a^2 - c^2}$,

$\alpha \| \beta \Rightarrow BO = DO_1 = \sqrt{a^2 - c^2}$

ΔABO_1 dan: $pr_\beta b = CO_1 = \sqrt{b^2 - a^2 + c^2}$.

14- masala. Tekislikni kesib o‘tmaydigan berilgan kesmaning uchlari tekislikdan 0,3 sm va 0,5 sm masofada yotadi (23- rasm). Berilgan kesmani $3:7$ nisbatda bo‘luvchi nuqta tekislikdan qanday masofada yotadi?



23- rasm

Berilgan: $AB \cap \alpha = \emptyset$, $AA_1 \perp \alpha$, $BB_1 \perp \alpha$, $A_1, B_1 \in \alpha$, $AA_1 = 0{,}3$ m, $BB_1 = 0{,}5$ m. $C \in AB$, $AC : CB = 3 : 7$, $CC_1 \perp \alpha$, $C_1 \in \alpha$.

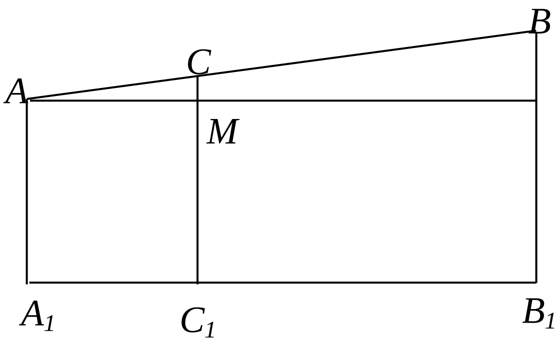
Topish kerak: $CC_1 - ?$

Yechish: A_1, B_1 va C_1 nuqtalar bir to'g'ri chiziqda yotadi. U holda masala planimetrik masalaga keltiriladi (24- rasm).

$\Delta ACM \sim \Delta ABE$ (3-alomatga ko'ra) $\frac{CM}{BE} = \frac{AC}{AB}$;

$BE = BB_1 - AA_1 = 0{,}2$.

Demak, $CM = BM \cdot \frac{AC}{AC+CB} = 0{,}2 \cdot \frac{1}{1+\frac{7}{3}} = \frac{0{,}6}{10} = 0{,}06$.



24- rasm

U holda, $CC_1 = 0{,}3 + 0{,}06 = 0{,}36$ sm

Izoh: Agar $AA_1 = 0{,}6$ va $BB_1 = 0{,}36$ m bo'lsa, $CC_1 = 0{,}44$ m boladi.

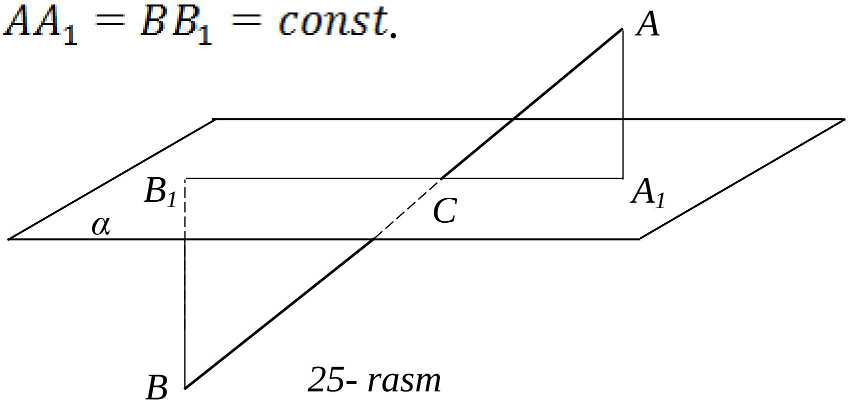
15- masala. Kesmaning o'rtasidan tekislik o'tkazilgan. Kesimning uchlari bu tekislikdan baravar uzoqlikda yotishini isbotlang (25- rasm).

Berilgan: AB — kesma; $c \in AB$, $c \in \alpha$, $AB \cap \alpha = C$, $AC = BC$

Ko'rsatish kerak: $\rho(\alpha, A) = \rho(\alpha, B) - const$.

Yechish: $AA_1 \perp \alpha$, $BB_1 \Rightarrow$ **3.4-** teoremadan, AB kesma va AA_1,

BB_1 perpendikularlar bir tekislikda yotadi. $AC = BC$, $\angle ACA_1 = \angle BCB_1$ va $\angle A_1 = \angle B_1$ =90°, $\Delta ACA_1 = \Delta BCB_1$ bo‘lganidan, $AA_1 = BB_1 = const$.



25- rasm

16- masala. Parallelogramm diagonallari orqali tekislik o‘tkazilgan. Parallelogramm ikkinchi diagonalining uchlari tekislikdan baravar uzoqlikda yotishini isbotlang.

Yechish: Parallelogramm diagonallari kesishish nuqtasida teng ikkiga bo‘linadi, shuning uchun tekislik ikkinchi diagonalning o‘rtasidan o‘tadi. Demak, 16- masalaning natijasiga ko‘ra, talab qilingan faraz isbotlandi.

17- masala. Tekislikni kesib o‘tmaydigan AB kesmaning uchlari, tekislikdan a va b masofada tursa, AB kesmaning o‘rtasidan tekislikkacha masofa topilsin.

Berilgan: α, AB, $AB \cap \alpha = \emptyset$, $AA_1 = a$, $BB_1 = b$, $c \in AB$, $AC = BC$, $AA_1 \perp \alpha$, $BB_1 \perp \alpha$, $CC_1 \perp \alpha$.

Topish kerak: $CC_1 -?$

Yechish: $AA_1 \perp \alpha$, $BB_1 \perp \alpha$ va $CC_1 \perp \alpha \Rightarrow AA_1 || BB_1 || CC_1$, (**3.4-** teoremaga ko‘ra).

Demak, AA_1, BB_1 va $CC_1 \in \beta$, $\beta \neq \alpha$.

U holda, 2-§ dagi 2- misol natijasiga ko‘ra, $CC_1 = \frac{a+b}{2}$

18- masala. Oldingi masalani AB tekislik bilan kesishgan hol uchun yeching.

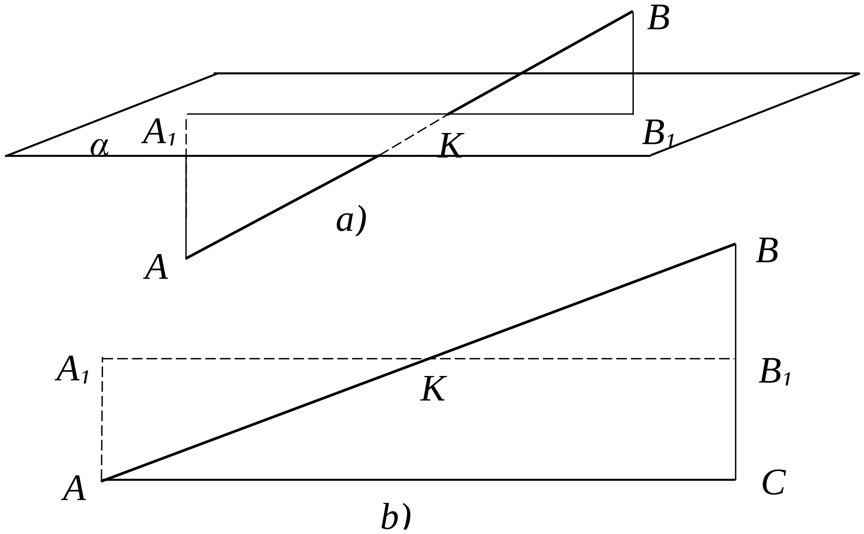
Berilgan: (16- masala berilishidan farqli), $B \cap \alpha = C$, $C \in \alpha$.

Topish kerak: $CC_1 -?$

Yechish: 16- masala va 2-§ dagi 3- masala natijalariga ko‘ra,

$$CC_1 = \frac{|a+b|}{2}.$$

19- masala. Tekislikni kesib o‘tuvchi uzunligi 1 m bo‘lgan kesmaning uchlari tekislikdan 0,5 m va 0,3 m uzoqlikda. Kesmaning tekislikdagi proyeksiyasi topilsin (26-a rasm).



26- rasm

Berilgan: α, AB, $AB \cap \alpha = K$, $AA_1 \perp \alpha$, $BB_1 \perp \alpha$.
$AA_1 = 0{,}3$ m, $BB_1 = 0{,}5$ m, $AB = 1$ m.

Topish kerak: $A_1B_1 -?$

Yechish: $AA_1 \| BB_1$ bo‘lganligidan, 4- teoremaga ko‘ra.

Demak, masala planimetrik masalaga keltirildi (26-b rasm).
$AC \| A_1B_1$, $B_1C \| AA_1$, bo‘lgan yordamchi C nuqta olamiz.

Natijada ABC uchburchak hosil bo‘ladi. $B_1C = AA_1 = 0{,}3$.

$BC = BB_1 + AA_1 = 0{,}8$ m. Pifagor teoremasiga ko‘ra,

$A_1B_1 = AC = \sqrt{1 - 0{,}64} = 0{,}6$ m.

20- masala. Uzunligi 15 m bo‘lgan telefon simi yerdan balandligi 8 m bo‘lgan telefon simyog‘ochidan uyga qarab 20 m balandlikda tortilgan. Sim osilib turmagan deb faraz qilib, simyog‘ochdan uygacha masofani toping (27- rasm).

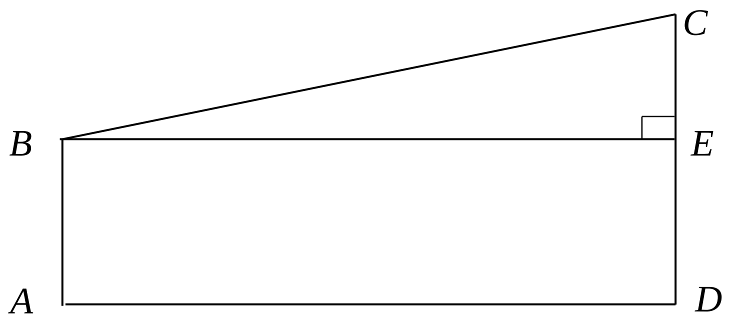
Berilgan: $AB = 8$ m, $BC = 15$ m, $CD = 20$ m.

Topish kerak: $AD -?$

Yechish: $BE \| AD$ to 'g 'r i chiziq o‘tkazamiz. U holda,
$CE = CD - ED = CD - AB = 12$ m.

ΔBCE dan, Pifagor teoremasiga ko‘ra,
$BE = \sqrt{BC^2 - CE^2} = \sqrt{225 - 144} = \sqrt{81} = 9.$

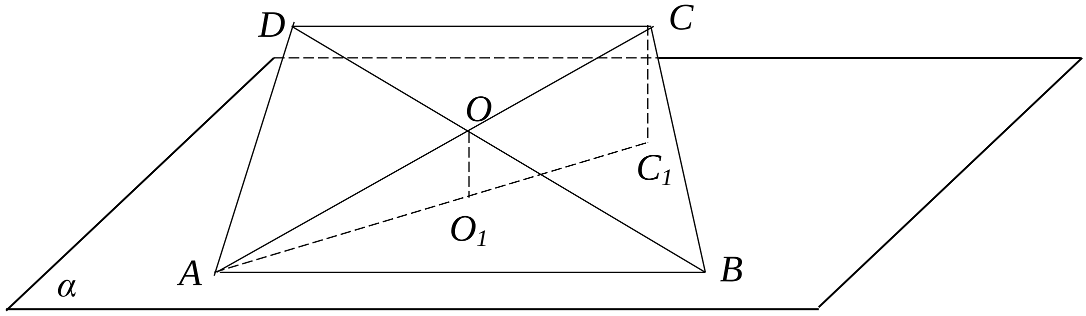
Demak, $BE = AD = 9$ m.



27- rasm

21- masala. Trapetsiyaning bitta asosi orqali ikkinchi asosidan a masofada yotuvchi tekislik o‘tkazilgan. Agar trapetsiyaning asoslari $m{:}n$ kabi nisbatda bo‘lsa, trapetsiyaning diagonallari kesishish nuqtasidan shu tekislikkacha masofa topilsin.

Berilgan: $ABCD$ — trapetsiya, α — tekislik. $AB{:}CD = m{:}n$, $AC \cap BD = O$ (28- rasm).



28- rasm

Topish kerak: $\rho(O,\ \alpha) - ?$

Izoh. Bu yerda p(•, α) — (•) dan α tekislikkacha masofa.

Yechish: C va O nuqtalardan α tekislikkacha perpendikular to‘g‘ri chiziqlar o‘tkazamiz. Masala shartiga ko‘ra $\rho(CD, \alpha) = a$. A va C_1, nuqtalarni tutashtirib, ACC_1 va AOO_1 uchburchaklar hosil qilamiz. $\Delta ACC_1 \sim \Delta AOO_1$ (chunki, $OO_1 \| CC_1$ $\angle A$ umumiy). U holda, $\frac{OO_1}{CC_1} = \frac{AD}{AC}$, bu yerdan

$$OO_1 = \frac{AD}{AC} \cdot CC_1 = \frac{AD}{AC} \cdot a \qquad (1)$$

Ikkinchi tomondan, $\Delta AOB \sim \Delta DOC$, (chunki, $\angle DOC = \angle AOB$, $\angle ODC = \angle OBA$, u holda, yoki $\frac{AO}{OC} = \frac{m}{n}$. Bu yerdan,

$$\frac{AO}{OC} = \frac{AO}{AC - AO} = \frac{1}{\frac{AC}{AO} - 1} = \frac{m}{n}, \quad \frac{AC}{AO} - 1 = \frac{m}{n}, \quad \frac{AC}{AO} = \frac{m+n}{m} \Rightarrow \frac{AO}{AC} = \frac{m}{m+n}.$$

buni (1) ga qo‘yib, $OO_1 = \frac{a \cdot m}{m+n}$ ekanini topamiz.

22- masala. A nuqtadan kvadratning uchlarigacha masofa a ga teng. Kvadratning tomoni b ga teng bo‘lsa, a nuqtadan kvadrat tekisligigacha masofa topilsin.

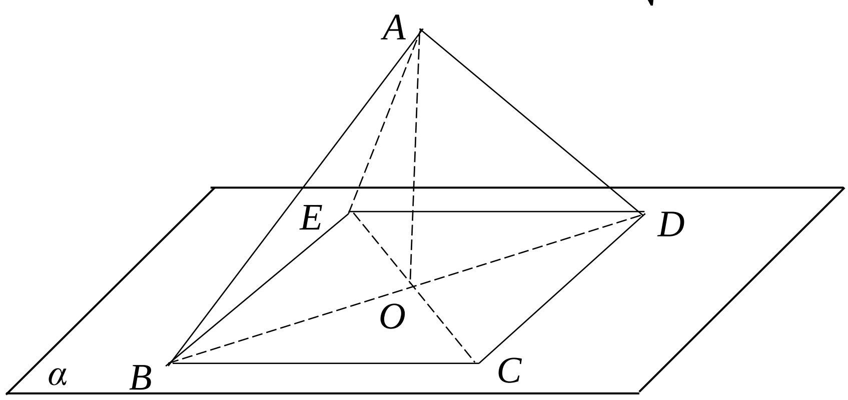
Berilgan: α — tekislik, $BCDE$ — kvadrat, $A \notin \alpha$, $BCDE \in \alpha$, $\rho(A,B) = \rho(A,D) = \rho(A,E) = a$, $BC = CD = DE = BE = b$ (29- rasm).

Topish kerak: $\rho(A,\alpha)-?$

Yechish: A nuqta kvadratning uchlaridan baravar uzoqlikda bo‘lgani uchun, A nuqtadan α tekislikka tushirilgan perpendikular $BCDE$ kvadratning diagonallari kesishish nuqtasiga tushadi. $AO \perp \alpha$, $AO \perp BD$, demak,

ΔABO — to‘g‘ri burchakli uchburchak: $BO = \frac{BD}{2} = \frac{\sqrt{2}b}{2} = \frac{b}{\sqrt{2}}$.

Demak, $AO = \rho(o,\alpha) = \sqrt{AB^2 - BO^2} = \sqrt{a^2 - \frac{b^2}{2}}$.



29- rasm

23- masala. Teng tomonli uchburchakning (ABC) uchidan uchburchak tekisligigacha AD perpendikular tushirilgan. Agar $AD = 13$ sm, $BC = 6$ sm bo‘lsa, D nuqtadan BC tomongacha masofa topilsin (30- rasm).

Berilgan: ΔABC — teng tomonli, α — tekislik. $ABC \in \alpha$, $AD \perp \alpha$, $AD = 13$ sm, $AB = BC = AC = 6$ sm.

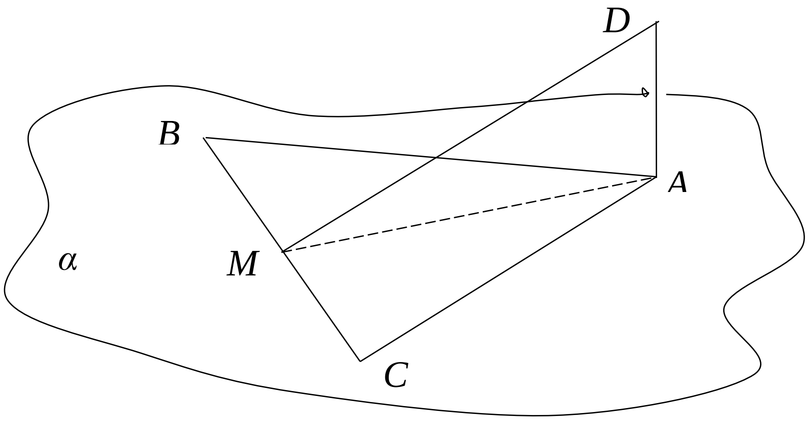
Topish kerak: $DM-?$, $M \in BC$.

Yechish: $DM \perp BC$ izlangan masofa. Uch perpendikular haqidagi teoremaga ko‘ra, $MA \perp BC$. Bizga ma’lumki, teng tomonli uchburchak balandligi shu tomonni teng ikkiga bo‘ladi.

Demak, $BC = 6 \Rightarrow BM = CM = 3$. U holda,

$$MA = \sqrt{AB^2 - BM^2} = \sqrt{27}.$$

ΔADM dan: $DM = \sqrt{AD^2 + MA^2} = \sqrt{169 + 27} = 14$ sm; $DM = 14$ sm.



30- rasm

MUSTAQIL ISHLASH UCHUN

1. Tekislikdan 1 m masofada yotgan nuqtadan ikkita teng og‘ma o‘tkazilgan. Agar og‘malar o‘zaro perpendikular va tekislikka o‘tkazilgan perpendikular bilan 60° ga teng burchaklar tashkil etishi ma’lum bo‘lsa, og‘malarning asoslari orasidagi masofani toping.

2. To‘g‘ri burchakli ABC uchburchakning C to‘gri burchagi uchidan gipotenuzaga parallel va undan 1 m masofada tekislik o‘tkazilgan. Katetlarning bu tekislikdagi proyeksiyasi 3 m va 5 m bo‘lsa, gipotenuza topilsin.

3. Asosi 6 m va yon tomoni 5 m bo‘lgan teng yonli uchburchak berilgan. Ichki chizilgan doiraning markazidan uchburchak tekisligiga uzunligi 2 m bo‘lgan perpendikular chiqarilgan. Bu perpendikularning oxiridan uchburchakning tomonlarigacha masofa topilsin.

4. Teng yonli uchburchakning asosi va balandligi 4 m ga teng. Berilgan nuqta uchburchak tekisligidan 6 m masofada va uning uchlaridan baravar uzoqlikda yotadi. Shu masofa topilsin.

5. Tekislikka parallel bo‘lgan AB kesmaning uchlaridan AC perpendikular va AB ga perpendikular ravishda BD og‘ma o‘tkazilgan. Agar $AB = a,\ AC = b,\ BD = c$ bo‘lsa, CD masofa topilsin.

6. To‘g‘ri burchagi C bo‘lgan ABC to‘g‘ri burchakli uchburchakning A uchidan uchburchak tekisligiga AD perpendikular chiqarilgan. Agar $AC = a$, $BC = b$, $AD = c$ bo‘lsa, D nuqtadan B, C uchlarigacha bo‘lgan masofa topilsin.

7. ABC uchburchakning C to‘g‘ri burchagi uchidan uchburchak tekisligiga CD perpendikular chiqarilgan. Agar $AB = a$, $BC = b$, $CD = c$ bo‘lsa, D nuqtadan uchburchakning gipotenuzasigacha bo‘lgan masofa topilsin.

8. a to‘g‘ri chiziq va α tekislik berilgan. α tekislikka perpendikular va a to‘g‘ri chiziqni kesib o‘tuvchi hamma to‘g‘ri chiziqlar α tekislikka perpendikular bo‘lgan bitta tekislikda yotishini isbotlang.

9. Ikki o‘zaro perpendikular tekislikda yotuvchi A va B nuqtalardan tekisliklarning kesishish to‘g‘ri chizig‘iga AC, BD perpendikular tushirilgan. Agar $AC = a$, $AD = b$, $CD = c$ bo‘lsa, AB kesmaning uzunligi topilsin.

10. Teng tomonli ABC uchburchakning A, B uchlaridan uchburchak tekisligiga AA_1, BB_1 perpendikularlar chiqarilgan. Agar $AB = 2$ m, $CA_1 = 3$ m, $CB_1 = 7$ m bo‘lsa va A_1B_1 kesma uchburchak tekisligini kesib o‘tmasa, C uchdan A_1B_1 kesmaning o‘rtasigacha bo‘lgan masofa topilsin.

11. O‘zaro perpendikular bolgan α va β tekisliklar c to‘g‘ri chiziq bo‘yicha kesishadi. α tekislikda $a||b$ to‘g‘ri chiziq, β tekislikda $b||c$ to‘g‘ri chiziq o‘tkazilgan. Agar a va c to‘g‘ri chiziqlar orasidagi masofa 1,5 m, b va c to‘g‘ri chiziqlar orasidagi masofa 0,8 m ga teng bo‘lsa, a, b to‘g‘ri chiziqlar orasidagi masofani toping.

12. A nuqtadan kvadratning hamma tomonlarigacha masofa a ga teng. Kvadratning diagonali d ga teng bo‘lsa, A nuqtadan kvadrat tekisligigacha masofa topilsin.

13. Kvadratning uchi dan uning tekisligiga perpendikular chiqarilgan. Bu perpendikularning uchidan kvadratning boshqa uchlarigacha masofa a va b ga teng, $(a < b)$. Perpendikularning uzunligi va kvadratning tomonini toping.

14. Berilgan to‘g‘ri burchak tekisligidan tashqarida yotgan M nuqta burchakning uchidan a masofada, uning tomonlaridan esa b masofada yotadi. M nuqtadan burchak tekisligigacha bo‘lgan masofa topilsin

15. Berilgan nuqtadan tekislikka uzunligi 2 m dan ikkita teng og‘ma tushirilgan. Agar og‘malar orasidagi burchak 60°, ularning proyeksiyalari esa perpendikular bo‘lsa, nuqtadan tekislikkacha masofa topilsin.

IV. FAZODA DEKART KOORDINATALARI VA VEKTORLARGA DOIR MASALALAR

1- masala. $A(3,3,3)$ nuqtadan: 1) koordinata tekisliklarigacha; 2) koordinata o‘qlarigacha; 3) koordinata boshigacha bo‘lgan masofalarni toping.

Berilgan: $A(3,3,3)$.

Topish kerak: 1) $AA_{xy},\ AA_{xz},\ AA_{yz}$;

2) $AA_x,\ AA_y,\ AA_z$;

3) $AO-?$

Yechish:

1. a) $A_{xy} \in OXY \Rightarrow z = 0 \Rightarrow A_{xy} = (3,3,0)$,

$AA_{xy} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{(3-3)^2 + (3-3)^2 + (0-3)^2} = 3$;

b) $A_{xz} \in OXZ \Rightarrow y = 0 \Rightarrow A_{xz} = (3,0,3)$,

$AA_{xz} = \sqrt{(3-3)^2 + (0-3)^2 + (3-3)^2} = 3$;

d) $A_{yz} \in OYZ \Rightarrow x = 0 \Rightarrow A_{yz} = (0,3,3)$,

$AA_{yz} = \sqrt{(3-0)^2 + (3-3)^2 + (3-3)^2} = 3$.

2. a) $AA_x \in OX \Rightarrow z = 0,\ y = 0 \Rightarrow A_x = (3,0,0)$,

$AA_x = \sqrt{(3-3)^2 + (0-3)^2 + (3-0)^2} = 3\sqrt{2}$;

b) $AA_y \in OY \Rightarrow z = 0,\ x = 0 \Rightarrow A_y = (0,3,0)$,

$AA_y = \sqrt{(3-0)^2 + (3-3)^2 + (3-0)^2} = 3\sqrt{2}$;

d) $AA_z \in OZ \Rightarrow x = 0,\ y = 0 \Rightarrow A_z = (0,0,3)$,

$AA_z = \sqrt{(3-0)^2 + (3-0)^2 + (3-3)^2} = 3\sqrt{2}$.

3. Koordinatalar boshida $x = 0,\ y = 0,\ z = 0$ bo‘lgani uchun $O(0,0,0)$ bo‘ladi. ___________

Bundan: $AO = \sqrt{(3-0)^2 + (3-0)^2 + (3-0)^2} = 3\sqrt{3}$.

2- masala. $B(-3,2,-5)$ nuqta Oxz tekislikidan qanday masofada yotibdi.

Berilgan: $B(-3,2,-5)$.

Topish kerak: 1) BB_{xz}

Yechish:

1. $B_{xz} \in OXZ \Rightarrow y = 0 \Rightarrow A_{xz} = (-3,0,-5)$,

$BB_{xz} = \sqrt{(-3+3)^2 + (0-2)^2 + (-5+5)^2} = 2$.

Javob: 2.

3- masala. $A(x,0,0)$ nuqta $B(1,2,3)$, $C(-1,3,4)$ nuqtalardan baravar uzoqlikda bo‘lsa, x ni toping.

Berilgan: $AB(1,2,3)$, $C(-1,3,4)$, $AB = AC$.

Topish kerak: A nuqtaning koordinatalarini.

Yechish: $A \in OX \Rightarrow y = 0,\ z = 0$,

$AB^2 = (x-1)^2 + 4 + 9 = x^2 - 2x + 14$,

$AC^2 = (x+1)^2 + 3 + 16 = x^2 + 2x + 20$,

$AC = AB \Rightarrow x^2 - 2x + 14 = x^2 + 2x + 20 \Rightarrow 4x = -6$,

$\Rightarrow x = -\frac{3}{2}$.

Demak, $C(0,0,0)$.

4- masala. $A(1,2,3)$ nuqta berilgan bo‘lib, u koordinatalar boshidan baravar uzoqlashgan fazoviy nuqtalarning geometrik o‘rni tenglamasini tuzing.

Berilgan: $A(1,2,3)$, $O(0,0,0)$, $DO = DA$.

Topish kerak: $D(x,y,z)$ nuqtalar to‘plami.

Yechish: $OD^2 = (x-0)^2 + (y-0)^2 + (z-0)^2$,

$DA^2 = (x-1)^2 + (y-2)^2 + (z-3)^2$,

$DO = DA \Rightarrow x^2 + y^2 + z^2 = x^2 - 2x + 1 + y^2 - 4y + 4 +$
$+z^2 - 6z + 92x + 4y + 6z - 14 = 0$, $x + 2y + 3z = 7$.

5- masala. Uchlari: 1) $A(0,2,-3)$, $B(-1,1,1)$, $C(2,-2,-1)$, $D(3,-1,-5)$; 2) $A(2,1,3)$, $B(1,0,7)$, $C(-2,1,-5)$, $D(-1,2,1)$ nuqtalardagi $ABCD$ to‘rtburchakning parallelogramm ekanini isbotlang.

1) *Berilgan:* $A(0,2,-3)$, $B(-1,1,1)$, $C(2,-2,-1)$, $D(3,-1,-5)$.

Ishot qilish kerak: $ABCD$ — parallelogramm.

Yechish: Parallelogrammning diagonallari bir nuqtada kesishib, shu nuqtada teng ikkiga bo‘linganliklari uchun AC va BD kesmalarning o‘rtalari bir nuqtada ekanligini ko‘rsatish kifoya.

AC kesmaning o‘rtasi

$x=\frac{0+2}{2}=1, \quad y=\frac{2-2}{2}=0, \quad z=\frac{-3-1}{2}=-2.$

BD kesmaning o‘rtasi

$x=\frac{3-1}{2}=1, \quad y=\frac{1-1}{2}=0, \quad z=\frac{1-5}{2}=-2.$

Demak, $ABCD$ to‘rtburchakning diagonallari: $(1,0,-2)$ nuqtada teng ikkiga bo‘lingani uchun $ABCD$ parallelogramm.

2) *Berilgan:* $A(2,1,3)$, $B(1,0,7)$, $C(-2,1,-5)$, $D(-1,2,1)$

Ishot qilish kerak: $ABCD$ parallelogramm.

Yechish: Yuqoridagidek mulohaza yuritib, AC va BD kesmalarining o‘rtalari bir nuqtadan iborat ekanligini ko‘rsatamiz.

AC kesmaning o‘rtasi

$x=\frac{2-2}{2}=0, \quad y=\frac{1+1}{2}=1, \quad z=\frac{3+5}{2}=4.$

BD kesmaning o‘rtasi

$x=\frac{1-1}{2}=0, \quad y=\frac{0+2}{2}=1, \quad z=\frac{1+7}{2}=4.$

Demak, AC va BD kesmalarning o 'rtalari $(0,2,4)$ nuqta. Bu nuqtada ular teng ikkiga bo‘lingani uchun $ABCD$— parallelogramm.

6- masala. $ABCD$ parallelogrammning uchta uchi koordinatalari berilgan, to‘rtinchi D uchining koordinatalarini toping:

1) $A(2,3,2)$, $B(0,2,4)$, $C(4,1,0)$;
2) $A(1,-1,0)$, $B(0,1,-1)$, $C(-1,0,1)$;
3) $A(1,-1,0)$, $B(0,1,-1)$, $C(-1,0,1)$;

1) *Berilgan:* $A(2,3,2)$, $B(0,2,4)$, $C(4,1,0)$; $ABCD$ — parallelogramm.

Topish kerak: $D(x,y,z)$.

Yechish: Parallelogrammning diagonallari kesishish nuqtasida teng ikkiga bo‘linishini bilamiz. Kesmaning o‘rtasini topish formulasidan:

$$\frac{x+0}{2}=\frac{2+4}{2}, \quad \frac{y+2}{2}=\frac{3+1}{2}, \quad \frac{z+4}{2}=\frac{2+0}{2}$$

$$x=6, \quad y=2, \quad z=-2$$

Demak, $D(6,2,-2)$

2) *Berilgan:* $A(1,-1,0)$, $B(0,1,-1)$, $C(-1,0,1)$; $ABCD$ — parallelogram.

Topish kerak: $D(x,y,z)$.

Yechish: Parallelogrammning diagonallari kesishish nuqtasida teng ikkiga bo‘linishini bilamiz. Kesmaning o‘rtasini topish formulasidan:

$$\frac{x+0}{2}=\frac{1-1}{2},\quad \frac{y+1}{2}=\frac{0-1}{2},\quad \frac{z+2}{2}=\frac{1-1}{2}$$

$$x=0,\quad y=-2,\quad z=-2$$

Demak, $D(0,-2,-2)$

3) *Berilgan*: $A(1,-1,0)$, $B(0,1,-1)$, $C(-1,0,1)$; $ABCD$ — parallelogramm

Topish kerak: $D(x,y,z)$.

Yechish: Yuqoridagidek mulohaza yuritib, x,y,z larni topamiz:

$$\frac{x+1}{2}=\frac{4-4}{2},\quad \frac{y-3}{2}=\frac{2+2}{2},\quad \frac{z+2}{2}=\frac{1-1}{2}$$

$$x=-1,\quad y=7,\quad z=-2$$

Demak, $D(-1,\ 7,-2)$

7- masala. Parallel ko‘chirishda $A(1,2,-1)$ nuqta $A'(1,-1,0)$ nuqtaga o‘tadi. Koordinatalar boshi qanday nuqtaga o‘tadi?

Berilgan: $A(1,2,-1)$, $A'(1,-1,0)$.

Topish kerak: $O'(a,b,c)$.

Yechish: Nuqtani parallel ko‘chirish formulasi $x'=x+a$, $y'=y+b$, $z'=z+c$ dan a, b, c larni topamiz.

$$1=1+a \qquad -1=2+b \qquad 0=-1+c$$

bundan: $a=0$ $\qquad b=-3$ $\qquad c=1$

Demak, koordinatalar boshi ($O(0,0,0)$ bo‘lgani uchun) $O'(0,-3,1)$ nuqtaga o‘tadi.

$AA_z=\sqrt{(3-0)^2+(3-0)^2+(3-3)^2}=3\sqrt{2}$.

8- masala. $A(2,0,-3)$ va $B(3,4,0)$ nuqtalar orasidagi masofani toping.

Berilgan: $A(2,0,-3)$ va $B(3,4,0)$.

Topish kerak: AB .

Yechish: Ikki nuqta orasidagi masofani topish formulasidan foydalanamiz $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$AB=\sqrt{(3-2)^2+(4-0)^2+(0+3)^2}=\sqrt{26}$.

Bundan:$AB = \sqrt{(3-2)^2 + (4-0)^2 + (0+3)^2} = \sqrt{26}$.

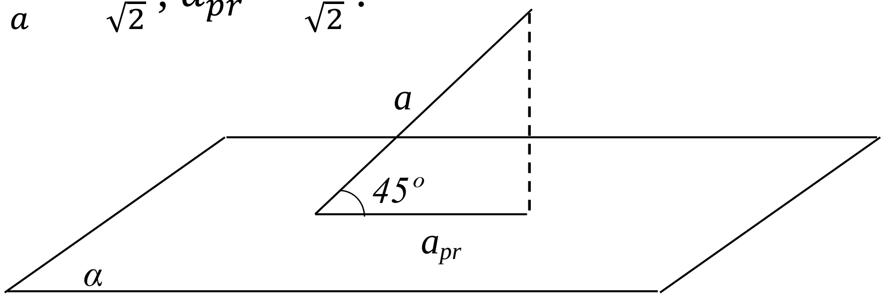
9- masala. Og‘ma a ga teng. Agar og‘ma tekislik bilan: 1) 45°, 2) 60°, 3) 30° ga teng burchak tashkil etsa, bu og‘maning tekislikdagi proyeksiyasi nimaga teng?

1) *Berilgan:* a og‘ma $(a\^\alpha) = 45°$ (31- rasm).

Topish kerak: $a_{pr}-?$

Yechish: Kosinus ta’rifiga asosan $cos45° = \frac{a_{pr}}{a}$,

$\frac{a_{pr}}{a} = \frac{1}{\sqrt{2}}$, $a_{pr} = \frac{a}{\sqrt{2}}$.

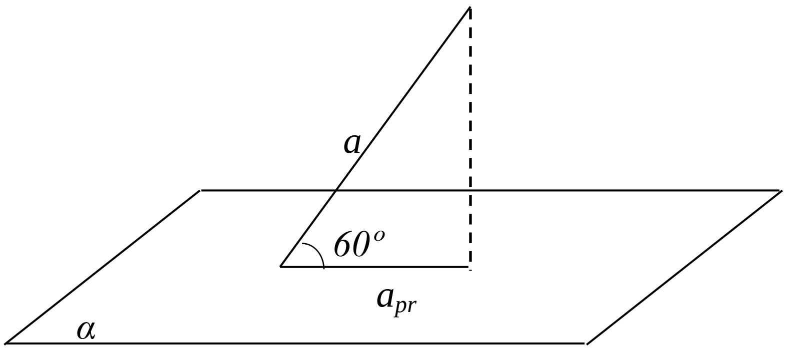


31- rasm

2) *Berilgan:* a og‘ma $(a\^\alpha) = 60°$ (32- rasm).

Topish kerak: $a_{pr}-?$

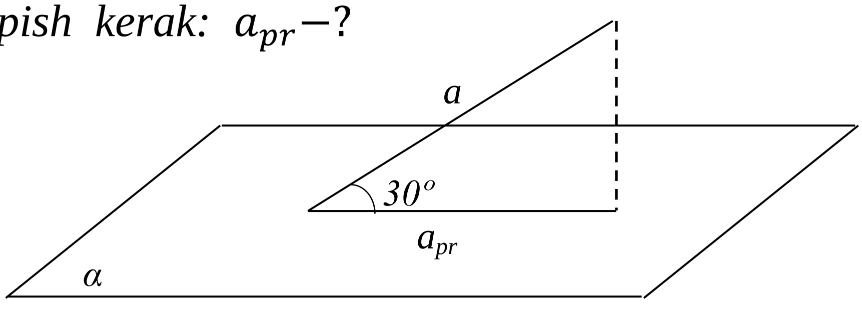
Yechish: Kosinus ta’rifiga asosan $cos60° = \frac{a_{pr}}{a}$, $\frac{a_{pr}}{a} = \frac{1}{2}$, $a_{pr} = \frac{a}{2}$.



32- rasm

3) *Berilgan:* a og‘ma $(a\^\alpha) = 30°$ (33- rasm).

Topish kerak: $a_{pr}-?$



33- rasm

Yechish: Kosinus ta'rifiga asosan $cos30° = \frac{a_{pr}}{a}$, $\frac{a_{pr}}{a} = \frac{\sqrt{3}}{2}$, $a_{pr} = \frac{\sqrt{3}a}{2}$.

10- masala. Uzunligi 10 m ga teng kesma tekislikni kesib o'tib, uchlari tekislikdan 2 m va 3 m masofada turadi. Berilgan tekislik orasidagi burchakni toping (34- rasm).

Berilgan: $AB = 10$ m, $AA_1 \perp \alpha$, $BB_1 \perp \alpha$, $AA' = 3$ m, $BB' = 2$ m, $\angle BOB' = \angle AOA' = AB\wedge\alpha$.

Topish kerak: $\angle BOB' -?$

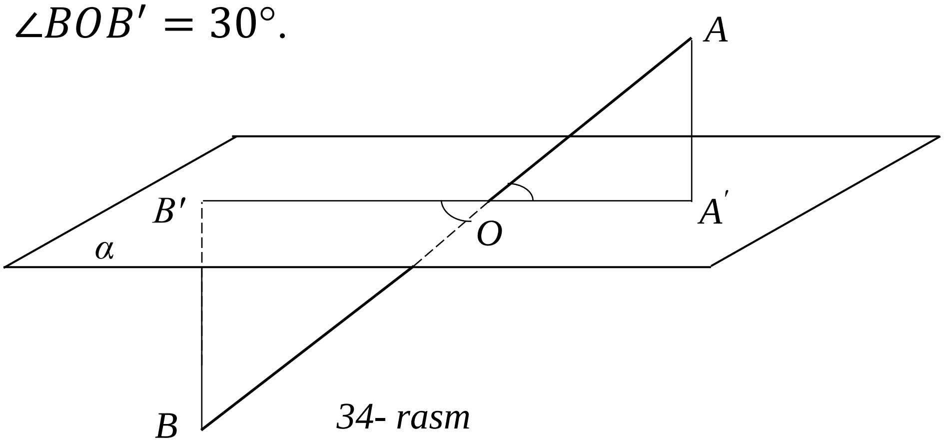
Yechish: Bu masalani yechishda sinuslar ta'rifidan foydalanamiz.

$$\left.\begin{aligned} sin\angle BOB' &= \frac{BB'}{BO} \\ sin\angle AOA' &= \frac{AA'}{AO} \end{aligned}\right\} \Rightarrow \angle AOA' = \angle BOB' \Rightarrow \frac{AA'}{AO} = \frac{BB'}{BO}.$$

dan $AO = x$ deb $BO = 10 - x$ ni topamiz va $\frac{2}{10-x} = \frac{3}{x} \Rightarrow 30 - 3x == 2x \Rightarrow 5x = 30$; $x = 6$, $sin\angle AOA' = \frac{AA'}{AO} = \frac{3}{6} = \frac{1}{2}$

$\angle AOA' = \angle BOB'$ dan $sin\angle BOB' = \frac{1}{2}$,

Demak: $\angle BOB' = 30°$.



34- rasm

11- masala. Teng yonli ikkita uchburchakning asoslari umumiy, ulaming tekisliklari o'zaro 60° burchakni tashkil etadi. Umumiy asos 6 m, birinchi uchburchakning yon tomoni 17 m ga teng, ikkinchi uchburchakning yon tomonlari perpendikular. Uchburchaklarning uchlari orasidagi masofani toping (35- rasm).

Berilgan: ABC uchburchak teng yonli, ABD uchburchak teng yonli va to'g'riburchakli, $\angle ADB = 90°$, $AC = BC = 17$, $AD = BD$, $AB = 16$, $\angle CAD = 60°$.

Topish kerak: $DC-?$

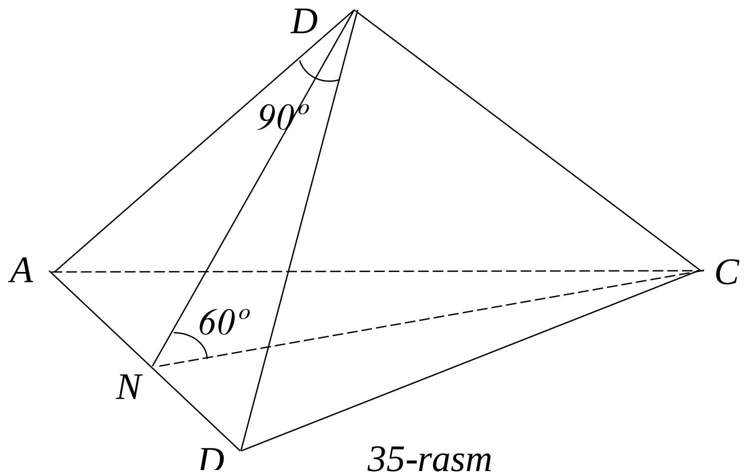
Yechish: ΔABD teng yonli va $\angle ADB = 90°$, bo‘lgani uchun Pifagor teoremasiga ko‘ra: $AB^2 = AD^2 + DB^2 \Rightarrow 2AD^2 = 16^2 \Rightarrow$ AD $= \frac{16}{\sqrt{2}} AD = 8\sqrt{2}$.

Endi teng yonli uchburchakning medianasi balandlik hamda bissektrisa bo‘lganligidan foydalanib, $AN = BN = \frac{AB}{2} = 8$ ni topamiz. ABC va ABD uchburchaklarning balandligini topamiz, bunda Pifagor teoremasidan foydalanamiz. Chunki NBD va NBC uchburchaklar to‘g‘ri burchakli:

$$ND = \sqrt{BD^2 - BN^2} = \sqrt{\left(8\sqrt{2}\right)^2 - 8^2} = 8,$$

$$NC = \sqrt{BC^2 - BN^2} = \sqrt{17^2 - 8^2} = 15.$$

Biz DC tomonni NDC uchburchak orqali topamiz. NDC uchburchakning ND va NC tomonlari va ular orasidagi $\angle CND = 60°$ burchak ma’lum. Kosinuslar teoremasidan DC ni topishimiz mumkin:



35-rasm

$$DC = \sqrt{CN^2 + DN^2 - 2CN \cdot DN\cos 60°} =$$

$$= \sqrt{15^2 + 8^2 - 2 \cdot 8 \cdot 15 \cdot \frac{1}{2}} = \sqrt{169} = 13$$

Demak, $DC = 13$ m.

12- masala. Umumiy asosi AB bo‘lgan teng yonli ABC va ABD uchburchaklar turli tekisliklarda yotadi, ular orasidagi burchak α ga teng (36- rasm). Agar:

1) $AB = 24$ m, $AC = 13$ m, $AD = 37$ m, $CD = 35$ m;

2) $AB = 32$ m, $AC = 65$ m, $AD = 20$ m, $CD = 63$ m bo‘lsa, α ni toping.

1) *Berilgan:* $AD = BD$, $AC = BC$, $AD = 37$ m, $AC = 13$ m, $AB = 24$ m, $CD = 35$ m, $\alpha = (\Delta ABD \wedge \Delta ABC)$.

Topish kerak: $cos\alpha -$?

Yechish: U chburchaklar teng yonli bo‘lgani uchun: $AN = \frac{AB}{2} == \frac{24}{2} = 12$,

$CN = \sqrt{AC^2 - AN^2} = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5$,

$DN = \sqrt{BD^2 - AN^2} = \sqrt{37^2 - 12^2} = \sqrt{1369 - 144} = \sqrt{1225} = 36$.

Kosinuslar teoremasiga ko‘ra:

$CD^2 = CN^2 + DN^2 - 2CN \cdot DN \cdot cos\alpha$.

$cos\alpha = \frac{1}{2} \cdot \frac{CN^2+DN^2-CD^2}{CN \cdot DN} = \frac{1}{2} \cdot \frac{5^2+35^2-35^2}{5 \cdot 35} = \frac{1}{2} \cdot \frac{25}{5 \cdot 35} = \frac{1}{2} \cdot \frac{1}{7} = \frac{1}{14}$.

Demak: $cos\alpha = \frac{1}{14}$.

2) *Berilgan:* $AB = 32$ m, $AC = 65$ m, $AD = 20$ m, $CD = 63$ m, $\alpha = (\Delta ABD \wedge \Delta ABC)$.

Topish kerak: $cos\alpha -$?

Yechish: Masalaning 1- qismidagidek fikr yuritib, quyidagilarni topamiz:

Uchburchaklar teng yonli bo‘lgani uchun:

$AN = \frac{AB}{2} = \frac{32}{2} = 16$,

$CN = \sqrt{AC^2 - AN^2} = \sqrt{65^2 - 16^2} = \sqrt{4225 - 256} = \sqrt{3969} =$

$= 63 DN = \sqrt{BD^2 - AN^2} = \sqrt{20^2 - 16^2} = \sqrt{400 - 256} =$

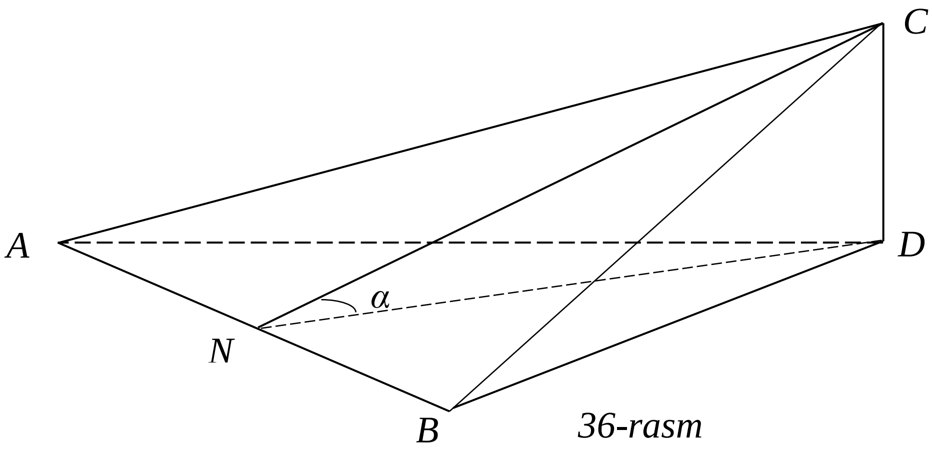
$= \sqrt{144} = 12$.

Kosinuslar teoremasiga ko‘ra:

$CD^2 = CN^2 + DN^2 - 2CN \cdot DN \cdot cos\alpha$.

$cos\alpha = \frac{1}{2} \cdot \frac{CN^2+DN^2-CD^2}{CN \cdot DN} = \frac{1}{2} \cdot \frac{63^2+12^2-63^2}{12 \cdot 63} = \frac{1}{2} \cdot \frac{144}{12 \cdot 63} = \frac{1}{2} \cdot \frac{12}{63} = \frac{2}{21}$.

Demak: $cos\alpha = \frac{2}{21}$.



36-rasm

13- masala. Agar kesishadigan ikkita tekislikning biridan olingan nuqta tekisliklar kesishgan to‘g‘ri chiziqdan ikkinchi tekislikka qaraganda ikki marta uzoqroqda joylashgan bo‘lsa, bu ikki tekislik orasidagi burchakni toping (37- rasm).

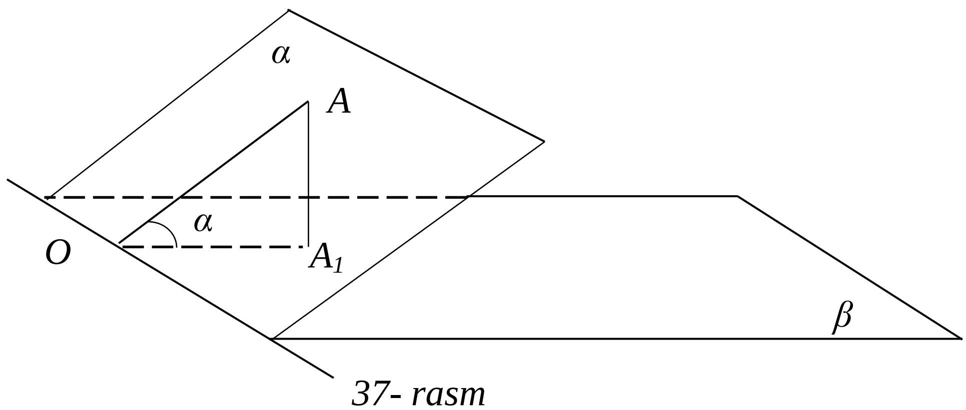
Berilgan: $OA = a,\ Oa = 2AA'$.

Topish kerak: $(\alpha \wedge \beta) - ?$

Yechish: To‘g‘ri burchakli OAA' uchburchakdan *x* burchakni topish uchun sinusning ta’rifidan foydalanamiz:

$sinx = \frac{AA`}{AO} = \frac{\frac{1}{2}a}{a} = \frac{1}{2}$, $\quad sinx = \frac{1}{2}$, $\quad x = 30°$

Demak, $(\alpha \wedge \beta) = 30°$.



37- rasm

14- masala. Tekislikdan a masofada yotgan nuqtadan tekislik bilan 45° va 30° li burchaklar, o‘zaro esa to‘g‘ri burchak tashkil etadigan ikkita og‘ma o‘tkazilgan. Og‘malarning uchlari orasidagi masofani toping (38- rasm).

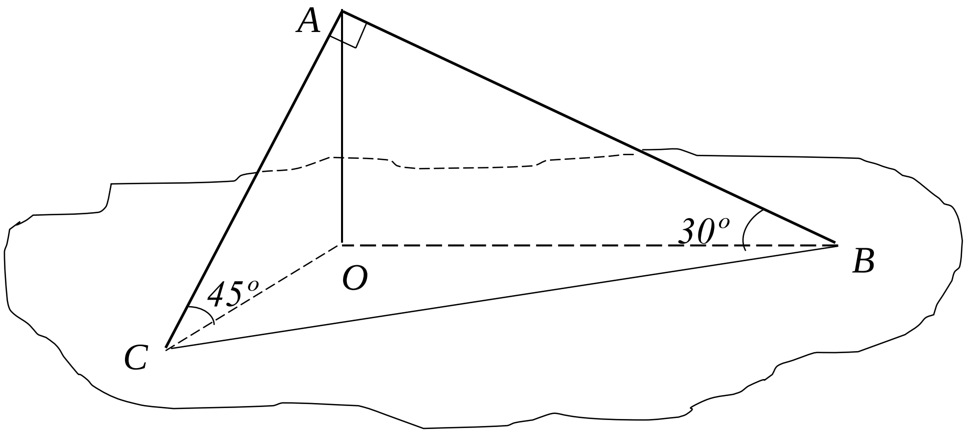
Berilgan: $OA = a,\ \ \angle OCA = 45°,\ \ \angle OBA = 30°,\ \ \angle CAB = 90°$.

Topish kerak: $CB - ?$

Yechish: Sinusning ta’rifiga ko‘ra $CA = \frac{AO}{sin45°} = a$.

Xuddi shuningdek: $BA = \frac{AC}{sin30°} = 2a$, $\angle ABC$ to‘g‘ri burchakli va CB gipotenuza bo‘lganligi uchun:

Pifagor teoremasiga ko‘ra, $CB = \sqrt{\left(\sqrt{2}a\right)^2 - (2a)^2} = a\sqrt{6}$.



38- rasm

15- masala. To‘g‘ri burchakli uchburchakning katetlari 7 m va 24 m ga teng. To‘g‘ri burchakning uchidan gipotenuza orqali o‘tuvchi va uchburchak tekisligi bilan 30° li burchak tashkil etuvchi tekislikkacha bo‘lgan masofani toping (39- rasm).

Berilgan: $\angle ABC = 90°$, $AC = 24$ m, $BC = 7$ m, α^$\Delta ABC = 30°$.

Topish kerak: $CD-$?

Yechish: Faraz qilaylik, α tekislik AB gipotenuza orqali o‘tib ABC uchburchak tekisligi bilan $\varphi = 30°$ burchak tashkil etsin. Chizmada φ burchakni hosil qilish uchun α tekislikda AB va CD to‘g‘ri chiziqlarga CN perpendikular tushiramiz, u holda $\angle CND = \varphi = 30°$ bo‘ladi. CD masofani topish uchun, CAD to‘g‘ri burchakli uchburchakning CN gipotenuzasini hisoblash kerak.

Ikkinchi tomondan CN, ABC to‘g‘ri burchakli uchburchakning balandligi hamdir. ABC to‘g‘ri burchakli uchburchakning AB gipo-tenuzasini hisoblab,

$AB = \sqrt{AC^2 + BC^2} = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25$.

ABC uchburchakning yuzini hisoblashning ikkita usulini yozamiz:

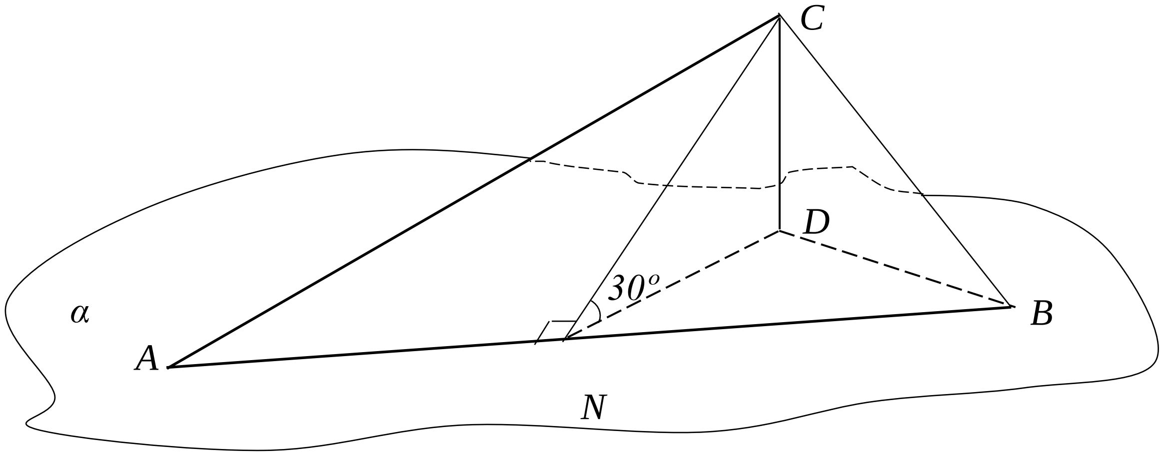
$S_{\Delta ABC} = \frac{1}{2} \cdot AC \cdot BC = \frac{1}{2} \cdot AB \cdot CN$.

Ya’ni, $\frac{1}{2} \cdot 24 \cdot 7 = \frac{1}{2} \cdot 25 \cdot CN$.

Bundan CN ni topamiz: $CN = 7 \cdot 24 : 25 = 168 : 25 = 6{,}72$.

Endi izlanayotgan CD masofani hisoblaymiz.
$CD = CNsin\varphi = CN \cdot sin30° = 6{,}72: 2 = 3{,}36$ m.

Javob: $CD = 3{,}36$ m.



39- rasm

16- masala. Agar $A(6,7,8)$, $B(8,2,6)$, $C(4,2,6)$, $D(2,8,4)$ va $M(3,5,2)$, $N(7,1,2)$, $P(3,-3,2)$, $K(-1,1,2)$ bo‘lsa, $ABCD$ va $MNPK$ to‘rtburchaklardan qaysi biri romb, qaysinisi kvadrat bo‘ladi?

Yechish: AB, AC va MN, MK vektorlarning koordinatasini topaylik:

$\overrightarrow{AB} = \overrightarrow{AB}(8-6,\ 2-7,\ 6-8) = AB(2,-5,-2)$,

$\overrightarrow{AB} = \overrightarrow{AB}(2-6,\ 8-7,\ 4-8) = AD(-4,1,-4)$,

$\overrightarrow{MN} = \overrightarrow{MN}(7-3,\ 1-5,\ 2-2) = MN(4,-4,0)$,

$\overrightarrow{MK} = \overrightarrow{MK}(-1-3,\ 1-5,\ 2-2) = MK(-4,-4,0)$.

Perpendikularlik shartidan:
$\overrightarrow{AB} \cdot \overrightarrow{AD} = 2 \cdot (-4) + (-5) \cdot 1 + (-2) \cdot (-4) = -8 - 5 + 8 = -5$.
Demak, AB va AC vektorlarning skalyar ko‘paytmasi -5 bo‘lgani uchun $ABCD$ to‘rtburchak romb.
Perpendikularlik shartidan:
$\overrightarrow{MN} \cdot \overrightarrow{MK} = 4 \cdot (-4) + (-4) \cdot (-4) + 0 \cdot 0 = -16 + 16 + 0 = 0$.
Demak, $\overrightarrow{MN}$ va $\overrightarrow{MK}$ vektorlarning skalyar ko‘paytmasi 0 bo‘lgani uchun $MNPK$ to‘rtburchak kvadrat.

17- masala. Uchta nuqta berilgan: $A(1,0,1)$, $B(-1,1,2)$, $C(0,2,-1)$.

Agar: 1) AB va CD vektorlar teng; 2) AB va CD vektorlaming yig‘indisi nolga teng ekani ma’lum bo‘lsa, $D(x,y,z)$ nuqtani toping.

Berilgan: $(1,0,1)$, $B(-1,1,2)$, $C(0,2,-1)$;

1) $AB = CD$ bo‘lganda, 2) $AB + CD = 0$ bo‘lganda.

Topish kerak: $D(x,y,z)$.

Yechish: AB va CD vektorlaming koordinatasini topaylik:
$AB = AB(-1-1,\ 1-0,\ 2-1) = AB(-2,1,1)$.
$CD = CD(x-0, y-2, z+1) = CD(x, y-2, z+1)$

1) $AB = CD$ bo‘lgani uchun AB va CD vektorlarning koordinatalari teng bo‘ladi. Bundan $x = -2$, $y - 2 = 1 \Rightarrow y = 3$, $z + 1 = 1 \Rightarrow z = 0$.

Shunday qilib, D izlanayotgan nuqtaning koordinatalari: $D(-2,3,0)$ bo‘ladi.

2) $AB + CD = 0$ bo‘lgani uchun $(O(0,0,0))$.
$x - 2 = 0$, bundan $x = 2$. $(y-2) + 1 = 0$, bundan $y = 1$,
$(z+1) + 1 = 0$, bundan $z = -2$.
Bulardan $D = D(2,1,-2)$ ekanligi kelib chiqadi.

18- masala. m va n ning qanday qiymatlarida berilgan vektorlar kollinear bo‘ladi:

1) $\vec{a}(2,n,3)$, $\vec{b}(3,2,m)$;

2) $\vec{a}(m,2,5)$, $\vec{b}(1,-1,m)$;

3) $\vec{a}(m,n,2)$, $\vec{b}(6,9,3)$.

Berilgan: 1) $\vec{a}(2,n,3)$, $\vec{b}(3,2,m)$;
2) $\vec{a}(m,2,5)$, $\vec{b}(1,-1,m)$;
3) $\vec{a}(m,n,2)$, $\vec{b}(6,9,3)$.

Topish kerak: $\vec{a} = \lambda\vec{b}$ dan foydalanib m va n ni.

Yechish: Kollinearlik shartidan mos proporsiyalami tuzamiz. Bulardan foydalanib, m va n ni tonamiz.

$$x_{\vec{a}}: x_{\vec{b}} = y_{\vec{a}}: y_{\vec{b}} = z_{\vec{a}}: z_{\vec{b}}$$

1) $2:3 = n:2 = 3:m$ shart bajariladi,

$2:3 = 3:m$ dan $m = 3\cdot 3:2 = \frac{9}{4}$,

$2:3 = n:3$ dan $n = 2\cdot 2:3 = \frac{4}{3}$;

2) $m:1 = 2:(-1) = 5:n$,

$m:1 = 2:(-1)$ dan $m = -2$,

$2:(-1) = 5:n$ dan $n = -\frac{5}{4}$;

3) $m:6 = n:9 = 2:3,$

$m:6 = 2:3$ dan $m = 6 \cdot 2:3 = 4$,

$n:9 = 2:3$ dan $n = 2 \cdot 9:3 = 6$;

19- masala. Uchta nuqta berilgan: $A(1,0,1)$, $B(-1,1,2)$, $C(0,2,-1)$. Z, o‘qida shunday $D(0,0,c)$ nuqtani topingki, bunda $\overrightarrow{AB}$, $\overrightarrow{CD}$ vektorlar perpendikular bo‘lsin.

Berilgan: $A(1,0,1)$, $B(-1,1,2)$, $C(0,2,-1)$; $\overrightarrow{AB} \perp \overrightarrow{CD}$, $D \in \overrightarrow{OZ}$.

Topish kerak: $D(0,0,c)$.

Yechish: $\overrightarrow{AB}$ va $\overrightarrow{CD}$ vektorlaming koordinatalarini topaylik. $\overrightarrow{AB} = \overrightarrow{AB}(-2,1,1)$, $\overrightarrow{CD} = \overrightarrow{CD}(0,-2,c+1)$.

Perpendikularlik shartidan:

$\overrightarrow{AB} \cdot \overrightarrow{CD} = (-2) \cdot 0 + 1 \cdot (-2) + 1 \cdot (c+1) = 0.$

Shuning uchun $c = 1$. So‘ralgan $D \in \overrightarrow{OZ}$ nuqtaning koordinatasi $D(0,0,1)$ bo‘ladi.

20- masala. $\vec{a}, \vec{b}$ vektorlar 60° li burchak tashkil etadi, $\vec{c}$ vektor esa ularga perpendikular. $\vec{a} + \vec{b} + \vec{c}$ vektorning modulini toping.

Berilgan: $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar, $(\vec{a}\hat{}\vec{b}) = 60°$, $\vec{a} \perp \vec{c}$, $\vec{b} \perp \vec{c}$

Topish kerak: $\left|\vec{a} + \vec{b} + \vec{c}\right|$–?

Yechish: Avvalo uch vektorning skalyar kvadratining umumiy formulasini yozamiz. Ikki vektor yig‘indisining kvadratini hisoblash formulasini, ikki marta qo‘llab, quyidagini hosil qilamiz:

$$\left(\vec{a} + \vec{b} + \vec{c}\right)^2 = \left(\left(\vec{a} + \vec{b}\right) + \vec{c}\right)^2 = \left(\vec{a} + \vec{b}\right)^2 + \vec{c}^2 + 2\left(\vec{a} + \vec{b}\right)\overrightarrow{\cdot c} =$$

$$= \left(\vec{a} + \vec{b} + 2 \cdot \vec{a} \cdot \vec{b}\right) + \vec{c}^2 + 2\vec{a} \cdot \vec{c} + 2\vec{b} \cdot \vec{c} =$$

$$= \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a} \cdot \vec{b} + 2\vec{a} \cdot \vec{c} + 2\vec{b} \cdot \vec{c} = \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2|\vec{a}| \cdot \left|\vec{b}\right|$$

21- masala. Birlik uzunlikdagi $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar juft-jufti bilan 60° li burchak tashkil etadi.

1) $\vec{a}$ va $\vec{b} + \vec{c}$;

2) $\vec{a}$ va $\vec{b} - \vec{c}$ vektorlar orasidagi burchakni toping.

Berilgan: $\vec{a}$, $\vec{b}$, $\vec{c}$, $(\vec{a}\hat{}\vec{b}) = 60°$, $(\vec{a}\hat{}\vec{c}) = 60°$, $(\vec{b}\hat{}\vec{c}) = 60°$.

Topish kerak: 1) $(\vec{a}\hat{}(\vec{b} + \vec{c})) = 60°$

2) $(\vec{a}\hat{}(\vec{b} - \vec{c})) = 60°$

Yechish: Barcha vektorlaming juft-jufti bilan skalyar ko‘paytmasi $(\vec{a}\cdot\vec{b})$, $(\vec{a}\cdot\vec{c})$, $(\vec{b}\cdot\vec{c})$ ga teng va $\left(\vec{a}\cdot\vec{b}\right)=1\cdot 1\cdot cos60°=\frac{1}{2}$.

1) $\vec{a}\cdot(\vec{b}+\vec{c})=(\vec{a}\cdot\vec{b})+(\vec{a}\cdot\vec{c})=\frac{1}{2}+\frac{1}{2}=1$,

$\left|\vec{b}+\vec{c}\right|=\sqrt{\vec{b}^2+\vec{c}^2+2\cdot\vec{b}\cdot\vec{c}}=\sqrt{1+1+2\cdot\frac{1}{2}}=\sqrt{3}$,

$cos\varphi=\frac{\vec{a}\cdot(\vec{b}+\vec{c})}{|\vec{a}|\cdot\left|\vec{b}+\vec{c}\right|}=\frac{1}{1\cdot\sqrt{3}}=\frac{1}{\sqrt{3}}$.

2) $\vec{a}\cdot\left(\vec{b}-\vec{c}\right)=\left(\vec{a}\cdot\vec{b}\right)-(\vec{a}\cdot\vec{c})=0$, bundan. (chunki $\left(\vec{a}\cdot\vec{b}\right)=(\vec{a}\cdot\vec{c})$)

$cos\varphi=\frac{\vec{a}\cdot(\vec{b}+\vec{c})}{|\vec{a}|\cdot\left|\vec{b}+\vec{c}\right|}=0,\quad \varphi=90°$.

22- masala. Uchta nuqta berilgan: $A(0,1,-1)$, $B(1,-1,2)$, $C(3,1,0)$. ACB uchburchakning ACB burchagining kosinusini toping.

Berilgan: $A(0,1,-1)$, $B(1,-1,2)$, $C(3,1,0)$.

Topish kerak: $\angle ACB-?$

Yechish: $\angle C=\angle ACB$ burchak $\overrightarrow{CA}=\vec{a}$ va $\overrightarrow{CB}=\vec{b}$ vektorlar orasidagi burchakka teng. $\vec{a}$ va $\vec{b}$ vektorlarni topamiz.

$\vec{a}=\vec{a}(0-3,1-1,-1-0)=(-3,0,-1)$

$\vec{b}=\vec{b}(1-3,-1-1,2-0)=(-2,-2,2)$.

Bundan $|\vec{a}|=\sqrt{(-3)^2+(-1)^2}=\sqrt{10}$,

$\left|\vec{b}\right|=\sqrt{(-2)^2+(-2)^2+2^2}=\sqrt{12}=2\sqrt{3}$,

$\vec{a}\cdot\vec{b}=6+0-2=4,\qquad cos\angle ACB=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot\left|\vec{b}\right|}=\sqrt{\frac{2}{15}}$.

23- masala. ABC uchburchakning A uchidan uchburchak tekisligiga AD perpendikular chiqarilgan. Agar ABD burchak α ga, ABC burchak esa β ga teng bo‘lsa, $\overrightarrow{BC}$ va $\overrightarrow{BD}$ vektorlar orasidagi φ burchakning kosinusini toping.

Berilgan: $\angle ABC=\beta$, $\angle ABD=\alpha$, $AD\perp(\Delta ABC)$.

Topish kerak: $\angle DBC=\varphi-?$

Yechish: Masalaning berilishida keltirilgan. D nuqtadan BC tomonga $DE\perp BC$ perpendikular tushiramiz va BD tomonni m ga teng deb olamiz. U holda BAD uchburchakdan BA tomonni topib,

$BA = BD \cdot cos\angle ABC = m \cdot cos\alpha$ va ΔBAE dan BE tomonni topamiz:

$BE = BA \cdot cos\beta = m \cdot cos\alpha \cdot cos\beta$.

Bundan ko‘rinadiki, $AE \perp BC$ bo‘lgani uchun

$cos\varphi = (BE\char`^BD) = m \cdot cos\alpha \cdot cos\beta : m = cos\alpha \cdot cos\beta.$

Javob: $cos\varphi = cos\alpha \cdot cos\beta.$

24- masala. Og‘ma tekislik bilan 45° li burchak tashkil etadi. Og‘ma asosidagi tekislikda og‘maning proyeksiyasiga 45° burchak ostida to‘g‘ri chiziq o‘tkazilgan. Shu to‘g‘ri chiziq bilan og‘ma orasidagi φ burchakni toping.

Berilgan: $\angle ACB = 45°$, $\angle BCA_1 = 45°$.

Topish kerak: $\angle ACA_1 = \varphi -?$

Yechish: AC og‘ma bo‘lsin, BC — uning proyeksiyasi, CD proyeksiyalar tekisligida yotgan to‘g‘ri chiziq va BC to‘g‘ri chiziq bilan 45° li burchak tashkil etadi. $\angle ACD = \varphi$ burchakni topish talab qilinmoqda. Bu burchakni to‘g‘ri burchakli ACA_1 uchburchakning burchagi sifatida qarash mumkin. Bu yerda $AA_1 \perp CD$. Agar deb olsak, $\angle ACB = \angle A_1CB = 45°$, shartdan quyidagilarni topamiz:

$BC = AC \cdot cos\angle ACB = a \cdot cos45° = \frac{a}{\sqrt{2}}$,

$A_1C = BC \cdot cos\angle BCA_1 = \frac{a}{\sqrt{2}} \cdot cos45° = \frac{a}{2}$.

(Uch perpendikular haqidagi teoremaga ko‘ra $BA_1 \perp CD$, demak BCA_1 uchburchak ham to‘g‘ri burchakli bo‘ladi.)

Shunday qilib, $\varphi -$ to‘g‘ri burchakli ACA_1 uchburchakning burchagi, $AC = a$ uning gipotenuzasi va φ burchakka yopishgan kateti $A_1C = \frac{|a|}{2}$.

$cos\varphi = \frac{A_1C}{AC} = \frac{\frac{|a|}{2}}{|a|} = \frac{1}{2}$ va so‘ralgan burchak $\varphi = 60°$.

25- masala. Qanday shart bajarilganda $ax + by + cz + d = 0$ tenglama bilan berilgan tekislik OXY tekislikka parallel bo‘ladi?

Berilgan: $(\alpha): ax + by + cz + d = 0.$

Topish kerak: $a-?$, $b-?$, $c-?$ $\alpha \| OXY$.

Yechish: Berilgan tekislikni α deb olaylik. Agar α tekislik OXY tekisligiga parallel bo‘lsa, u holda Z o‘qiga va $\vec{e}(0, 0, 1)$ ga perpendikular bo‘ladi. **4.1-** teoremaga ko‘ra bir tekislikka perpendikular ikki to‘g‘ri chiziq parallel, bunda $\vec{n}(a,b,c)$ vektorlar α tekislikka perpendicular va $\overrightarrow{e_3}$ bilan kollinear bo‘lishi kerak.

Demak, $\vec{n}=\lambda\overrightarrow{e_3}(0,0,\lambda)$ $\alpha\|xy$ tekislik tenglamasi $a=b=0$ shartni qanoatlantirishi kerak, bundan ko‘rinadiki, tekislik tenglamasi $cz+d=0$ ko‘rinishda bo‘lishi kerak va $c\neq 0$. Xuddi shuningdek, $\alpha\|xy$ da koordinatalar boshi $O(0,0,0)$ nuqtadan o‘tmaydi. Shuning uchun $c\neq 0$.

Shunday qilib, agar $\alpha\|xy$ bo‘lsa, tekislik tenglamasida $a=b=0$, $c\neq 0$ va $d\neq 0$ bo‘lishi kerak.

26- masala. A nuqtadan o‘tib, AB to‘g‘ri chiziqqa perpendikular bo‘lgan tekislik tenglamasini tuzing, bunda:

1) $A(-1,1,2)$, $B(2,0,1)$,
2) $A(1,0,-1)$, $B(4,3,-3)$,
3) $A(3,-4,5)$, $B(2,1,2)$.

Berilgan: 1) $A(-1,1,2)$, $B(2,0,1)$,
2) $A(1,0,-1)$, $B(4,3,-3)$,
3) $A(3,-4,5)$, $B(2,1,2)$.

Topish kerak: $A\in\alpha$, $AB\perp\alpha$, α — tekislikni.

Yechish: Agar $AB=AB(a,b,c)$, $A(x_0,y_0,z_0)$ bo‘lsa, u holda izlanayotgan tenglama $a(x-x_0)+b(y-\ y_0)+c(z-\ z_0)=0$ ko‘rinishda bo‘ladi.

Bu tenglamadan foydalanib, tekislikni topamiz.

1. $AB=AB(2+1,\ 0-1,\ 1-2)=AB(3,-1,-1)$ va $A(-1,\ 1,\ 2)$ tenglama:

$3(x+1)-(y-\ 1)-(z-\ 2)=0$ yoki $3x-y-z+6=0$.

2. $AB=AB(3,\ 3,\ -2)$ va $A(1,\ 0,-1)$ tenglama:
$3(x-1)+3(y-0)-2(z-1)=0$ yoki $3x+3y-2z-5=0$.

3. $AB=AB(-1,\ 5,\ -3)$ va $A(3,\ -4,\ 5)$ tenglama:
$(3-x)+5(y+4)-3(z-5)=0$ yoki $-x+5y-3z+38=0$.

27- masala. $ax+by+cz+d=0$ tekislikning koordinata o‘q-laridan ajratgan kesmalami toping: bunda $a,\ b,\ c,\ d$ lar nolga teng emas.

Berilgan: α — tekislik, $a,\ b,\ c,\ d\neq 0$.

Topish kerak: $OX\cap\alpha$, $OY\cap\alpha$, $OZ\cap\alpha$.

Yechish: Tekislikning x o‘qi bilan kesishgan nuqtasini topaylik. Bu nuqta $x(x,0,0)$, tenglamani qanoatlantiradi, shuning uchun

$ax + d = 0$, bundan $x = -\frac{d}{a}$.

Bundan ko‘rinadiki, y va z o‘qlarini kesishda kesmalarning uzunliklari: $y = \left|\frac{d}{a}\right|$, $z = \left|\frac{d}{c}\right|$

28- masala. $ax + by + cz + d = 0$, $ax + by + cz + d_1 = 0$ tenglamalar bilan berilgan tekisliklar $d \neq d_1$ shart bajarilganda, umumiy nuqtaga ega bo‘lmasligini isbotlang.

Isboti: Agar tekisliklar umumiy nuqtaga ega bo‘lganda, ular umumiy (x_0, y_0, z_0) yechimga ega bo‘lgan bo‘lar edi. U holda bir tenglamadan ikkinchi tenglamani ayirib, $d - d_1 = 0$ tenglikni hosil qilamiz: $d \neq d_1$ bu shartga ziddir.

29- masala. Quyidagi tenglamalar bilan berilgan uchta tekislikning kesishish nuqtasini toping:

$x + y + z = 1$, $x - 2y = 0$, $2x + y + 3z = 0$,

Yechish: Bu masalani yechish berilgan uchta tenglamalar sistemasini yechishga olib keladi.

Birinchi va ikkinchi tenglamalardan x va z larniy orqali ifodalaymiz:

$x = 2y$, $z = 1 - x - y = 1 - 3y$, uchinchi tenglamaga x va z larning qiymatlarini qo‘yib topamiz:

$2(2y) + y + 3(1 - 3y) + 1 = 0, \quad -4y + 4 = 0, \quad y = 1.$

U holda $x = 2y = 2$, $z = 1 - 3y = -2$.

Javob: $M(2,\ 1, -2)$.

30- masala. Qanday shart bajarilganda tenglama bilan berilgan tekislik: 1) z o‘qiga parallel bo‘ladi, 2) z o‘qi orqali o‘tadi.

Yechish: Agar $\alpha: ax + by + cz + d = 0$ tekislik faqat va faqatgina ixtiyoriy $\vec{A}(x,\ y,\ z)$ nuqta bilan birga $B(x,\ y, z + 1)$ nuqta orqali o‘tgandagina, bu tekislik z o‘qiga parallel yoki z o‘qi orqali o‘tadi.

Bunday ikki nuqtadan o‘tuvchi AB to‘g‘ri chiziq z o‘qiga parallel, bundan $\alpha \| OZ$. U holda $ax + by + cz + d = 0$ va $c = 0$ bo‘lgandagina bir vaqtning o‘zida bajarilishi mumkin. (bir xil x, y, z uchun).

Demak, α tekislik tenglamasi $ax + by + d = 0$ ko‘rinishda bo‘lishi kerak. α tekislik OZ o‘qini kesib o‘tmasligi uchun, ya’ni unga parallel bo‘lishi uchun $d \neq 0$ bo‘lish yetarli. Agar $d = 0$

bo‘lsa, α tekislik tenglamasi $ax + by = 0$ ko‘rinishni oladi va $OZ \in \alpha$ bo‘ladi.

Javob: 1) $c = 0,\ d \neq 0$, 2) $c = d = 0$.

Ikkala holda ham a va b ning kamida bittasi noldan farqli bo‘lishi kerak.

MUSTAQIL ISHLASH UCHUN

1. $A(1{,}2{,}3)$ nuqta berilgan. Bu nuqtadan koordinata o‘qlarigacha va koordinata tekisliklariga tushirilgan perpendikularlar asoslarini toping.

2. $A(0{,}0{,}1)$, $B(0{,}1{,}0)$, $C(0{,}0{,}1)$ nuqtalardan bir xil masofada va YZ tekisligidan 2 birlik masofada yotuvchi nuqtalarni toping.

3. Uchlari: 1) $A(6{,}7{,}8)$, $B(8{,}2{,}6)$, $C(4{,}3{,}2)$, $D(2{,}8{,}4)$;
2) $A(0{,}2{,}0)$, $B(1{,}0{,}0)$, $C(2{,}0{,}2)$, $D(1{,}2{,}2)$ nuqtalar bo‘lgan $ABCD$ to‘rtburchakning romb ekanligini isbotlang.

4. Uchlari $A(a, c, -b)$ va $B(-a, d, b)$ nuqtalarda bo‘lgan kesmaning o‘rtasi Y o‘qida yotishini ko‘rsating.

5. $A(1, 2, 3)$, $B(0, -1, 2)$, $C(1, 0, -3)$ nuqtalar berilan. Bu nuqtalarga koordinatalar boshiga nisbatan simmetrik nuqtalarni toping.

6. A nuqta, B nuqta, C nuqta D nuqtaga o‘tadigan parallel ko‘chirish mavjudmi? Bunda:

1) $A(2, 1, 0)$, $B(1, 0, 1)$, $C(3, -2, 1)$, $D(2, -3, 0)$;
2) $A(-2, 3, 5)$, $B(1, 2, 4)$, $C(4, -3, 6)$, $D(7, -2, 5)$;
3) $A(0, 1, 2)$, $B(-1, 0, 1)$, $C(3, -2, 2)$, $D(2, -3, 1)$;
4) $A(1, 1, 0)$, $B(0, 0, 0)$, $C(-2, 2, 1)$, $D(1, 1, 1)$

7. Tekislikdan a masofada yotgan nuqtadan tekislik bilan 45° li va o‘zaro 60° li burchak tashkil etgan ikkita og‘ma o‘tkazilgan. Og‘malarning uchlari orasidagi masofani toping.

8. Teng yonli to‘g‘ri burchakli uchburchakning bir kateti orqali ikkinchi katetiga 45° burchak ostida tekislik o‘tkazilgan. Gipotenuza bilan tekislik orasidagi burchakni toping.

9. Tekislikdan a masofada yotuvchi nuqtada tekislikka 30° li burchak ostida ikkita og‘ma o‘tkazilgan bo‘lib, ularning proyeksiyalari 120° li burchak tashkil etadi. Og‘maning uchlari orasidagi masofani toping.

10. n ning qanday qiym atlarida berilgan vektorlar perpendikular bo‘ladi.

1) $\vec{a}(2,-1,3)$, $\vec{b}(1,3,n)$; 3) $\vec{a}(n,-2,1)$, $\vec{b}(n,2n,4)$;

2) $\vec{a}(n,-2,1)$, $\vec{b}(n,-n,1)$; 4) $\vec{a}(4,2n,-1)$, $\vec{b}(-1,1,n)$.

11. Ikkita nuqta berilgan: $A(1,0,2)$ va $B(-1,1,1)$, AB vektorga kollinear va y bilan bir xil yo‘nalgan $\vec{e}(a,b,c)$ birlik vektorning koordinatalarini toping.

12. $P(k,\ l,\ m)$ nuqta berilgan. Koordinatalar boshi O orqali o‘tib, OP to‘g‘ri chiziqqa perpendikular bo‘lgan tekislik tenglamasini toping.

13. To‘g‘ri chiziq $2x+3y+z=1$, $x+y+z=1$

tekisliklarning kesishmasidan iborat. Shu to‘g‘ri chiziqqa parallel biror vektorni ko‘rsating.

14. Tekislik tashqarisidagi nuqtadan perpendikular va y bilan α burchak tashkil etuvchi ikkita teng og‘ma o‘tkazilgan. Og‘malar orasidagi burchak β ga teng, proyeksiyalar orasidagi φ burchakni toping.

15. $\vec{a}(2,1,-2)$ vektorga kollinear birlik vektorni toping.

16. Qanday shart bajarilganda $\vec{a}(a_1,a_2,a_3)$ vektor Z o‘qqa parallel bo‘ladi?

17. $a_1x+b_1y=d_1$, $a_2x+b_2y=d_2$ tenglamalar bilan berilgan tekisliklarning kesishish to‘g‘ri chizig‘i z o‘qiga parallel ekanini isbotlang.

18. $ax+by+cz+d=0$ tekislikka parallel har qanday tekislikning $ax+by+cz+d_1=0$ ko‘rinishdagi tenglama bilan ifodalanishini isbotlang.

19. Tekislik $ax+by+cz+d=0$ tenglama bilan berilgan. $P(k,\ l,\ m)$ nuqtadan va koordinatalar boshidan o‘tuvchi to‘g‘ri chiziq tekislikka perpendikular bo‘lishi uchun $P(k,\ l,\ m)$ nuqtaning koordinatalari qanday shartni qanoatlantirishi kerak?

20. $x+y+z=1$, $2x+y+3z+1=0$, $x+2z+1=0$ tenglamalar bilan berilgan tekislik bitta ham umumiy nuqtaga ega emas ekanligini isbotlang.

21. Qanday shart bajarilganda, $ax+by+cz+d=0$

tenglama bilan berilgan tekislik xy tekislikka perpendikular boladi?

22. Tekislik $2x + 3y + z = 1$ tenglama bilan berilgan. Tekislikka parallel birorta vektorni ko‘rsating.

V. KO‘PYOQLAR

Ko‘pyoqlarning sirtlari haqida ayrim formulalarni keltiramiz.

1. Prizma (to‘g‘ri prizma):

a) $S_{yon\,s} = pl,\ p$ — asosining perimetri, l — yon qirrasi.

b) $S_{t\,s} = S_{yon\,s} + 2S_{asos} = pl + 2S_{asos}$

2. Parallelepiped (to‘g‘ri burchakli):

$d^2 = a^2 + b^2 + c^2$, d — diagonali, $a,\ b,\ c$ — chiziqli o‘lchovlari.

3. Piramida (muntazam):

a) $S_{yon\,s} = \frac{P \cdot f}{2}$, P — asosining perimetri, f — apofemasi.

b) $S_{t\,s} = S_{yon\,s} + S_{asos} = \frac{P \cdot f}{2} + S_{asos}$

1- masala. To‘g‘ri burchakli parallelepipedda $AB = 4$, $AD = 3$ va $AA_1 = 5$ bo‘lsa, DBD_1 burchakni toping(40- rasm).

Berilgan: $ABCDA_1B_1C_1D_1$ — to‘g‘ri burchakli parallelepiped, $AB = 4$, $AD = 3$ va $AA_1 = 5$.

Topish kerak: $\angle DBD_1 = \alpha - ?$

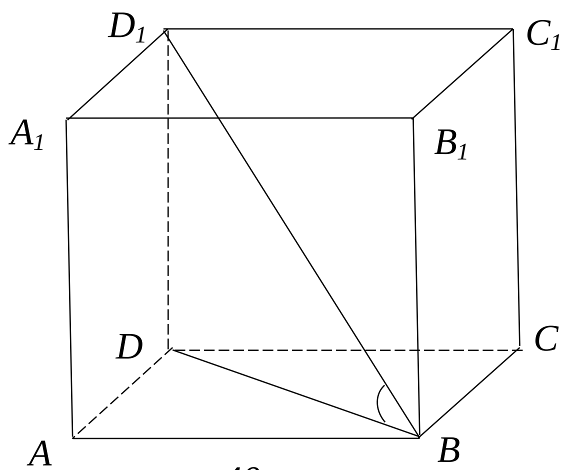
Yechish: ΔABD va ΔDBD_1 — to‘g‘ri burchakli uchburchak.

ΔABD dan: $BD = \sqrt{AB^2 + AD^2} =$
$= \sqrt{16 + 9} = \sqrt{25} = 5$,

$AA_1 = DD_1 = 5$,

$tg\alpha = \frac{DD_1}{BD} = \frac{5}{5} = 1,\ \alpha = 45°$.

Javob: $\alpha = 45°$.



40- rasm

2- masala. To‘g‘ri parallelepiped asosining tomonlari 3 sm va 5 sm, asosining diagonallaridan biri 4 sm ga teng. Parallelepiped kichik diagonallaridan biri asos tekisligi bilan 60° li burchak tashkil etadi. Uning diagonallari uzunligini toping (41- rasm).

Berilgan: $ABCDA_1B_1C_1D_1$ — to‘g‘ri burchakli parallelepiped, $AB = 5$, $AD = 3$ va $BD = 4$.

Topish kerak: $AC_1 - ?$, $BD_1 - ?$

Yechish: Asos parallelogrammdir,

Kosinuslar teoremasiga ko‘ra

ΔABD, dan $BD^2 = AD^2 + AB^2 - 2AD \cdot AB \cdot cos\alpha$,

$4^2 = 5^2 + 3^2 - 2 \cdot 5 \cdot 3 \cdot cos\alpha$,

$-18 = -30 \cdot cos\alpha$,

$cos\alpha = {}^3/_5$,

$\angle ABC = \cos(\pi - \alpha) = -cos\alpha = -\frac{3}{5}$,

ΔABC, dan $AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos(\pi - \alpha)$,

$AC^2 = 5^2 + 3^2 + 2 \cdot 5 \cdot 3 \cdot \frac{3}{5}$,

$AC = 2\sqrt{13}$,

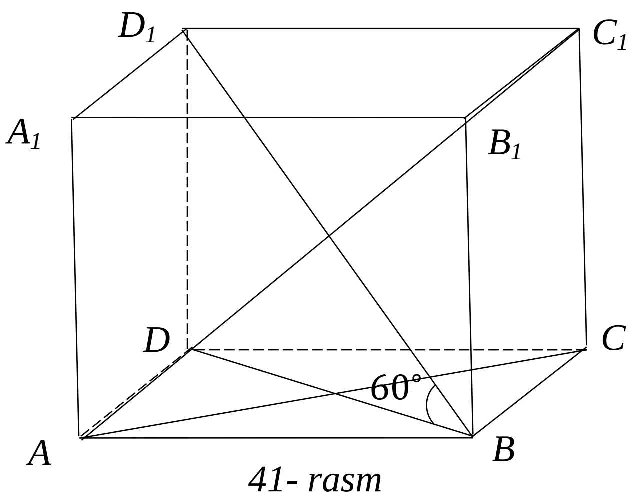
$cos60^\circ = \frac{BD}{BD_1}$, $\frac{1}{2} = \frac{4}{BD_1}$, $BD_1 = 8$.

$sin60^\circ = \frac{DD_1}{BD_1}$, $\frac{\sqrt{3}}{2} = \frac{DD_1}{8}$, $DD_1 = 4\sqrt{3}$.

$DD_1 = CC_1$ va ΔACC_1 — to‘g‘ri burchakli uchburchak bolganidan

ΔACC_1dan: $AC_1 = \sqrt{AC^2 + {CC_1}^2} = \sqrt{52 + 48} = \sqrt{100} = 10$.

Javob: $BD_1 = 8$, $AC_1 = 10$.



41- rasm

3-masala. Uchburchakli to‘g‘ri prizmaning balandligi 50 sm, asosining tomonlari 40 sm, 13 sm va 37 sm. Prizmaning to‘la sirtini toping (42- rasm).

Berilgan: $ABCA_1B_1C_1$ — to‘g‘ri prizma, $AB = 40$ sm, $BC = 13$ sm, $AC = 37$ sm va $AA_1 = h = 50$ sm.

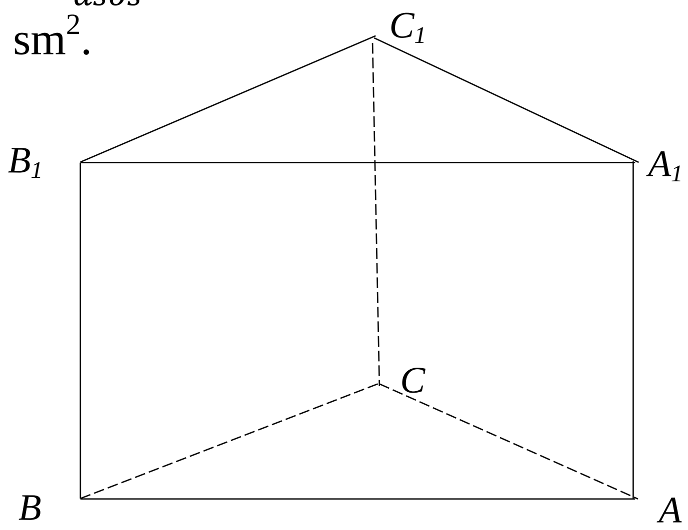
Topish kerak: $S_{\text{to‘la s}}$ −?

Yechish: To‘g‘ri prizmaning yon yog‘i yoyilmasi to‘g‘ri to‘rtburchakligidan $S_{yon\ s} = (AB + BC + AC) \cdot AA_1 = p \cdot h$

$S_{yon\ s} = (40 + 13 + 37) \cdot 50$ =4500 sm^2.

Asosining yuzi $S_{\Delta ABC} = \sqrt{p(p-a)(p-b)(p-c)} =$
$= \sqrt{45(45-40)(45-13)(45-37)} = \sqrt{45 \cdot 5 \cdot 32 \cdot 8} = 240$ sm^2.
$S_{\text{to‘la s}} = S_{yon\ s} + 2S_{asos} = 4500 + 480 = 4980$ sm^2.

Javob: 4980 sm^2.



42-rasm

4- masala. $AA_1 = 3$, $BB_1 = 4$, $A_1B_1 = 6$, $AB = 7$ bo‘lganda, α ikki yoqli burchakni toping (43- rasm).

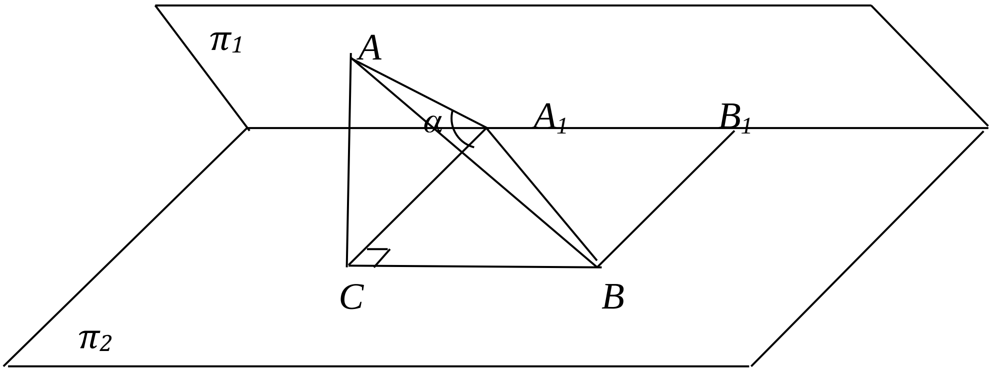
Berilgan: $AA_1 \perp A_1B_1$, $BB_1 \perp A_1B_1$, $AA_1 = 3$ sm, $BB_1 = 4$ sm, $A_1B_1 = 6$ sm, $AB = 7$ sm.

Topish kerak: π_1^π_2 tekisliklar orasidagi burchak α−?

Yechish: $A_1C \perp A_1B_1 \Rightarrow \alpha = (\pi_1$^$\pi_2) = \angle AA_1C$, $A_1C \| BB_1$, $BC \| A_1B_1 \Rightarrow A_1C = BB_1 = 4$ sm, $BC = A_1B_1 = 6$ sm.

$AA_1 \perp A_1B_1$, $A_1C \perp A_1B_1$ va $A_1B_1 \| BC \Rightarrow$ BC $\perp (ACA_1) \Rightarrow$ $AC \perp BC$.

Demak, ABC uchburchak to‘g‘ri burchakli:
$AC = \sqrt{AB^2 - BC^2} = \sqrt{49 - 36} = \sqrt{13}$.
Kosinuslar teoremasiga ko‘ra (ΔACA_1 uchun):
$AC^2 = {AA_1}^2 + A_1C^2 - 2 \cdot AA_1 \cdot A_1C \cdot cos\alpha$,
$13 = 9 + 16 - 2 \cdot 3 \cdot 4 \cdot cos\alpha$, $13 = 25 - 24 \cdot cos\alpha$,
$cos\alpha = \frac{12}{24} = \frac{1}{2} = 60°$.



43- rasm

5- masala. Uchyoqli burchakning ikkita yassi burchaklari o‘tkir va α ga teng, uchinchi burchagi esa γ ga teng. Yassi α burchaklar qarshisida yotgan ikkiyoqli φ burchaklarni γ burchak yotgan tekislik bilan qarshisidagi qirra orasidagi β burchakni toping (44-rasm).

Berilgan: $\angle BAC = \gamma$, $\angle BAD = \angle DAC = \alpha$.

Topish kerak: 1) $(\pi_2,\ \pi_1) = (\pi_3,\ \pi_1) = \varphi -?$

2) $(\pi_1,\ AD) = \beta -?$

Yechish: AD qirrada ixtiyoriy bir nuqta, masalan, E ni tanlab, undan EO perpendikular tushiramiz. E nuqtadan AB va AC qirralarga EN va EM perpendikular o‘tkazamiz. U holda $\angle ENO = \angle EMO = \varphi$ pekanligi ko‘rinadi (buni ko‘rsatish o‘quvchiga havola).

$\angle ENO = \angle EMO$, chunki $\angle ENO = \angle EMO$, EO — urnumiy.

Demak, $NO = MO$, $\Delta NOA = \Delta MOA$, chunki, $NO = MO$, AO — umumiy va uchburchaklar to‘g‘ri burchakli.

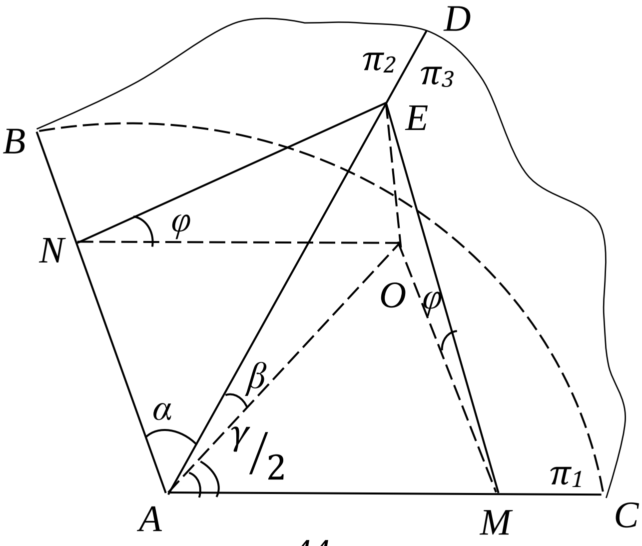
Demak, $NA = MA$ va $\angle NAO = \angle MAO = \frac{\gamma}{2}$.

Yangi parametr kiritamiz: $NA = a$, u holda ΔNOA dan:

$AO = \frac{a}{cos\frac{\gamma}{2}}$, $= atg\frac{\gamma}{2}$, ΔENA dan: $AE = \frac{a}{cos\alpha}$, $NE = atg\alpha$

ΔENO dan: $ctg\varphi = \frac{NO}{NE} = \frac{tg\frac{\gamma}{2}}{tg\alpha}$.

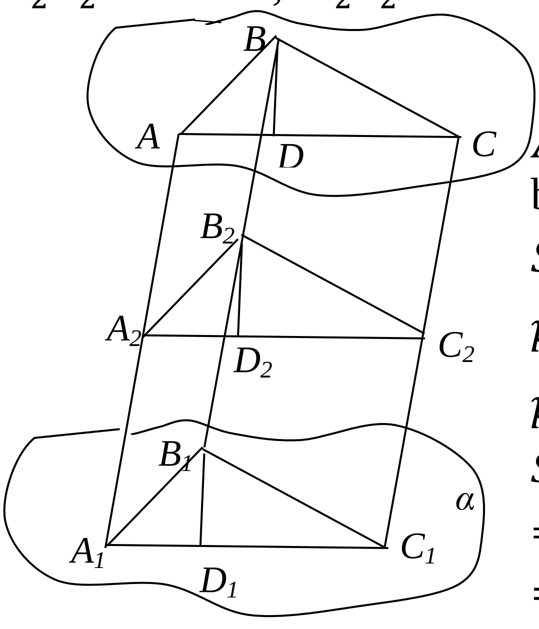
ΔEAO dan: $cos\beta = \frac{AO}{AE} = \frac{cos\alpha}{cos\frac{\gamma}{2}}$.



44- rasm

6- masala. Uchburchakli og‘ma prizmada yon qirralar orasidagi masofalar 37 sm, 13 sm va 40 sm ga teng. Katta yon yog‘i bilan uning qarshisidagi yon qirra orasidagi masofani toping (45- rasm).

Berilgan: $ABCA_1B_1C_1$ — og‘ma prizma. Bundan: $A_2B_2 \perp BB_1$, AA_1, $B_2C_2 \perp BB_1$, CC_1, $A_2C_2 \perp CC_1$, AA_1. $A_2C_2 = 40$ sm, $A_2B_2 = 13$ sm, $B_2C_2 = 37$ sm.



45- rasm

Topish kerak: $\rho(ACC_1A_1, BB_1)$−?

Yechish: Qidirilayotgan masofa $A_2B_2C_2$ uchburchakning B_2D_2 balandligidir. Shuning uchun

$S_{\Delta A_2B_2C_2} = \sqrt{p(p-a)(p-b)(p-c)}$,

$p = \frac{a+b+c}{2} \Rightarrow$

$p = \frac{37+40+13}{2} = \frac{90}{2} = 45$,

$S_{\Delta A_2B_2C_2} =$

$= \sqrt{45(45-37)(45-40)(45-13)} =$

$= \sqrt{45 \cdot 8 \cdot 5 \cdot 32} = 15 \cdot 16 = 240$

$S_{\Delta A_2B_2C_2} = 240$, $S_{\Delta A_2B_2C_2} = \frac{1}{2} \cdot A_2C_2 \cdot B_2D_2$,

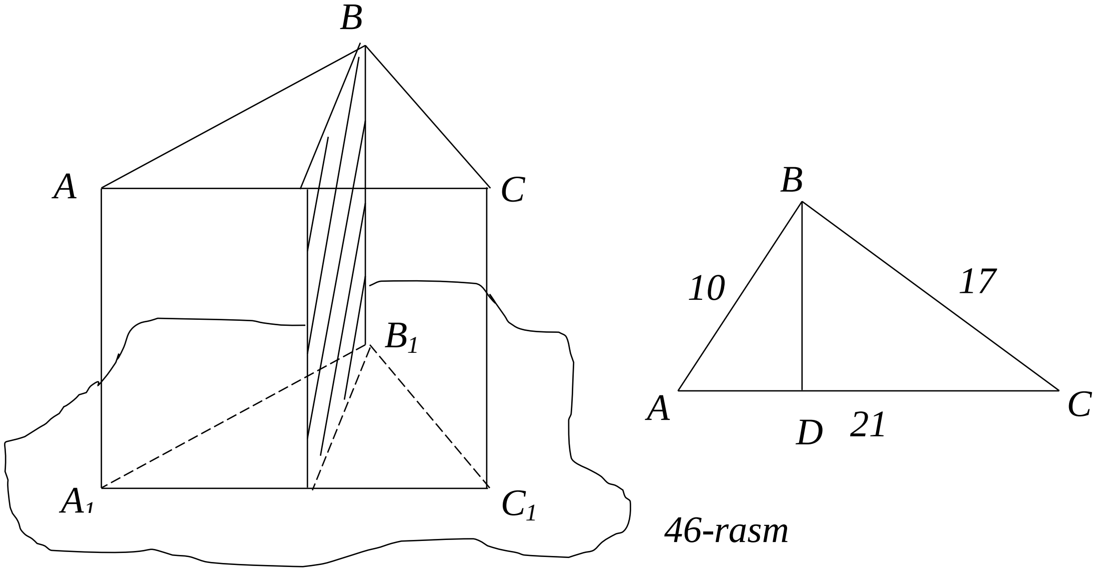
$240 = \frac{1}{2} \cdot B_2D_2$, $B_2D_2 = \frac{240}{20}$, $B_2D_2 = 12$ sm.

Demak, katta yon yog‘i bilan uning qarshisidagi, yon qirra orasidagi masofa $B_2D_2 = 12$ sm.

7- masala. Uchburchakli to‘g‘ri prizmada asosining tomonlari 10 sm, 17 sm va 21 sm, prizmaning balandligi esa 18 sm ga teng. Yon qirra va asosning kichik balandligi orqali o‘tkazilgan kesimning yuzini toping (46- rasm).

Berilgan: $AB = 10$ sm , $BC = 17$ sm, $AC = 21$ sm
$AA_1 = BB_1 = CC_1 = h = 18$ sm.

Topish kerak: $S_{kesim}-?$



46-rasm

Yechish: BB_1D_1D — to‘g‘ri burchakli to‘rtburchak, chunki prizma to‘g‘ri. $S_{BB_1D_1D} = BD \cdot 18,\ p = \frac{a+b+c}{2} = p = \frac{10+17+21}{2} = 24,$

$$S_{\Delta A_2B_2C_2} = \sqrt{p(p-a)(p-b)(p-c)} =$$

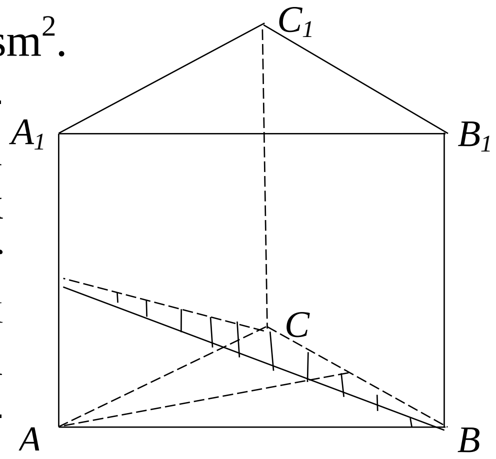
$$= \sqrt{24(24-17)(24-10)(24-21)} = \sqrt{24 \cdot 7 \cdot 14 \cdot 3} = 84 \text{ sm}^2.$$

$S = \frac{1}{2} \cdot AC \cdot BD,\ 84 = \frac{1}{2} \cdot 21 \cdot BD,$

$BD = \frac{2 \cdot 84}{21} = \frac{168}{21} = 8$ sm,

$S_{BB_1D_1D} = BD \cdot BB_1 = 8 \cdot 18 = 144$ sm^2.

8- masala. Uchburchakli muntazam prizma ostki asosining tomoni orqali yon yoqlari bilan α burchak tashkil etuvchi to‘g‘ri chiziqlar bo‘yicha kesib o‘tuvchi tekislik o‘tkazilgan. Bu tekislikning prizma asosiga og‘ish burchagini toping (47- rasm).



47- rasm

Berilgan: $ABCA_1B_1C_1$ — muntazam prizma, $BD\hat{}CD = \angle BCD = = \alpha$.

Topish kerak: $(BCD\hat{}ABC) = x-?$

Yechish: $DE \perp BC$ o‘tkazamiz. E nuqta BC tomon o‘rtasida bo‘ladi, chunki ABC uchburchakning muntazamligidan $BD = CD$, yangi $AB = a$ parametr kiritamiz. U holda: $BE = \frac{a}{2}$, $AE = \frac{\sqrt{3}a}{2}$ (muntazam uchburchak balandligi),

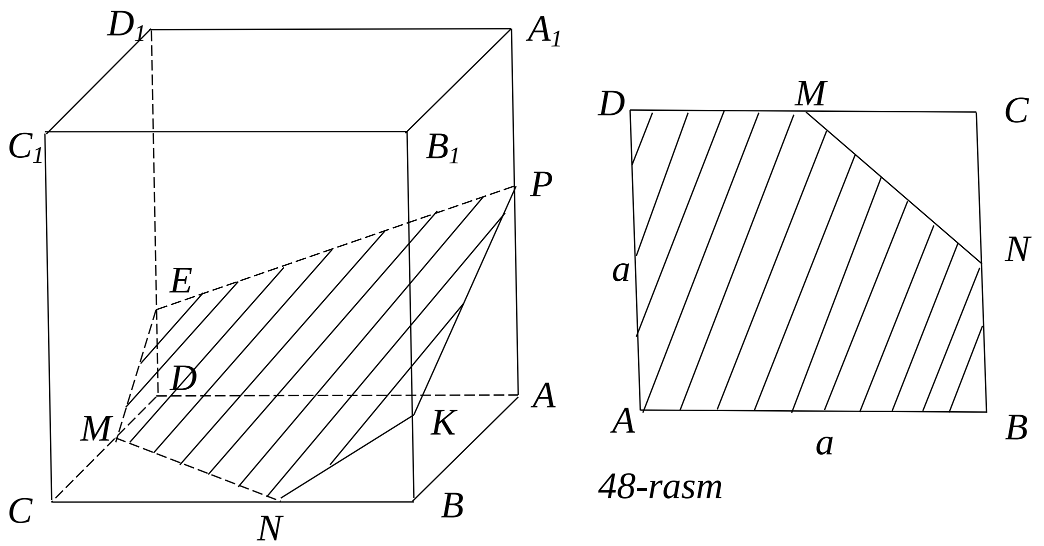
$(BCD\hat{}ABC) = x-?$

ΔBED dan: $DE = \frac{a}{2tg\frac{\alpha}{2}}$.

ΔBED dan: $cosx = \frac{AE}{DE} = \frac{\sqrt{3}a}{2} : \frac{a}{2tg\frac{\alpha}{2}} = \sqrt{3}tg\,\frac{\alpha}{2}$.

Javob: $x = \arccos(\sqrt{3}tg\,\frac{\alpha}{2})$.

9- masala. To‘rtburchakli muntazam prizmada asosining ikkita qo‘shni tomonlarining o‘rtalari orqali uchta yon qirrani kesib o‘tadigan va asos tekisligiga α burchak ostida og‘ishgan tekislik o‘tkazilgan. Asosining tomoni a ga teng. Hosil bo‘lgan kesimning yuzini toping (48- rasm).



48-rasm

Berilgan: $ABCDA_1B_1C_1D_1$— muntazam prizma, $CM = DM = \frac{a}{2}$, $BN = CN = \frac{a}{2}$.

Topish kerak: $S_{MNKPE}-?$

Yechish: Hosil bo‘lgan kesimni prizma asosiga proyeksiyasi $ABNMD$ ko‘pburchakka tushadi. Shuning uchun $S_{MNKPE} = \frac{S_{ABNMD}}{cos\alpha}$,

$S_{\Delta MCN} = \frac{1}{2} \cdot \frac{a}{2} \cdot \frac{a}{2} = \frac{a^2}{8}$, $S_{ABNMD} = a^2 - \frac{a^2}{8} = \frac{7a^2}{8}$.

Demak, $S_{MNKPE} = \frac{7a^2}{8cos\alpha}$.

Javob: $S_{MNKPE} = \frac{7a^2}{8cos\alpha}$.

10- masala. Har bir qirrasi a ga teng, asosining burchagi 60° ga teng bo‘lgan to‘g‘ri parallelepipedning diagonallarini toping (49- rasm).

Berilgan: $ABCDA_1B_1C_1D_1$— to‘g‘ri parallelepiped, $AB = BC = AA_1 = \cdots = DD_1 = a$, $\angle BAD = 60°$.

Topish kerak: $AC_1 -?$, $BD_1 -?$

Yechish: $ABCD$ asosini qaraymiz. Bu rombdir.

Kosinuslar teoremasiga ko‘ra

$AC^2 = AD^2 + CD^2 - 2AD \cdot CD \cdot cos120°$,

$AC^2 = a^2 + a^2 - 2a \cdot a \cdot cos120°$,

$AC^2 = 2a^2 + 2a^2 \cdot cos60° = AC^2 = 2a^2 + 2a^2 \cdot \frac{1}{2}$,

$AC^2 = 2a^2 + a^2 = 3a^2$, $AC = a\sqrt{3}$, $A_1C_1 = AC = a\sqrt{3}$,

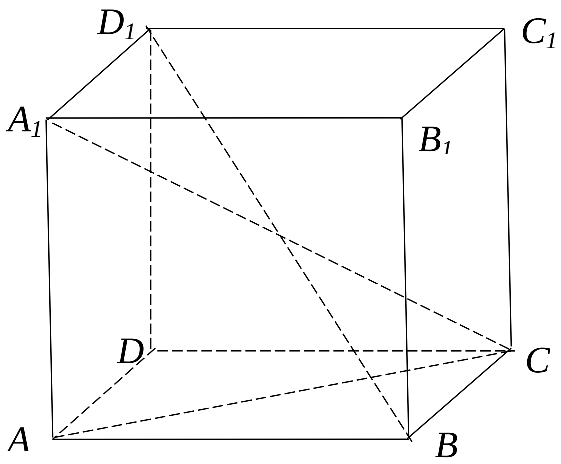
ΔAA_1C_1, dan: $AC_1{}^2 = AA_1{}^2 + A_1C_1{}^2 = a^2 + 3a^2 = 4a^2$, $AC_1 = 2a$.

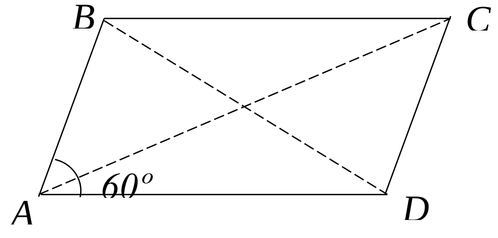
BD_1 ni topish uchun $d_1{}^2 + d_2{}^2 = 2(a^2 + b^2)$ formuladan foydalanamiz

$BD^2 + AC^2 = 2(AB^2 + AD^2)$, $BD^2 + 3a^2 = 2(a^2 + a^2)$,

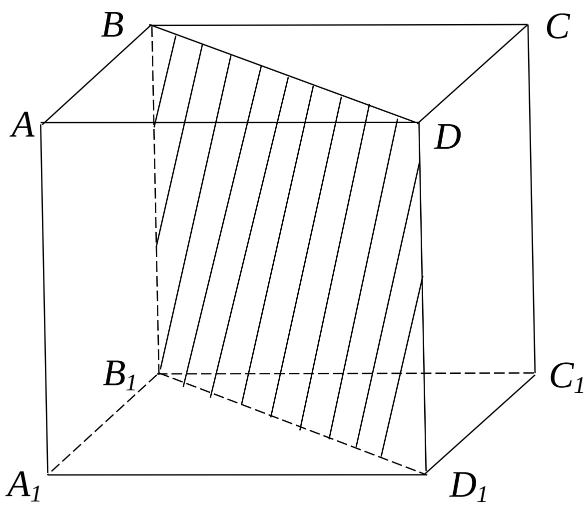
$BD^2 = 4a^2 - 3a^2$, $BD^2 = a^2$, $BD = a$, $B_1D_1 = BD = a$.

ΔBB_1D_1, dan: $BD_1{}^2 = BB_1{}^2 + B_1D_1{}^2 = a^2 + a^2 = 2a^2$, $BD_1 = a\sqrt{2}$.





49- rasm



50- rasm

11- masala. To‘g‘ri parallelepipedning yon qirrasi 1 m, asosining tomonlari 23 dm va 11 dm ga teng, asosining diametrlari esa $2:3$ kabi nisbatda. Diagonal kesimlarining yuzlarini toping (50-rasm).

Berilgan: $ABCDA_1B_1C_1D_1$ — to‘g‘ri parallelepiped, $AA_1 = 1$ m, $AB = 23$ dm, $AD = 11$ dm, $BD:AC = 2:3$.

Topish kerak: $S_{BDD_1B_1}-?$, $S_{ACC_1A_1}-?$

Yechish: $ABCD$ parallelgramm, ${d_1}^2 + {d_2}^2 = 2(a^2 + b^2)$ formulaga ko‘ra $AC^2 + BD^2 = 2(AB^2 + AD^2)$.

Masala shartiga ko‘ra, $BD = \frac{2}{3}AC$, u holda

$AC^2 + \frac{4}{9}AC^2 = 2(121 + 529)$Л $\frac{13}{9}AC^2 = 1300,$

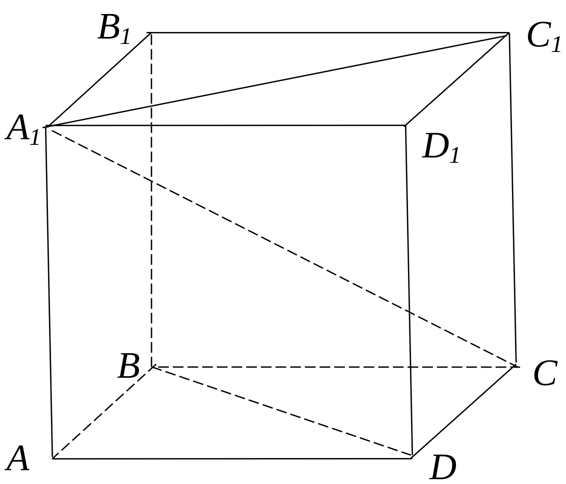
$AC^2 = 9 \cdot 100,\ AC = 30$ dm $= 3$ m.

Bundan, $BD = \frac{2}{3} \cdot 3 = 2$ m, u holda $S_{BDD_1B_1} = 2$ m^2,

$S_{ACC_1A_1} = 3$ m^2.

Javob: 2 m^2, 3 m^2.

12- masala. To‘g‘ri burchakli parallelepipedning diagonallarini uning uchta o‘lchoviga ko‘ra toping: 1) 1, 2, 2; 2) 2, 3, 6; 3) 6, 6, 7;



51- rasm

Berilgan: $ABCDA_1B_1C_1D_1$ — to‘g‘ri burchakli parallelepiped (51-rasm), $AD = 2$, $AB = 3$, $AA_1 = 6$.

Topish kerak: $A_1C-?$

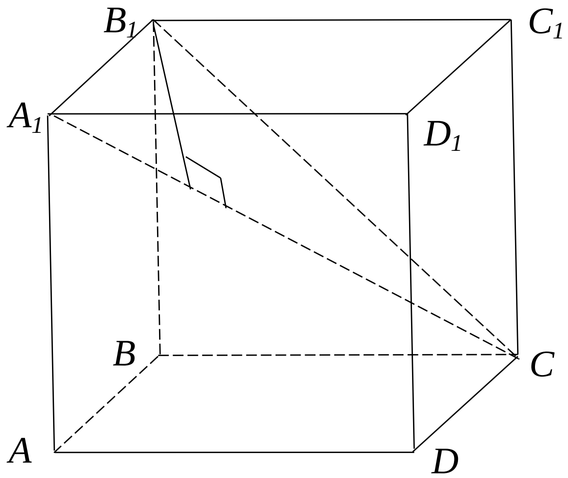
Yechish: To‘g‘ri burchakli parallelepiped diagonali uchala o‘lchov kvadratlari yig‘indisidan ildiz chiqarilganiga teng:

$$d = A_1C = \sqrt{4 + 9 + 36} = 7.$$

13- masala. Kubning qirra α ga teng. Kubning bir uchidan qolgan ikki uchini tutashtiruvchi diagonaligacha bo‘lgan masofani toping (52- rasm).

Berilgan: $ABCDA_1B_1C_1D_1$ — kub, $AB = a$, A_1C — diagonal, $B_1E \perp A_1C$.

Topish kerak: $B_1E-?$



52- rasm

Yechish: BCC_1B_1— yon yog‘iga B_1C diagonal o‘tkazamiz. Bu diagonal CD tomonga perpendikular. Chunki $ABCD$ va BCC_1B_1 tekisliklar perpendikular.

ΔBCB_1 dan: $B_1C = a\sqrt{2}$. ΔA_1B_1C dan: $A_1C = a\sqrt{3}$. ΔA_1B_1C to‘g‘ri burchakli uchburchakning yuzini ikki xil formula bilan topamiz.

$$S_{\Delta A_1B_1C} = \frac{1}{2}a^2\sqrt{2} = \frac{a^2\sqrt{2}}{2}, \quad S_{\Delta A_1B_1C} = \frac{1}{2}a\sqrt{3}\cdot B_1E,$$

bundan, $\frac{a^2\sqrt{2}}{2} = \frac{1}{2}a\sqrt{3}\cdot B_1E$, $B_1E = a\sqrt{\frac{2}{3}}$

Javob: $a\sqrt{\frac{2}{3}}$

14- masala. To‘g‘ri burchakli parallelepipedning uchala 10 sm, 22 sm, 16 sm o‘lchoviga ko‘ra sirtini toping.

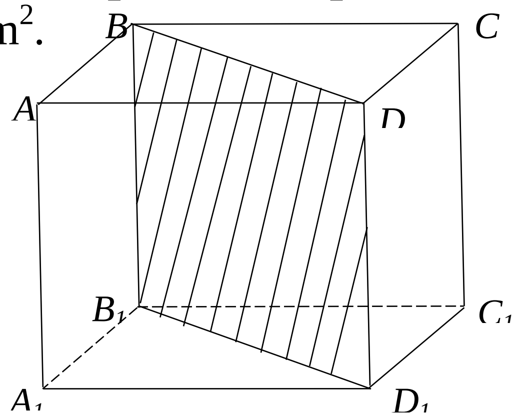
Berilgan: $ABCDA_1B_1C_1D_1$ — to‘g‘ri burchakli parallelepiped, $AD = 10$ sm, $AB = 16$ sm. $AA_1 = 22$ sm.

Topish kerak: $S_{t\,s}-?$

Yechish: $S_{t\,s} = 2(AD\cdot AB + AD\cdot AA_1 + AB\cdot AA_1) =$
$= 2(160 + 220 + 352) = 1464$ sm^2.

Javob: 1464 sm^2.

15- masala. Agar to‘g‘ri parallelepipedning balandligi h, asosining yuzi Q, diagonal kesimining yuzi esa M bo‘lsa, uning yon sirtini toping (53- rasm).



53- rasm

Berilgan: $ABCDA_1B_1C_1D_1$ — to‘g‘ri burchakli parallelepiped, $AA_1 = h,\ \ S_{ABCD} = Q,\ S_{BDD_1B_1}M.$

Topish kerak: $S_{yon\ s}-?$

Yechish: $S_{yon\ s} = 2(S_{ADD_1A_1} + S_{ABB_1A_1}),$ (1)

$S_{ADD_1A_1} = AA_1 \cdot AD = h \cdot AD,$ (2)

$S_{ABB_1A_1} = AA_1 \cdot AB = h \cdot AB,$ (3)

$AB = x,\ AD = y$ deb belgilasak, $S_{ABCD} = x \cdot y = Q.$

$x \cdot y = Q.$ (4)

$S_{BDD_1B_1} = BB_1 \cdot BD = h \cdot AB = M, \quad BD = \sqrt{x^2 + y^2},$

$h \cdot \sqrt{x^2 + y^2}$ (5)

$$\begin{cases} xy = Q \\ x^2 + y^2 = \left(\frac{M}{h}\right)^2 \end{cases} \Rightarrow \begin{cases} 2xy = 2Q \\ x^2 + y^2 = \left(\frac{M}{h}\right)^2 \end{cases} \Rightarrow$$

$$x^2 + 2xy + y^2 = 2Q + \left(\frac{M}{h}\right)^2 \Rightarrow\ (x + y)^2 = 2Q + \left(\frac{M}{h}\right)^2 \Rightarrow$$

$$x + y = \sqrt{\frac{2h^2Q + M^2}{h^2}}.$$

$(1) \Rightarrow S_{yon\ s} = 2(h \cdot AD + h \cdot AB) = 2h(x + y).$

$$S_{yon\ s} = 2h\sqrt{\frac{2h^2Q + M^2}{h^2}} = 2\sqrt{2h^2Q + M^2}.$$

Javob: $2\sqrt{2h^2Q + M^2}.$

16- masala. Piramidaning asosi teng yonli uchburchak bo‘lib, bu uchburchakning asosi 12 sm, yon tomoni esa 10 sm. Yon yoqlardan har biri asos bilan 45° dan bo‘lgan ikki yoqli burchaklar tashkil qiladi. Piramidaning balandligini toping.

Berilgan: ΔABC — teng yonli, $AB = 12$sm, $AC = CB = 10$ sm,

$(P_1 \hat{} P_2) = (P_1 \hat{} P_3) = (P_1 \hat{} P_4) = 45°.$

Topish kerak: $SO = H-?$

Yechish: ΔSEO — teng yonli, to‘g‘ri burchakli, chunki,

$\Delta SEO = 45°.$

Demak, $SO =$ EO. O nuqta ABC uchburchakka ichki chizilgan aylana markazi bo‘ladi.

$p = \frac{2S}{a+b+c}$, $\quad p = \frac{2S}{10+10+12} = \frac{2S}{32} \Rightarrow \quad S_{\Delta ABC}$ topish kerak. $CE = h$, $h = \sqrt{AC^2 - AE^2} = \sqrt{100 - 36} = \sqrt{64} = 8$, $\; CE = h = 8$ sm.

$S_{\Delta ABC} = \frac{1}{2} \cdot AB \cdot CE = \frac{1}{2} \cdot 12 \cdot 8 = 48$ sm^2

$p = \frac{2 \cdot 48}{32} = \frac{96}{32} = 3$ sm.

Demak, $EO = r = 3$ sm, $EO = SO = 3$ sm.

Javob: 3 sm

17- masala. Piramidaning balandligi 16 m, asosining yuzi 512 m^2 ga teng. Agar asosga parallel o‘tkazilgan kesimning yuzi 50 m^2 bo‘lsa, bu kesim asosdan qanday masofada bo‘ladi (54- rasm)?

Berilgan: $ABCD$ — piramida, $SO = H = 6$ m, $(ABCD) \| \alpha$, $S_{ABCD} = 512$ m^2.

Topish kerak: $OO_1 -?$

Yechish: $S_{ABCD} = S$ deb, $S_{A\prime B\prime C\prime D\prime} = S_1$ deb belgilab, o‘xshash figuralar yuzlaridagi k koeffitsiyentni aniqlaylik.

$S = R^2 \cdot S_1 \Rightarrow R^2 = \frac{S}{S_1} = \frac{512}{50} \Rightarrow R = \sqrt{\frac{512}{50}} = \frac{16}{5}$,

$SO = R \cdot SO_1 \Rightarrow SO_1 = \frac{SO}{R} = \frac{H}{R}$, $\quad SO_1 = \frac{16}{\frac{16}{5}} = 5$.

$SO = SO_1 + OO_1 \Rightarrow OO_1 = SO - SO_1$, $\quad OO_1 = 16 - 5 = 11$ m.

Demak, $OO_1 = 11$ m ekan.

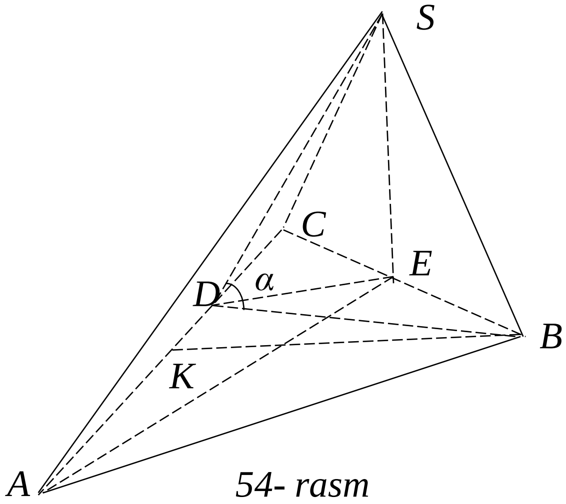
Javob: 11 m.

18- masala. Piramidaning asosi muntazam uchburchakdan iborat; yon yoqlaridan biri asosiga perpendikular, qolgan ikkitasi asosga α burchak ostida og‘gan. Yon qirralar asosga qanday og‘gan (54- rasm)?

Berilgan: ΔABC — muntazam uchburchak, $(BCS) \perp (ABC)$, $(ACS \widehat{\ } ABC) = (ABS \widehat{\ } ABC) = \alpha$.

Topish kerak: $(AS \widehat{\ } ABC) = x$, $(CS \widehat{\ } ABC) = y$.

Yechish: (ACS)) va (ABS) yoqlar asosga bir xil burchak ostida og‘gani uchun S uchidan BC tomonga tushirilgan perpendikular BC ning o‘rtasiga tushadi.



54- rasm

S uchidan AC tomonga SD perpendikular o‘tkazamiz. SE — perpendikular, SD — og‘ma, DE — proyeksiya. Uch perpendikular haqidagi teoremaga ko‘ra, $DE \perp AC$, u holda,
$\alpha = \angle SDE,\ AB = a$ parametr kiritamiz.

ΔCDE dan: $DE = CE \cdot sin60° = \frac{a\sqrt{3}}{4}$,

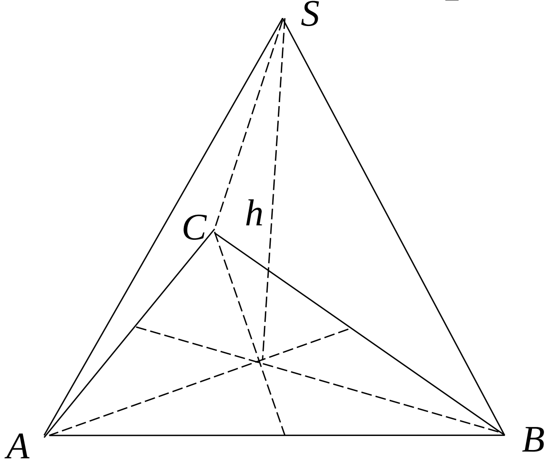
ΔBCK dan: $BK = BC \cdot sin60° = \frac{a\sqrt{3}}{2}$, $AE = BK$.

ΔDES dan: $SE = DE \cdot tg\alpha = \frac{a\sqrt{3}}{4} \cdot tg\alpha$.

$SE(ABC)$ ga perpendikularligidan $y = (CS\hat{\ }ABC) = \angle SCE$ va $x = (AS\hat{\ }ABC) = \angle SAE$ bo‘ladi.

$\Delta SCE \Rightarrow tgy = \frac{SE}{CE} = \frac{\sqrt{3}}{2} tg\alpha$. $\Delta ASE \Rightarrow tgx = \frac{SE}{AE} = \frac{1}{2} tg\alpha$.

Javob: $x = arctg(\frac{1}{2} tg\alpha),\ y = arctg(\frac{\sqrt{3}}{2} tg\alpha)$.



55- rasm

19- masala. Piramidaning asosi gipotenuzasi a ga teng bo‘lgan to‘g‘ri burchakli uchburchakdan iborat. Har bir yon qirrasi asos tekisligi bilan α burchak tashkil qiladi. Piramidaning balandligini toping (55- rasm).

Berilgan: ΔABC — to‘g‘ri burchakli, $\angle C = 90°$, $AB = a$,

$(AS\hat{\ }ABC) = (CS\hat{\ }ABC) = (BS\hat{\ }ABC) = \beta$

Topish kerak: $H-$?

Yechish: Piramidaning har bir yon qirrasi asos tekisligi bilan bir xil burchak tashkil etgani uchun S nuqtadan tushirilgan SO perpendikular, ABC to‘g‘ri burchakli bo‘lganidan O nuqta uchburchakga tashqi chizilgan aylana markazi bo‘ladi.

Demak, $R = AO = \frac{a}{2}$; $SO \perp (ABC)$ ekanligidan, $AO = pr_{(ABC)}AS$, demak $\beta = (AS\hat{\ }ABC) = \angle SAO$.

$\Delta AOS \Rightarrow \frac{SO}{AO} = tg\beta,\ SO = \frac{a}{2} tg\beta,\ SO = H$.

Javob: $H = \frac{a}{2} tg\beta$.

20- masala. Uchburakli muntazam piramida asosining tomoni orqali unga qarshi yotgan yon qirraga perpendikular tekislik o‘tkazilgan. Agar asosining tomoni a, piramidaning balandligi h bo‘lsa, kesim yuzini toping (56- rasm).

Berilgan: ABC muntazam uchburchak, $AB = AC = BC = a$, $DO \perp (ABC)$, $DO = h$; $(ABE) \perp CD$.

Topish kerak: $S_{\Delta ABC} = S_{kesim}-?$

Yechish: ΔABE teng yonli, chunki piramida muntazam: $BE = AE$, bundan $AK = KB$, u holda:

$S_{\Delta ABE} = \frac{1}{2} \cdot AB \cdot KE = \frac{a}{2} \cdot KE$. (1)

$KE = \frac{a\sqrt{3}}{2}$, bundan $OC = \frac{2}{3} \cdot KC = \frac{a}{\sqrt{3}}$.

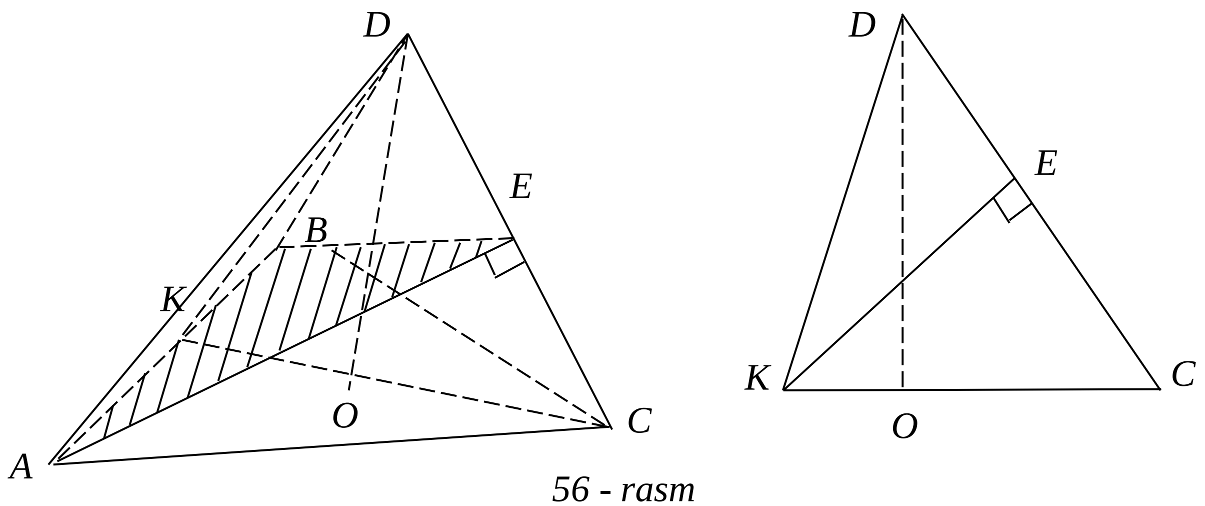
ΔCDO dan: $CD = \sqrt{OD^2 + OC^2} = \sqrt{h^2 + \frac{a^2}{3}} = \sqrt{\frac{3h^2+a^2}{3}}$.

ΔCDK ning yuzini ikki xil usul bilan topamiz:

$S_{\Delta DKC} = \frac{1}{2} \cdot OD \cdot KC = \frac{1}{2} \cdot CD \cdot KE \Rightarrow OD \cdot KC = CD \cdot KE \Rightarrow$

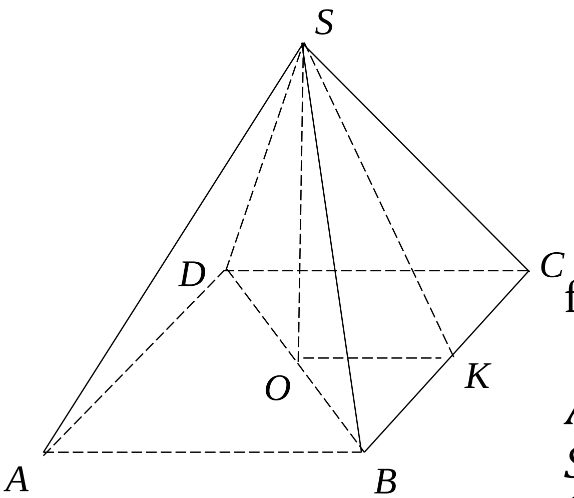
$KE = \frac{OD \cdot KC}{CD} = \frac{h \cdot \frac{a\sqrt{3}}{2}}{\sqrt{\frac{3h^2+a^2}{3}}} = \frac{3ah}{2\sqrt{3h^2+a^2}}$.

Demak, (1) ga ko‘ra $S_{kesim} = \frac{3a^2h}{4\sqrt{3h^2+a^2}}$



56 - rasm

21- masala. To‘rtburchakli muntazam piramidaning yon sirti 14,76 m^2 ga, to‘la sirti 18 m^2 ga teng. Piramida asosining tomonini va balandligini toping (57- rasm).



57- rasm

Berilgan: $SABCD$ — muntazam piramida, $S_{yon\ s} = 14{,}76$ m^2, $S_{t\ s} = 18$ m^2.

Topish kerak: $AB-?$, $SO = h-?$

Yechish: $S_{t\ s} = S_{asos} + S_{yon\ s}$ formulaga ko'ra:

$$S_{asos} = S_{t\ s} - S_{yon\ s} \qquad (1)$$

$AB = x$ deb belgilaymiz. U holda, $S_{asos} = a^2$ va (1) ga ko'ra

$x^2 = 18 - 14{,}76 = 3{,}24$, $x = 1{,}8$ m

Ikkinchi tomondan , $SK = \sqrt{h^2 + \frac{x^2}{4}} = \sqrt{h^2 + 0{,}81}$.

U holda

$$S_{yon\ s} = 4 \cdot \frac{1}{2} \cdot BC \cdot SK = 2 \cdot 1{,}8 \cdot \sqrt{h^2 + 0{,}81} = 3{,}6 \cdot \sqrt{h^2 + 0{,}81}$$

$\Rightarrow 3{,}6 \cdot \sqrt{h^2 + 0{,}81} = 14{,}76$ yoki $\sqrt{h^2 + 0{,}81} = 4{,}1 \Rightarrow$
$h^2 + 0{,}81 = 16{,}81$, $h^2 = 16$, $h = 4$ m.

Javob: 1,8 m, 4 m.

22- masala. Asosining yuzi Q ga, yon qirrasi S ga teng bo'lgan muntazam piramida asosidagi ikki yoqli burchaklarini toping (58- rasm).

Berilgan: $SABCD$ — muntazam piramida, $S_{yon\ s} = S$, $S_{asos} = Q$.

Topish kerak: $(ABCD \ldots \hat{} ABS) = x-?$

Yechish: Faraz qilaylik $ABCD \ldots$ n burchakli ko'pburchak berilgan bo'lsin.

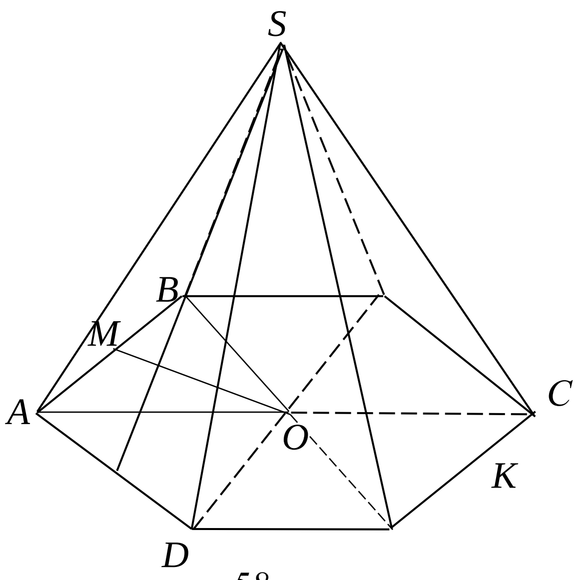
$SO \perp (ABCD \ldots)$, $SM \perp AB$, $MO \perp AB$, $(ABCD \ldots \hat{} ABS) = x$, $(ABO) = pr_{ABCD} ABS$.

Bundan, $S_{asos} = S_{yon\ s} cosx$, u holda $cosx = \frac{S}{a}$.

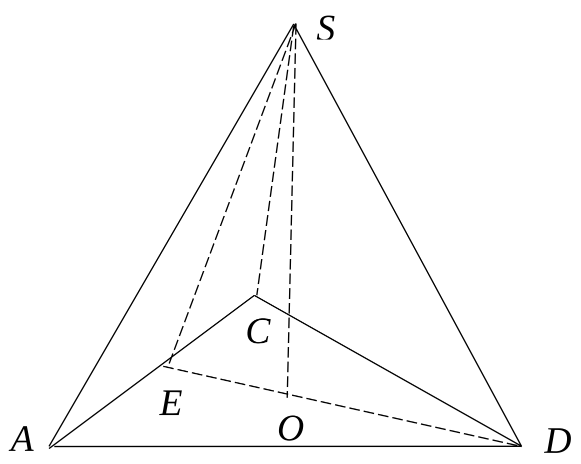
Javob: $arccosx = \frac{S}{a}$.

23- masala. Uchburchakli muntazam piramidaning yon qirrasi 10 sm ga, yon sirti esa 144 sm^2 ga teng bo'lsa, asosining tomonini va apofemasini toping (59- rasm).

Berilgan: $SABC$ — muntazam piramida, $AS = 10$ sm,
$S_{yon\ s} = 144$ sm^2, $SE \perp AC$ (apofema).



58-rasm



59- rasm

Topish kerak: $AB = x$; $SE = a$.

Yechish: ABC — muntazam uchburchak, $AB = x$ va $SE = a$ belgilashlarni kiritamiz.

ABC uchburchakdan: $BE = \frac{\sqrt{3}}{2}x$, O nuqta medianalarning kesishish nuqtasi.

Demak, $OB = \frac{2}{3} \cdot BE = \frac{\sqrt{3}}{3}x$, $OE = \frac{\sqrt{3}}{6}x$.

SOB uchburchakdan: $SO = \sqrt{BS^2 - BO^2} = \sqrt{100 - \frac{x^2}{3}} = \sqrt{\frac{300-x^2}{3}}$

SOE uchburchakdan: $SE = \sqrt{SO^2 + EO^2} = \sqrt{\frac{300-x^2}{3} + \frac{x^2}{12}} = \frac{\sqrt{400-x^2}}{2}$

Bundan, $S_{yon\ s}$ ni topamiz:

$S_{yon\ s} = 3 \cdot \frac{1}{2} \cdot AC \cdot SE = \frac{3}{2} \cdot x \cdot \frac{\sqrt{400-x^2}}{2} = \frac{3x\sqrt{400-x^2}}{2} = 144 \Rightarrow$

$\Rightarrow 3x\sqrt{400 - x^2} = 488$, $x\sqrt{400 - x^2} = 192$,

$x^2(400 - x^2) = 192^2$,

$(x^2)_{1,2} = 200 \pm \sqrt{200^2 - 19x^2} = 200 \pm \sqrt{8 \cdot 392} = 200 \pm 56$.

1) $(x^2)_1 = 256 \Rightarrow x_{1,2} = \pm 16$, $x = 16$,

2) $(x^2)_2 = 144 \Rightarrow x_{3,4} = \pm 12$, $x = 12$.

U holda,

$a_1 = SE = \frac{\sqrt{400-256}}{2} = \frac{12}{2} = 6$, $a_2 = SE = \frac{\sqrt{400-144}}{2} = \frac{16}{2} = 8$.

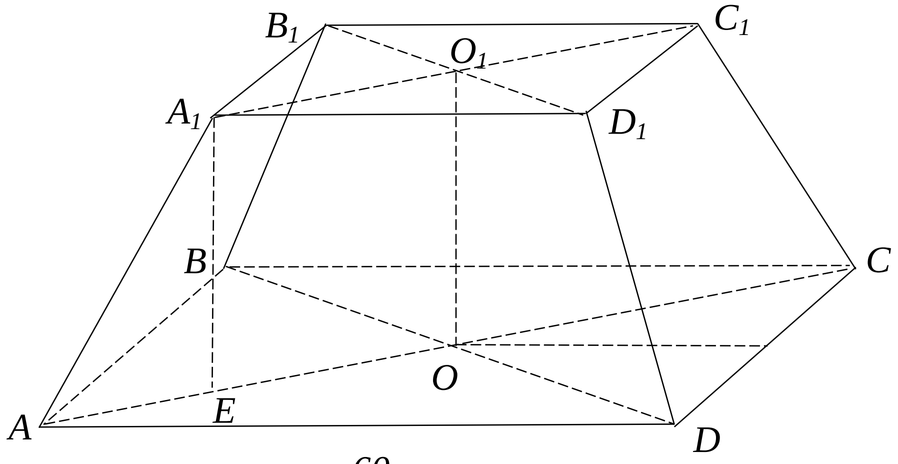
Demak, 16 sm va 6 sm yoki 12 sm va 8 sm.

24- masala. To‘rtburchakli muntazam kesik piramidaning balandligi 7 sm ga teng. Asoslarining tomonlari 10 sm va 2 sm ga teng. Piramidaning yon qirrasini toping (60- rasm).

Berilgan: $ABCD$, $A_1B_1C_1D_1$— kvadrat, $AB = 10$ sm, $A_1B_1 = 2$ sm, $OO_1 = 7$ sm.

Topish kerak: AA_1−?

Yechish: $(ABCD)$ tekislikka A_1E perpendikular o‘tkazamiz. $A_1E = OO_1 = 7$ sm, AC va A_1C_1 kvadrat diagonallari: $AC = 10\sqrt{2}$;



60- rasm

$A_1C_1 = 2\sqrt{2}$. Bundan $AO = 5\sqrt{2}$; $A_1O_1 = \sqrt{2}$;
$AE = AC - A_1O_1 = 4\sqrt{2}$.

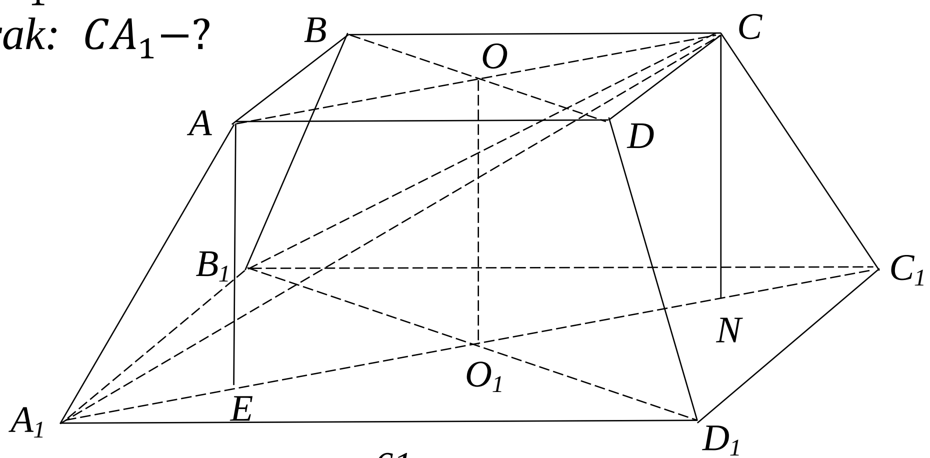
ΔAA_1E dan: $AA_1 = \sqrt{AE^2 + A_1E^2} = \sqrt{32 + 49} = 9$.

Javob: 9 sm.

25- masala. To‘rtburchakli muntazam kesik piramidaning balandligi 2 sm ga, asoslarining tomonlari 3 sm va 5 sm ga teng. Piramidaning diagonalini toping (61- rasm).

Berilgan: $ABCD$ va $A_1B_1C_1D_1$— kvadrat, $OO_1 = h = 2$ sm, $AB = 3$ sm, $A_1B_1 = 5$ sm.

Topish kerak: CA_1−?



61- rasm

Yechish: $ABCD$ — kvadrat, $A_1B_1C_1D_1$— kvadrat, A_1C_1 va AC larni topamiz. ΔABD va $\Delta A_1B_1D_1$ lardan
$BD=\sqrt{AB^2+AD^2}=\sqrt{3^2+3^2}=\sqrt{18}=3\sqrt{2},\ B_1D_1=\sqrt{5^2+5^2}=$
$=5\sqrt{2}.$
$A_1E=x$ bo‘lsin, u holda $EN=AC$ ekanligidan $AN=A_1E+EN==x+3\sqrt{2},\quad A_1C_1=2x+EN,\ 5\sqrt{2}=2x+3\sqrt{2}\ ,\ x=\sqrt{2}.$

$AN=4\sqrt{2}.$

ΔA_1NC dan: $CA_1=\sqrt{32+4}=6$ sm.

Javob: 6 sm.

26- masala. Uchburchakli muntazam kesik piramida katta asosining tomoni Q, kichik asosining tomoni b. Yon qirrasi asosi tekisligi bilan 45° li burchak tashkil qiladi. Piramidaning yon qirrasidan va o‘qidan o‘tuvchi kesimning yuzini toping (62- rasm).

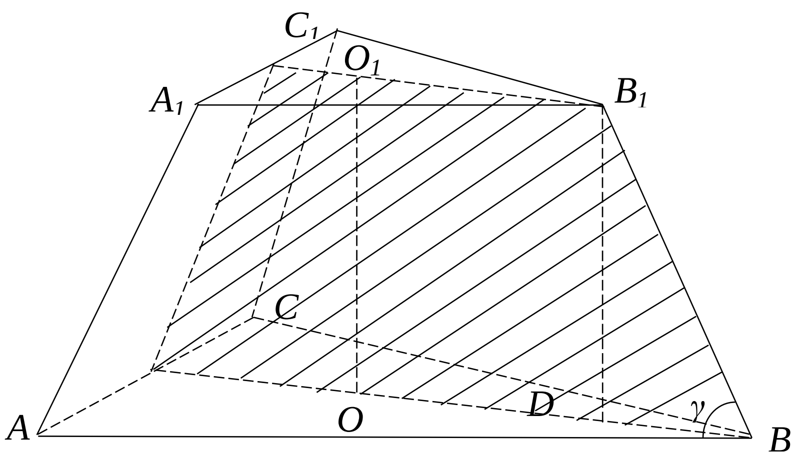
Berilgan: ABC va $A_1B_1C_1$ muntazam uchburchaklar, $AB=a$, $A_1B_1=b,\ \angle B_1BO=\varphi=45°$.

Topish kerak: $AA_1-?,\ S_{kesim}=S_{BB_1E_1E}-?$

Yechish: 47-masalaga o‘xshash BO va B_1O_1 ni topamiz:
$BO=\frac{a}{\sqrt{3}}\ ,\ B_1O_1=\frac{b}{\sqrt{3}}.$ U holda $BF=BO-B_1O_1=\frac{a-b}{\sqrt{3}},\ \varphi=45°,$ dan $BF=\frac{a-b}{\sqrt{3}}.$ Bundan

$BB_1=BF\sqrt{2}=(a-b)\sqrt{\frac{2}{3}},\ AA_1=BB_1=(a-b)\sqrt{\frac{2}{3}}.$

ABC va $A_1B_1C_1$ uchburchaklardan: $BE=\frac{\sqrt{3}}{2}a,\ B_1E_1=\frac{\sqrt{3}}{2}b$



62-rasm

U holda $S_{BB_1E_1E} = \frac{BE - B_1E_1}{2} \cdot BF = \frac{\sqrt{3}(a+b)}{4} \cdot \frac{a-b}{\sqrt{3}} = \frac{a^2-b^2}{4}$.

Javob: $(a-b)\sqrt{\frac{2}{3}}$, $\frac{a^2-b^2}{4}$.

27- masala. Kub yoqlarining markazlari oktaedrning uchlari bo‘lishini, oktaedr yoqlarining markazlari esa kubning uchlari bo‘lishini isbotlang (63- rasm).

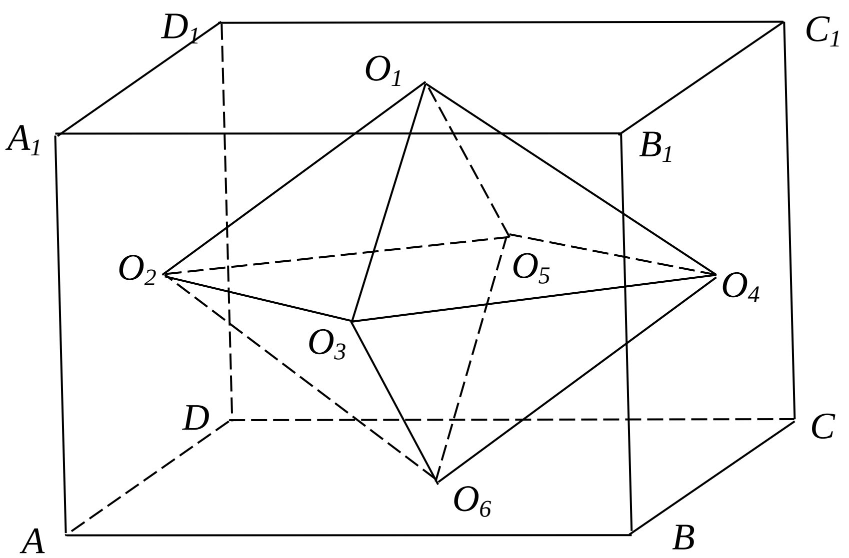
Berilgan: $ABCDA_1B_1C_1D_1$ — kub.

Isbotlash kerak: $O_1O_2O_3O_4O_5O_6$ — oktaedr.

Isbot: Kubning 6 ta yog‘i bor. Faraz qilaylik, $AB = a$ bo‘lsin u holda $O_3E = O_4E = \frac{a}{2}$.

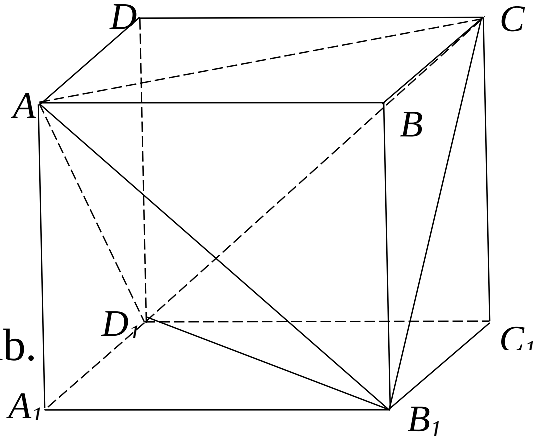
ΔEO_3O_4 dan: $O_3O_4 = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{a}{\sqrt{2}}$.

Xuddi shuningdek, qolgan qirralarni ham $\frac{a}{\sqrt{2}}$ ga tengligini isbotlash mumkin. Demak, qirralari teng 6 ta uchga ega bo‘lgan figura hosil bo‘ladi. Bu oktaedrdir. Endi aksinchasini isbotlaymiz. Oktaedrning 8 ta yog‘i mavjud, ularning markazlarini tutashtirib, yuqoridagi hisoblashlarni qilib, bu kubning uchlari ekanligini ko‘rsatish mumkin. Isbot bo‘ldi.



63- rasm

28- masala. Kubning qarama-qarshi yoqlaridagi o‘zaro parallel bo‘lmagan ikkita diagonalining uchlari tetraedrning uchlari ekanligini isbotlang (64- rasm).



64- rasm

Berilgan: $ABCDA_1B_1C_1D_1$ — kub.

Isbotlash kerak: ACD_1B_1 — tetraedr.

Isbot: Faraz qilaylik, $AB = a$, yasashdan hosil bo‘lgan qirralari: AC_1, CD_1, AD_1, AB_1, CB_1, B_1D_1 — kub yoqlarining diagonallaridir va o‘zaro teng.

Qirralari o‘zaro teng burchaklar tashkil etadi. Demak, ACD_1B_1— tetraedr. Shuni isbot qilish talab etilgan edi.

29- masala. Oktaedrning ikki yoqli burchaklarini toping (65- rasm).

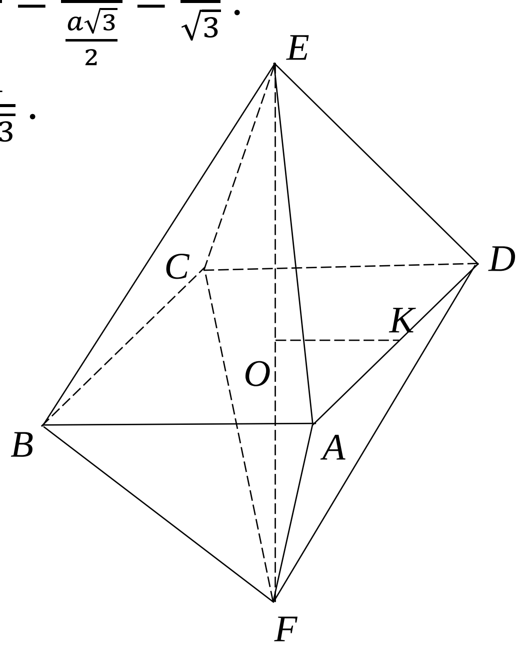
Berilgan: $ABCDEF$— oktaedr.

Topish kerak: $(ADE\hat{}ADF) = \varphi-?$

Yechish: Faraz qilaylik, $AB = a$ bo‘lsin, ΔADE va ΔADF ni qaraylik. $EK \perp AD$, $FK \perp AD$, demak $\varphi = \angle EKF$, ΔEKF ni qaraymiz. Bu teng yonli uchburchak, chunki $EK = FK = \frac{a\sqrt{3}}{2}$ u holda, ΔEKO dan: $cos\frac{\varphi}{2} = \frac{KO}{EK} = \frac{\frac{a}{2}}{\frac{a\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$.

Bu yerdan $\varphi = 2arccos\frac{1}{\sqrt{3}}$.

Javob: $\varphi = 2arccos\frac{1}{\sqrt{3}}$.



65- rasm

MUSTAQIL ISHLASH UCHUN

1. Og‘ma prizmaning yon qirrasi 15 sm ga teng va asos tekisligiga 30° burchak ostida og‘gan. Prizmaning balandligini toping.

2. To‘rburchakli muntazam prizma asosining yuzi 144 sm^2, balandligi 14 sm. Prizmaning diagonalini toping.

3. To‘rtburchakli muntazam prizma asosining tomoni 15 ga, balandligi 20 ga teng. Asosining tomonidan uni kesib o‘tmaydigan prizma diagonaligacha bo‘lgan eng qisqa masofani toping.

4. Asosining a tomoni va l yon qirrasiga ko‘ra:

1) uchburchakli; 2) o‘rtburchakli; 3) oltiburchakli muntazam prizmaning to‘la sirtini toping.

5. To‘g‘ri parallelepiped asosining tomonlari 6 m va 8 m bo‘lib, 30° li burchak tashkil etadi, yon qirrasi 5 m ga teng. Shu parallelepipedning to‘la sirtini toping.

6. Piramidaning balandligi 16 m, asosining yuzi 512 m^2 ga teng. Agar asosga parallel o‘tkazilgan kesimning yuzi 50 m^2 bo‘lsa, bu kesim asosdan qanday masofada bo‘ladi.

7. Piramidaning asosi katetlari 6 sm va 8 sm bo‘lgan to‘g‘ri burchakli uchburchak. Hamma yon yoqlari asos tekisligi bilan 60° li burchak hosil qiladi. Piramidaning balandligini toping.

8. To‘g‘ri burchakli muntazam piramidaning baland-ligi 7 sm ga, asosining tomoni esa 8 sm ga teng. Yon qirrasini toping.

9. To‘rtburchakli muntazam piramidaning uchidagi yassi burchagi α ga teng. Piramida asosidagi ikki yoqli x burchakni toping.

10. 1) Uchburchakli, 2) to‘rtburchakli, 3) oltiburchakli muntazam piramida asosining a tomoni va b balandligiga ko‘ra apofemasini toping.

11. To‘rtburchakli muntazam piramidaning yon sirti 14,76 m^2 ga, to‘la sirti 18 m^2 ga teng. Piramida asosining tomonini va balandligini topingю

12. Asosini tomoni bo‘yicha va diagonal kesimi asosi-ga tengdosh ekanini bilgan holda, to‘rtburchakli muntazam piramidaning yon sirtini toping.

13. To‘rtburchakli muntazam piramidaning yon qirrasi 5 sm ga, to‘la sirti 16 sm^2 ga teng bo‘lsa, c shu piramida asosining tomonini toping.

14. Piramidaning asosi diagonallari 6 m va 8 m ga teng romb, piramidaning balandligi romb diagonallarining kesishgan nuqtasidan o‘tadi va 1 m ga teng. Piramidaning yon sirtini toping.

15. Piramidaning asosi — tomonlari 40 sm, 25 sm, 25 sm bo‘lgan teng yonli uchburchak. Piramidaning balandligi 40 sm li tomon qarshisida yotgan burchakning uchidan o‘tadi va 8 sm ga teng. Piramidaning yon sirtini toping.

16. Piramidaning asosi kvadrat, balandligi asosining uchlaridan biri orqali o‘tadi. Agar piramida asosining tomoni 20 dm ga, balandligi esa 21 dm ga teng bo‘lsa, uning yon sirtini toping.

17. Uchburchakli muntazam kesik piramida asoslarining tomonlari 4 dm va 1 dm. Yon qirrasi 2 dm. Piramidaning balandligini toping.

18. Uchburchakli kesik muntazam piramida asoslari-ning tomonlari 2 sm va 6 sm. Yon yog‘i katta asosi bilan 60° burchak tashkil qiladi. Balandligini toping.

19. Uchburchakli muntazam kesik piramida asoslarining tomonlari 8 m va 5 m, balandligi 3 m. Ostki asosining tomoni va ustki asosining unga qarshi yotgan uchi orqali kesim o‘tkazing. Kesimning yuzini va kesim bilan ostki asos tomoni orasidagi ikki yoqli burchakni toping.

20. To‘rtburchakli muntazam kesik piramida asosining tomonlari 8 m va 2 m. Balandligi 4 m ga teng. To‘la sirtini toping.

VI. AYLANISH SIRTLARI

Bu bo‘limda asosan aylanish jismlari haqida gap yuritiladi. Ayrim belgilashlarni ko‘rsatib o‘tamiz.

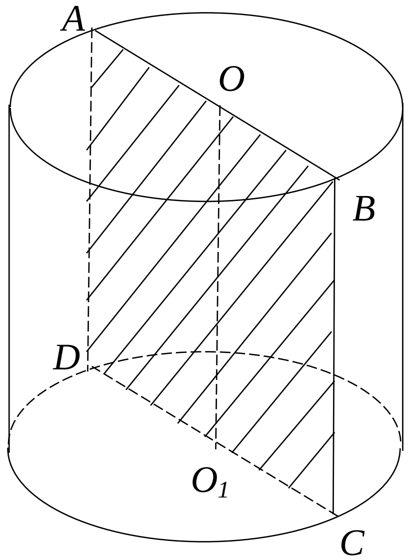
1) $\rho(A,B)$ — A va B nuqtalar orasidagi masofa;

2) $sh(O,R)$ — markazi O nuqtada, radiusi R bo‘lgan shar;

3) $S(R,H)$ — asosining radiusi R, balandligi H bo‘lgan silindr;

4) $K(R,H)$ — asosining radiusi R, balandligi H bo‘lgan konus;

5) $K(R,r,H)$ — asosining radiuslari R va r, balandligi H bo‘lgan kesik konus.



66-rasm

1- masala. Silindrning o‘q kesimi yuzi Q ga teng kvadrat. Silindr asosining yuzini toping (66- rasm).

Berilgan: $ABCD$ — o‘q kesim va u kvadrat, $S_{ABCD} = Q$.

Topish kerak: S_{asos}—?

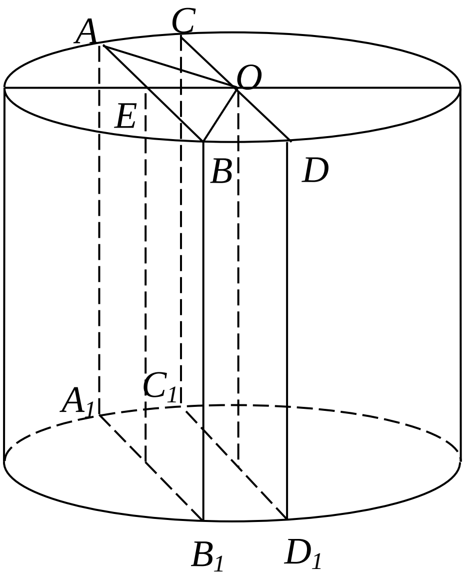
Yechish: $ABCD$ kvadratligidan $AB = 2R = AD = H$, bundan, $H = 2RQ = 2RH$, yoki $Q = 2R \cdot 2R$ yoki $R^2 = \frac{Q}{4}$, u holda

$$S_{asos} = \pi R^2 - \pi \frac{Q}{4} = \frac{\pi Q}{4}.$$

Javob: $\frac{\pi Q}{4}$.

2- masala. Silindrning balandligi 6 dm, asosining radiusi 5 dm. Berilgan 10 dm uzunlikdagi kesmaning oxirlari ikkala asos aylanalarida yotadi.

Bu kesmadan o‘qqacha bo‘lgan eng qisqa masofani toping (67- rasm).



67-rasm

Berilgan: $R = 5$ dm, $H = 6$ dm, $AB = 10$ dm.

Topish kerak: $\min\rho(AB, OO_1)$—?

Yechish: AB va OO_1 chiziqlar to‘g‘ri chiziqlardir. Ayqash to‘g‘ri chiziqlar orasidagi eng qisqa masofa ular orqali o‘tkazilgan parallel tekisliklar orasidagi masofa bo‘ladi.

Demak, AB orqali ABB_1A_1 tekislik, OO_1 orqali CDD_1C_1 tekislik o‘tkazamiz.

O nuqtadan ABB_1A_1 tekislikning AB tomoniga OE perpendikular o‘tkazamiz. AOB uchburchak teng yonli, $AO = BO = R = 5$ dm. AB kesma ABB_1A_1 to‘g‘ri to‘rtburchakning diagonali.

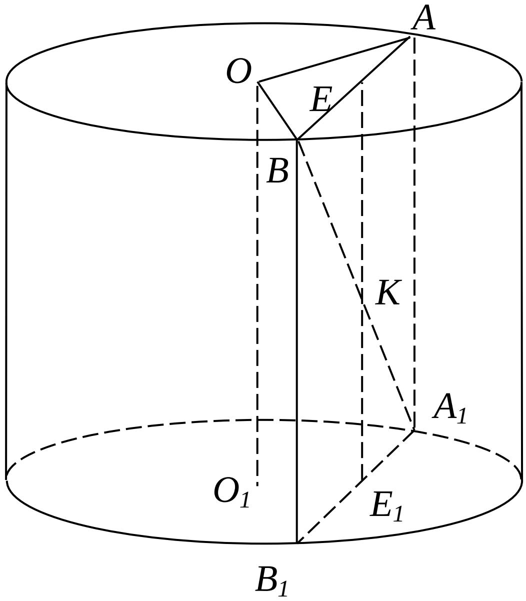
U holda, to‘g‘ri burchakli uchburchak ABB_1 dan:
$AB_1 = \sqrt{AB^2 - BB_1{}^2} = \sqrt{100 - 36} = 8$ dm.

Demak, ΔAOE dan: $OE = \sqrt{AO^2 - AE^2} = \sqrt{25 - 16} = 3.$

Shunday qilib, $\min\rho(AB, OO_1) = OE = 3$ dm.

Javob: 3 dm.

3- masala. Teng tomonli (diametri balandligiga teng) silindrning yuqori asos aylanasidagi ikki nuqta ostki asos aylanasidagi nuqta bilan tutashtirilgan. Bu nuqtalarga o‘tkazilgan radiuslar orasidagi burchak 60° ga teng. O‘tkazilgan to‘g‘ri chiziq bilan silindr o‘qi orasidagi x burchakni toping (68- rasm).



68-rasm

Berilgan: $2R = H,\ (AO\wedge OO_1) = 60°$. $(AB\wedge OO_1) = x-?$

Yechish: AO va BO_1 to‘g‘ri chiziqlar ayqash to‘g‘ri chiziqlardir.
Ayqash to‘g‘ri chiziqlar orasidagi burchak deb, ularni bitta tekislikka parallel ko‘chirishda hosil bo‘lgan to‘g‘ri chiziqlar orasidagi burchakka aytiladi.

Demak, $(AO\wedge BO) = \angle AOB_1 = 60°$ va

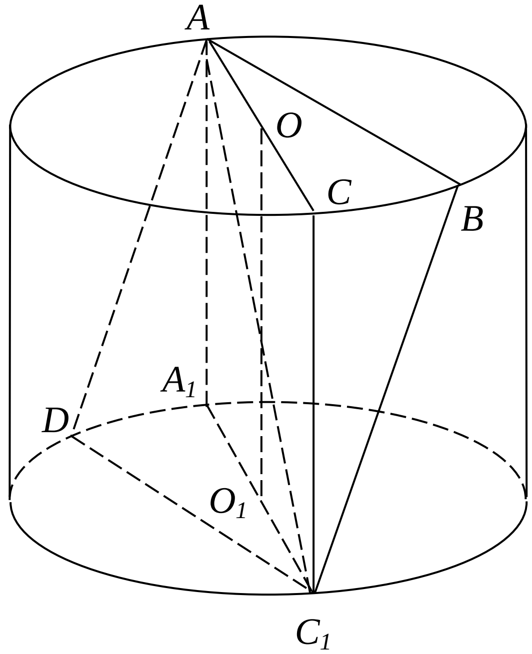
$$(AB\wedge OO_1) = (AB\wedge EE_1) = \angle AKE = x-?$$

ΔAOB_1 dan: kosinuslar teoremasiga ko‘ra

$AB_1{}^2 = R^2 + R^2 - 2R^2 cos60° = 2R^2 - 2R^2 \cdot \frac{1}{2} = R^2$ bundan: $AB_1 = R,\ AE_1 = \frac{AB_1}{2} = \frac{R}{2}$.

Masala shartiga ko‘ra, $BB_1 = H = 2R,$ и holda $KE_1 = \frac{BB_1}{2} = R,$ bundan, ΔAEK dan (bu uchburchak to‘g‘ri burchakli):

$$tgx = \frac{AE_1}{KE_1} = \frac{\frac{R}{2}}{R} = \frac{1}{2}.$$



69-rasm

Demak, $= arctg\frac{1}{2}$.

Javob: $arctg\frac{1}{2}$.

4- masala. Silindrning balandligi 2 m, asosining radiusi 7 m. Bu silindrga kvadrat og'ma qilib shunday ichki chizilganki, kvadratning uchlari silindr asoslarining aylanalarida yotadi. Kvadratning tomonini toping.

Berilgan: $R = 7$ m, $H = 2$ m, $ABCD$ — kvadrat.

Topish kerak: $AB-?$

Yechish: A nuqtadan AA_1 yasovchi, C nuqtadan CC_1 yasovchi o'tkazamiz. AA_1va CC_1 kesmalar asos tekisliklariga perpendikular.

Demak, AA_1C_1C — to'g'ri to'rtburchak (69- rasm). U holda ACC_1 uchburchakdan:

$AC = \sqrt{{AC_1}^2 + {CC_1}^2} = \sqrt{(2R)^2 + H^2} = \sqrt{196 + 4} = \sqrt{200} = 10\sqrt{2}$ m.

Ikkinchi tomondan, AC kesma $ABCD$ kvadratning diagonalidir.

Demak, $AC = \sqrt{2}AB$, $10\sqrt{2} = \sqrt{2}AB$.

Shunday qilib, $AB = 10$ m.

Javob: 10 m.

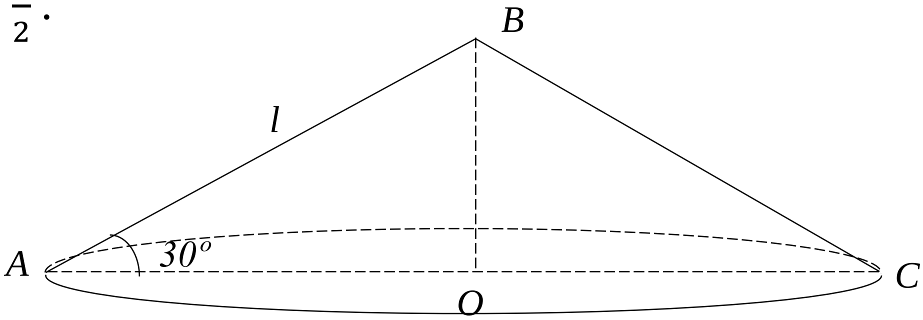
5- masala. Konusning l yasovchisi asos tekisligiga 30° burchak ostiga og'gan. Konusning balandligini toping (70- rasm).

Berilgan: $AB = l,\ \angle BAO = 30°$.

Topish kerak: $H-?$

Yechish: $\angle ABO$ dan: $\frac{AO}{AB} = sin\angle ABO$. Bundan: $H = ABsin30° = \frac{l}{2}$

Javob: $\frac{l}{2}$.



70- rasm

6- masala. Konus asosining radiusi R, o‘q kesimi to‘g‘ri burchakdan iborat. O‘q kesimining yuzini toping.

Berilgan: ΔABC — to‘g‘ri burchakli, $BO = R$.

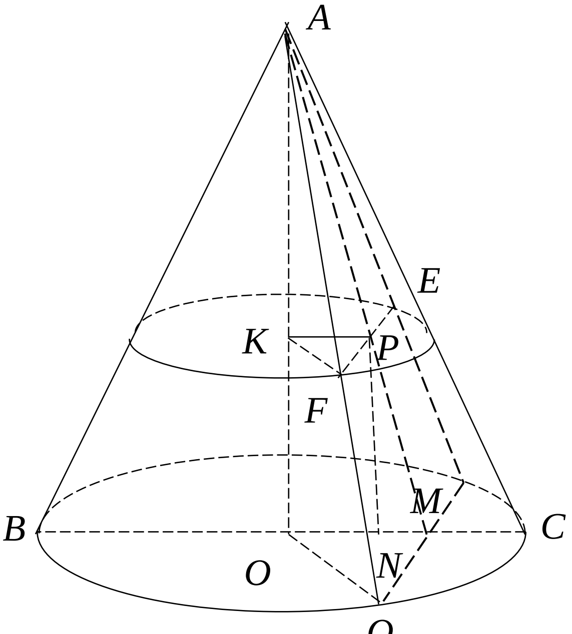
Topish kerak: $S_{\Delta ABC}-?$

Yechish: Konusning o‘q kesimi ΔABC teng yonli, $AB = AC$. Masala shartiga ko‘ra $\angle A = 90°$. Demak, ABC uchburchakda, $\angle B = \angle C = 45°$. Bundan $AO = BO = R$.

Demak, $S_{\Delta ABC} = \frac{1}{2} \cdot BC \cdot AC = \frac{1}{2} \cdot 2R \cdot R = R^2$.

Javob: R^2.

7- masala. Konusning yasovchisi 13 sm, balandligi 12 sm. Konus asosiga parallel to‘g‘ri chiziq bilan kesilgan; undan asosgacha masofa 6 sm, baland-likkacha masofa esa 2 sm ga teng. Bu to‘g‘ri chi-ziq kesmasining konus ichiga olingan uzunligini toping (71-rasm).



71- rasm

Berilgan: $l = AB = 13$ sm, $H = AO = 12$ sm , $EF \|$ asos, $\rho(EF, \text{asos}) = 6$ sm, $\rho(EF, AO) = 2$ sm.

Topish kerak: $EF-?$

Yechish: EF asosga parallel ekanligidan $EF \| QM$. EF kesmaning o‘rtasidan, asosga perpendikular o‘tkazamiz. $PN \| AO$ ekanligidan, $KP = ON$, $KP = \rho(EF, AO) = 2$ sm.

Demak, $ON = 2$ sm.

ΔBOA dan $BO = R = \sqrt{l^2 - H^2} = \sqrt{169 - 144} = 5$ sm.

E va F nuqtalar orqali asosga parallel tekislik o‘tkazamiz.

Kesmada hosil bo‘lgan doira radiusi r bo‘lsin, ya’ni $KF = KE = r$. (BCA) va (FEA) konuslar o‘xshashligidan, ularning yuzlarining nisbati balandliklar kvadratlarining nisbati kabi bo‘ladi, ya’ni

$$\frac{\pi r^2}{\pi R^2} = \frac{(AK)^2}{(AO)^2}.$$

Bundan $AK = AO - KO = AO - \rho(EF,\text{asos}) = 12 - 6 = 6$ sm.

Demak, $\frac{r^2}{R^2} = \frac{36}{144}$ yoki $r^2 = \frac{25}{4}$; $r = \frac{5}{2}$

ΔKPF dan: $FP = \sqrt{KF^2 - KP^2} = \sqrt{\frac{25}{4} - 4} = \frac{3}{2}$ sm.

Shunday qilib, $EF = 2FP = 3$ sm.

Javob: 3 sm.

8-masala. Konus asosining radiusi R va balandligi h berilgan. Konusga ichki chizilgan kubning qirrasini toping (72- rasm).

Berilgan: $SO = H$, $AO = R$, $DEFPD_1E_1F_1P_1$ kub ichki chizilgan.

Topish kerak: $DE-?$

Yechish: Faraz qilaylik, $DE = x-?$

U holda $DO = \frac{\sqrt{2}x}{2} = D_1O_1$.

$OO_1 = DE = x$, chunki OO_1 — kub qirrasiga teng.

$\Delta SD_1O_1 \sim \Delta SDO$ (uchburchaklar o‘xshashligining 3- alomatiga ko‘ra), bundan, $\frac{\frac{\sqrt{2}x}{2}}{R} = \frac{H-x}{H}$, $\sqrt{2} \cdot x \cdot H = 2 \cdot R \cdot H - 2 \cdot R \cdot x$,

$x(\sqrt{2}H + 2R) = 2RH$, $x = \frac{2RH}{\sqrt{2}(H+\sqrt{2}R)} = \frac{\sqrt{2}RH}{H+\sqrt{2}R}$.

Demak, $DE = \frac{\sqrt{2}RH}{H+\sqrt{2}R}$.

9- masala. Konus asosining radiusi R va balandligi H berilgan. Unga yon yoqlari kvadratlardan iborat bo‘lgan uchburchakli muntazam prizma ichki chizilgan. Prizmaning qirrasini toping(73- rasm).

Berilgan: $SO = H$, $AO = R$, $DEFD_1E_1F_1$— muntazam prizma, DEE_1D_1 — kvadrat.

Topish kerak: $DD_1-?$

Yechish: Faraz qilaylik, $DE = x$ bo‘lsin. U holda DEE_1D_1 kvad-ratligidan, $DD_1 = DE = x$, $x-?$

ΔDEF — muntazamligidan, OE ni topamiz. O nuqta medianalar-ning kesishish nuqtasi.

Demak, $EO = \frac{2}{3} \cdot EK = \frac{2}{3} \cdot \frac{\sqrt{3}}{2} \cdot x = \frac{\sqrt{3}}{3}x$.

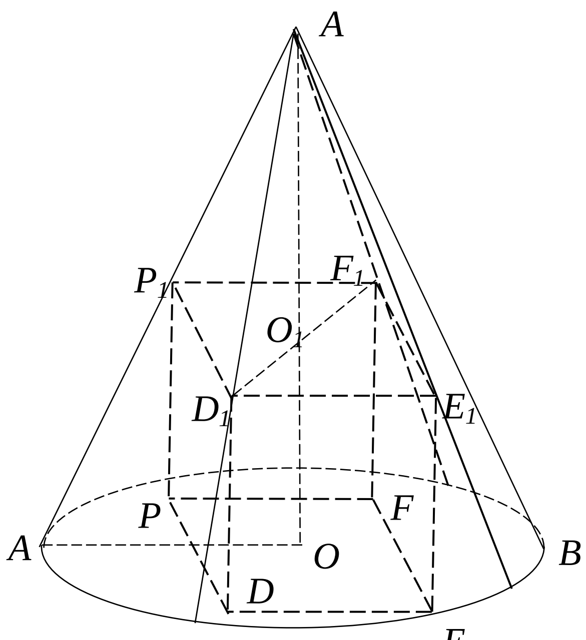
$\Delta SO_1E_1 \sim \Delta SOC$ bo‘1ganidan, $\frac{O_1E_1}{OC} = \frac{SO_1}{SO}$ bu yerda $O_1E_1 = OE$; $SO_1 = H - x$.

Demak, $\frac{\sqrt{3}x}{3R} = \frac{H-x}{H}$, $\sqrt{3} \cdot x \cdot H = 3 \cdot R \cdot H - 3 \cdot R \cdot x$

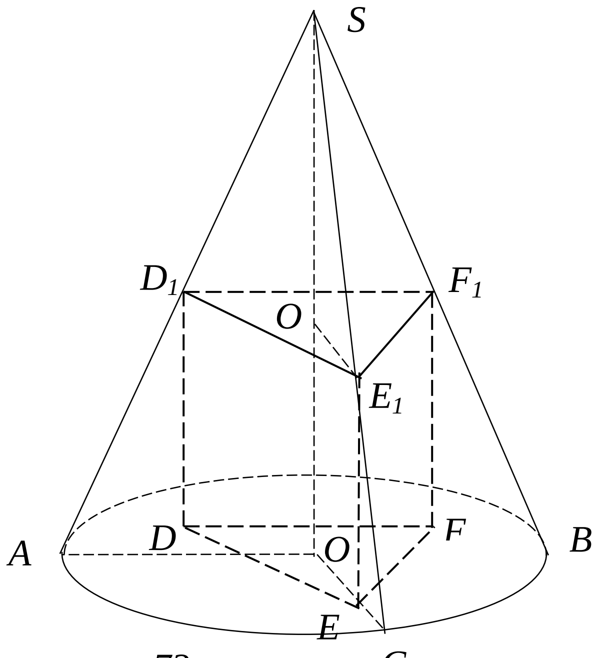
$x = \frac{3RH}{\sqrt{3}H+3R} = \frac{\sqrt{3}RH}{H+\sqrt{3}R}$.

14- masala. Kesik konusning yasovchisi $2a$ ga teng bo‘lib, asosga 60° burchak ostida og‘gan. Bir asosining radiusi ikkinchisinikidan ikki marta katta. Radiuslarning har birini toping.

Berilgan: $AA_1 = 2a$, $\angle A_1AO = 60°$, $AO = 2A_1O_1$.



72- rasm



73- rasm

Topish kerak: $AO = R-?$ $A_1O_1 = r-?$

Yechish: Masala shartiga ko‘ra, $R = 2r$.

ΔABA_1 dan: $\frac{AB}{AA_1} = cos60°$, $AB = AA_1 \cdot cos60° = 2a \cdot \frac{1}{2} = a$.

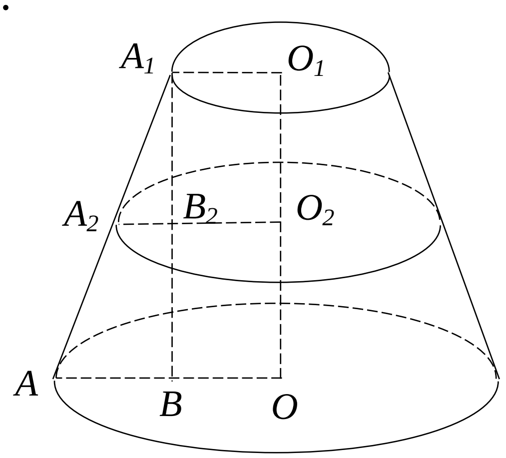
Ikkinchi tomondan, $AB = R - r$.

Shunday qilib, $\begin{cases} R - r = 2a \\ R = 2r \end{cases}$,

$r = a$, $R = 2a$.

Javob: a, $2a$.

15- masala. Kesik konus asoslarining yuzlari 4 dm^2 va 16 dm^2, balandligining o‘rtasidan asoslariga parallel tekislik o‘tkazilgan. Kesimning yuzini toping (74- rasm).



74-rasm

Berilgan: $S(O_1) = 4$ dm^2, $S(O) = 4$ dm^2, $OO_1 = O_2O_1$.

Topish kerak: $S(O_2)-?$

Yechish: $S(O_1) = \pi r^2$, S $S(O) = \pi R^2$, bundan: $r = \frac{2}{\sqrt{\pi}}$; $R = \frac{4}{\sqrt{\pi}}$; $AB = R - r = \frac{2}{\sqrt{\pi}}$.

ΔABA_1 da A_2B_2 uning o‘rta chizig‘i, demak, $A_2B_2 = \frac{AB}{2} = \frac{1}{\sqrt{\pi}}$, u holda $A_2O_2 = \frac{3}{\sqrt{\pi}}$.

Demak, $S(O_2) = \pi \cdot \frac{9}{\pi} = 9$ dm^2.

Javob: 9 dm^2.

16- masala. Kesik konus asoslarining yuzlari M va m. Asoslariga parallel o‘rta kesimning yuzini toping.

Berilgan: $S(O_1) = m$, $S(O) = M$, $OO_2 = O_2O_1$.

Topish kerak: $S(O_2)-?$

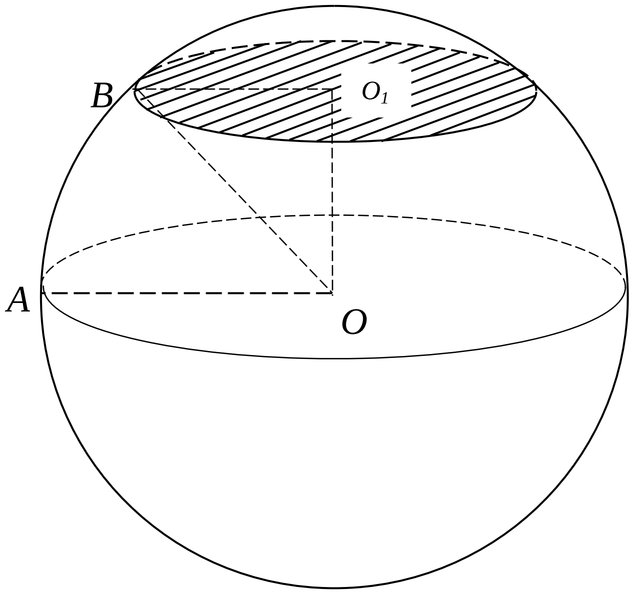
Yechish: $r = \sqrt{\frac{m}{\pi}}$; $R = \sqrt{\frac{M}{\pi}}$.

ΔABA_1 dan: A_2B_2 — o 'rta chiziq: $A_2B_2 = \frac{AB}{2} = \frac{1}{2\sqrt{\pi}}(\sqrt{M} - \sqrt{m})$, u holda

$$A_2O_2 = A_2B_2 + B_2O_2 = \frac{1}{2\sqrt{\pi}}\left(\sqrt{M} - \sqrt{m}\right) + \sqrt{\frac{m}{\pi}} = \frac{1}{2\sqrt{\pi}}\left(\sqrt{M} + \sqrt{m}\right).$$

Demak, $S(O_2) = \pi \cdot (A_2O_2)^2 = \frac{1}{4}\left(\sqrt{M} + \sqrt{m}\right)^2$.

Javob: $\frac{1}{4}\left(\sqrt{M} + \sqrt{m}\right)^2$.



75- rasm

17- masala. Radiusi 41 dm bo‘lgan shar markazidan 9 dm masofadagi tekislik bilan kesilgan. Kesimning yuzini toping (75- rasm).

Berilgan: $AO = R = 41$ dm, $OO_1 = 9$ dm.

Topish kerak: $S(O_1)-?$

Yechish: $BO_1 = r-?$

ΔBOO_1 dan: $r = \sqrt{BO^2 - OO_1{}^2} = \sqrt{41^2 - 9^2} = \sqrt{1600} = 40$ dm,

u holda $S(O_1) = \pi r^2 = 1600\pi$ dm$^2 = 16\pi$ m^2.

Javob: 16π m^2.

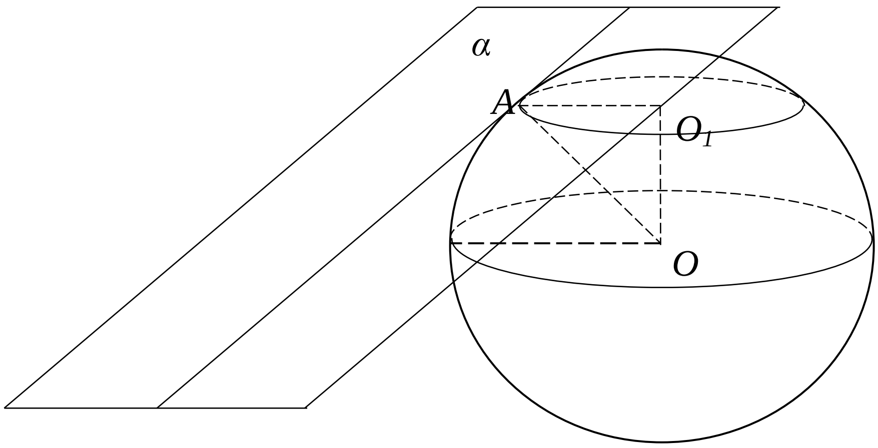
18- masala. Radiusi R bo‘lgan shar berilgan. Uning sirtidagi bir nuqtadan ikkita tekislik o‘tkazilgan: biri — sharga urinma tekislik, ikkinchisi — birinchisiga 30° li burchak ostida o‘tkazilgan. Kesimning yuzini toping (76- rasm).

Berilgan: $AO = R$, $R(\alpha$^kesim$) = 30°$.

Topish kerak: S_{kesim}–?

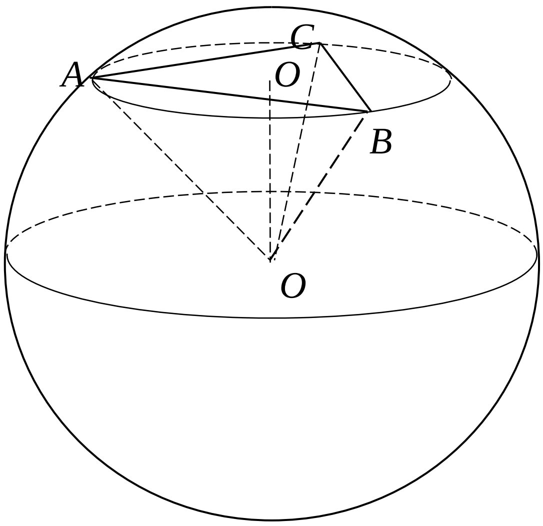
Yechish: α tekislik sharga urinma bo‘lgani uchun, uning urinish nuqtasiga o‘tkazilgan radius tekislikka perpendikulardir, ya’ni $AO \perp \alpha$, bundan, $\angle O_1AO = 60°$.

Demak, $AO_1 = r = \frac{R}{2}$, u holda $S_{kesim} = S(O_1) = \frac{\pi R^2}{4}$.



76- rasm

19- masala. Shar sirtida uchta nuqta berilgan; ular orasidagi to‘g‘ri chiziqli masofalar 6 sm, 8 sm, 10 sm. Sharning radiusi 13 sm ga teng. Markazdan shu nuqtalar orqali o‘tuvchi tekislikkacha maso-fani toping (77- rasm).



77- rasm

Berilgan: $A, B, C \in$ sh(OD), $R = 13$ sm, $AB = 6$ sm, $BC = 8$ sm, $AC = 10$ sm.

Topish kerak: $\rho\big(O, (ABC)\big)$–?

Yechish: A, B, C nuqtalar orqali o‘tuvchi aylanani qaraymiz. Bu aylana ABC uchburchakka tashqi chizilgan. Demak, uning radiusi $BO_1 = CO_1 = AO_1 = r = \frac{abc}{4S}$ formula bilan topiladi. Bu yerda $a = 6$, $b = 8$, $c = 10$.

$S = \sqrt{p(p-a)(p-b)(p-c)} = \sqrt{12 \cdot 6 \cdot 4 \cdot 2} = 24.$

Bundan: $r = \frac{6 \cdot 8 \cdot 10}{4 \cdot 24} = \frac{20}{4} = 5$ sm. $OO_1 \perp (ABC)$.

Demak, $\rho\big(O, (ABC)\big) = OO_1$

Shunday qilib, ΔOO_1C dan:

$OO_1 = \sqrt{OC^2 - O_1C^2} = \sqrt{R^2 - r^2} = \sqrt{169 - 25} = 12$ sm.

Javob: 12 sm.

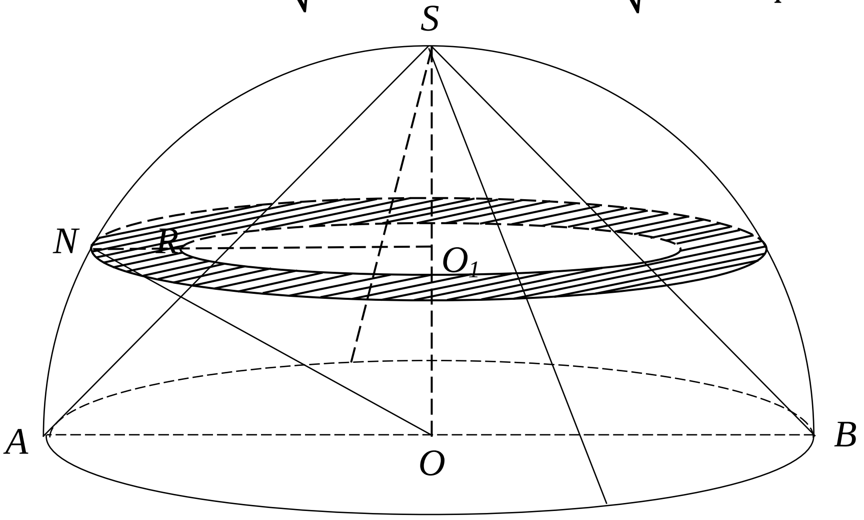
20- masala. Yarim shar va unga ichki chizilgan konus umumiy asosga va umumiy balandlikka ega. Balandlikning o‘rtasidan asosiga parallel tekislik o‘tkazilgan. Konusning yon sirti bilan yarim shar sirti orasiga olingan kesimning yuzi asos yuzining yarmiga teng bo‘lishini isbotlang(78- rasm).

Berilgan: sh(OD) — yarim shar, $SO = R$ — radius, $SO_1 = OO_1 = R/2$, $K(AO, SO)$ — konus.

Topish kerak: $S_{kesim} = \frac{S_{asos}}{2}$.

Yechish: Faraz qilaylik, $AO = R$ bo‘lsin. $N, S \in$ sh(O, R)dan: $NO = SO = R$ bo‘ladi.

ΔNOO_1 dan: $NO_1 = \sqrt{NO^2 - {OO_1}^2} = \sqrt{R^2 - \frac{R^2}{4}} = \frac{\sqrt{3}}{2}R.$



78- rasm

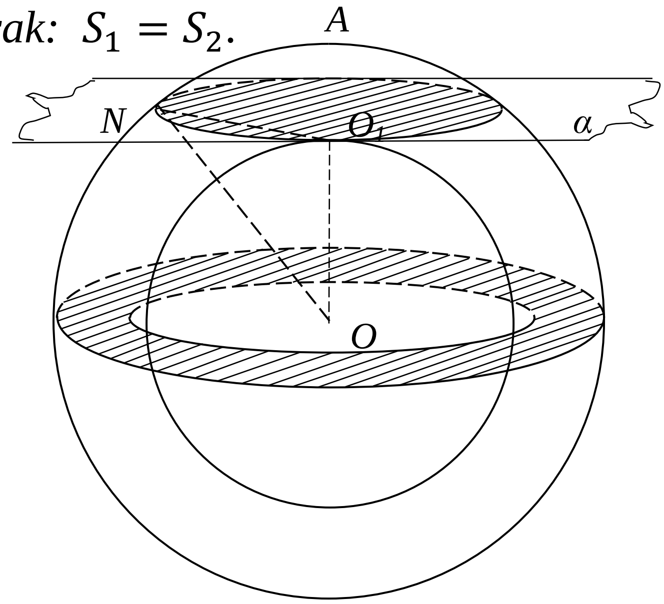
$\Delta AOC \sim \Delta KO_1C$ dan, $\frac{KO_1}{AO} = \frac{SO_1}{SO}$; $KO_1 = \frac{R \cdot \frac{R}{2}}{R} = \frac{R}{2}$. U holda $S_{kesim} = \pi\left({NO_1}^2 - {KO_1}^2\right) = \pi\left(\frac{3R^2}{4} - \frac{R^2}{4}\right) = \frac{\pi R^2}{2}$; $S_{asos} = \pi R^2$.

Demak, $S_{kesim} = \frac{S_{asos}}{2}$.

21- masala. Jismni ikkita konsentrik shar sirtlari chegaralab turibdi (ichi bo‘sh shar). Jismning bu sharlar markazidan o‘tadigan tekislik bilan kesimi ichki shar sirtiga urinuvchi kesimga tengdosh ekanini isbotlang (79- rasm).

Berilgan: $\text{sh}(O, R)$ va $\text{sh}(O, r)$ sharlar berilgan, α tekislik $\text{sh}(O, r)$ ga urinadi.

Isbotlash kerak: $S_1 = S_2$.



79- rasm

Yechish: $C \in \text{sh}(OD) \Rightarrow ON = R,\ O_1 \in \text{sh}(O, r) \Rightarrow OO_1 = r$.

Bundan, $NO_1 = \sqrt{R^2 - r^2}$; $S_2 = \pi \cdot {NO_1}^2 = \pi(R^2 - r^2)$.

Ikkinchi tomondan, $S = \pi \cdot R^2 - \pi \cdot r^2 = \pi(R^2 - r^2)$.

Demak, $S_1 = S_2$.

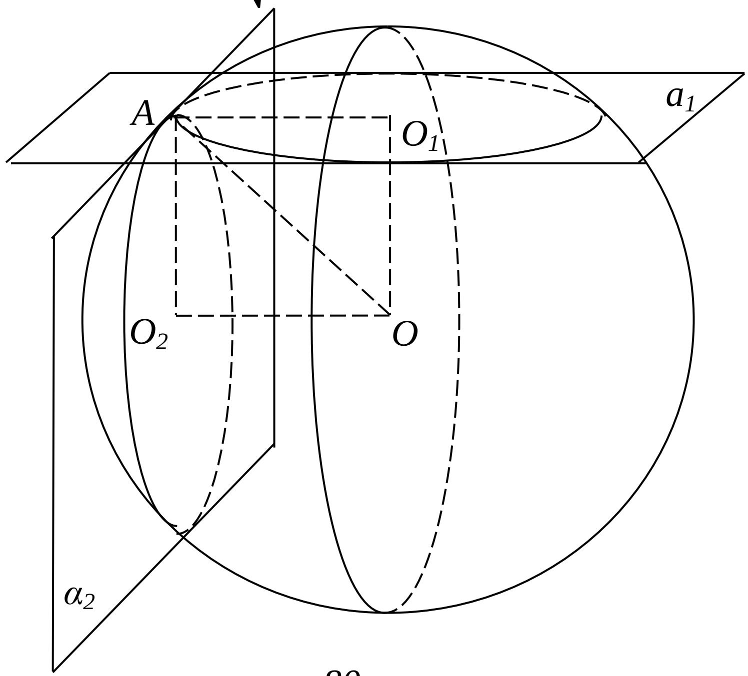
22- masala. Shar sirtiga urinma to‘g‘ri chiziq orqali o‘zaro perpendikular ikkita tekislik o‘tkazilgan bo‘lib, ular sharni radiusi r_1 va r_2 bo‘lgan aylanalar bo‘yicha kesadi. Sharning R radiusini toping (80- rasm).

Berilgan: $\text{sh}(O, R)$ shar, $A \in \text{sh}(O, R)$, $\alpha_1 \cap \text{sh}(O, R) - r_1$ radiusli aylana, $\alpha_2 \cap \text{sh}(O, R) - r_2$ radiusli aylana.

Topish kerak: $R-?$

Yechish: Chizmadan ko‘rinadiki, $OO_1 = r_1$, $OO_2 = r_2$,
$\alpha_1 \perp \alpha_2 \Rightarrow O_1A \perp O_2A$ va $AO_1 = OO_2 = r_1$, $AO_2 = OO_1 = r_2$
u holda,

ΔAO_1O dan: $AO = R = \sqrt{{AO_1}^2 + {OO_1}^2} = \sqrt{{r_1}^2 + {r_2}^2}$.



80-rasm

23- masala. Sharning radiusi 7 sm. Uning sirtida uzunligi 2 sm ga teng umumiy vatarga ega bo‘lgan ikkita teng aylana berilgan. Aylanalarning tekisliklari perpendikular ekanini bilgan holda ularning radiuslarini toping (81- rasm).

Berilgan: $\mathrm{sh}(O, R)$ — shar, $R = 7$ sm, $\alpha_1 \perp \alpha_2$,
$A, B \in \mathrm{sh}(O, R)$, $AB = 2$ sm, $\alpha_1 \cap \mathrm{sh}(O, R) - r_1$ radiusli aylana, $\alpha_2 \cap \mathrm{sh}(O, R) - r_2$ radiusli aylana.

Topish kerak: $r_1 -?$ $r_2 -?$

Yechish: O_1 va O_2 aylana markazlaridan AB vatarga O_1K va O_2K perpendikular o‘tkazamiz. Ular AB vatarni teng ikkiga bo‘ladi, ya’ni $AK = BK = 1$ sm, u holda,

ΔAKO_1 dan: $KO_1 = OO_1 = \sqrt{{AO_1}^2 - AK^2} = \sqrt{{r_1}^2 - 1}$. (1)

ΔAOO_1 dan: $OO_2 = \sqrt{R^2 - {AO_2}^2} = \sqrt{49 - {r_2}^2}$. (2)

(1) = (2) dan:
${r_1}^2 + {r_2}^2 = 50$ va $r_1 = r_2$ ekanligidan $r_1 = r_2 = 5$.

Javob: $r_1 = r_2 = 5$.

24- masala. (0,0,0), (4,0,0), (0,4,0) nuqtalardan o'tuvchi va radiusi 3 ga teng sfera tenglamasini toping.

Berilgan: A(0,0,0), B(4,0,0), C(0,4,0), $\mathrm{sh}(O,R)$ — shar.

Topish kerak: $R = 3$ bo'lgan sfera tenglamasi yozilsin.

Yechish: Sferaning umumiy tenglamasi:

$(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$.

$A \in \mathrm{sh}(O,R)$, $B \in \mathrm{sh}(O,R)$, $C \in \mathrm{sh}(O,R)$ va $R = 3$ bo'lgani uchun, bu yerda $O(a,b,c)$:

$$\begin{cases} a^2 + b^2 + c^2 = 9 & (1) \\ (4-a)^2 + b^2 + c^2 = 9 & (2) \\ a^2 + (4-b)^2 + c^2 = 9 & (3) \end{cases}$$

(1) va (2) ni tenglashtirib, topamiz:

$a^2 + b^2 + c^2 = (4-a)^2 + b^2 + c^2$;

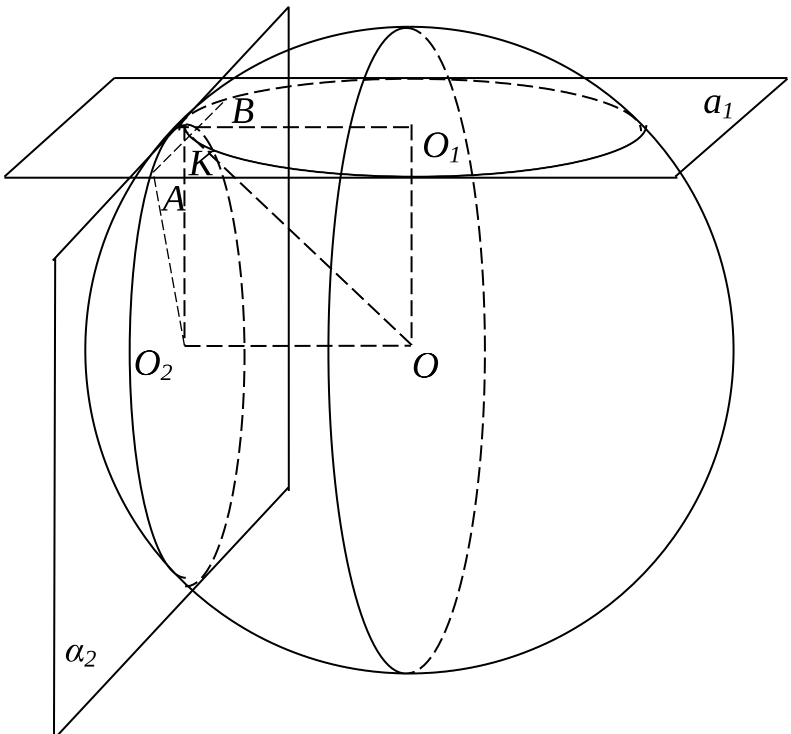
$a^2 = 16 - 8a + a^2$, $16 - 8a = 0$, $a = 2$.

(1) va (3) ni tenglashtirib, $(4-2)^2 + b^2 = 4 + (4-b)^2$,

$4 + b^2 = 4 + 16 - 8b + b^2$, $b = 2$ ni hosil qilamiz.

(1) dan: $c^2 = 9 - a^2 - b^2 = 9 - 4 - 4 = 1$, $c = 1$.

Demak, $(x-2)^2 + (y-2)^2 + (z-1)^2 = 9$.



81-rasm

25- masala. Qirrasi a ga teng bo‘lgan muntazam tetraedrga tashqi chizilgan sharning radiusini toping (82- rasm).

Berilgan: $ABCD$ — tetraedr, $AB = a$, $\mathrm{sh}(O, R)$ — shar tetraedrga tashqi chizilgan.

Topish kerak: $R-?$

Yechish: $DO = AO = R$. Faraz qilaylik, $OO_1 = x$, u holda

$$DO_1 = R + x. \qquad (1)$$

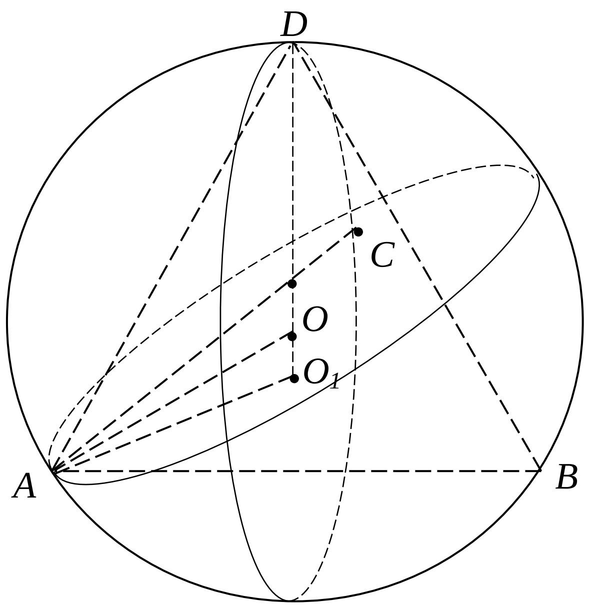
Ikkinchi tomondan, $AO_1 = \frac{2}{3}AE = \frac{2}{3} \cdot \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{3}a.$

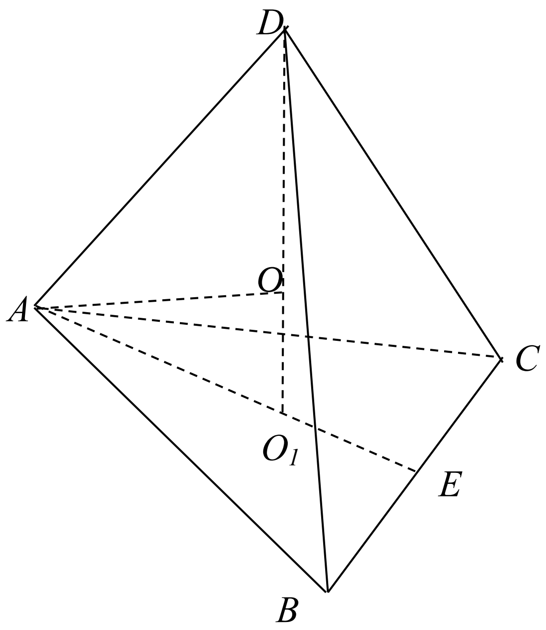
ΔAO_1D dan: $DO_1 = \sqrt{AD^2 - AO_1{}^2} = \sqrt{a^2 - \frac{a^2}{3}} = a\sqrt{\frac{2}{3}}$.

(1) ga ko‘ra $R + x = a\sqrt{\frac{2}{3}}$.

AO_1 kesma ABC uchburchakka tashqi chizilgan aylana radiusi, shu sababli $AO_1 = \frac{a}{\sqrt{3}}$; AOO_1 dan:

$$x = \sqrt{AO^2 - AO_1{}^2} = \sqrt{R^2 - \frac{a^2}{3}},$$





82-rasm

(3) ni (2) ga qo‘yib, R ni topamiz. $R + \sqrt{R^2 - \frac{a^2}{3}} = a\sqrt{\frac{2}{3}} \Rightarrow$

$\Rightarrow \sqrt{R^2 - \frac{a^2}{3}} = a\sqrt{\frac{2}{3}} - R \Rightarrow R^2 - \frac{a^2}{3} = \frac{2a^2}{3} - 2Ra\sqrt{\frac{2}{3}} + R^2 \Rightarrow$
$2Ra\sqrt{\frac{2}{3}} = a^2,\ R = \frac{a\sqrt{3}}{2\sqrt{2}}$.

Javob: $\frac{a\sqrt{3}}{2\sqrt{2}}$

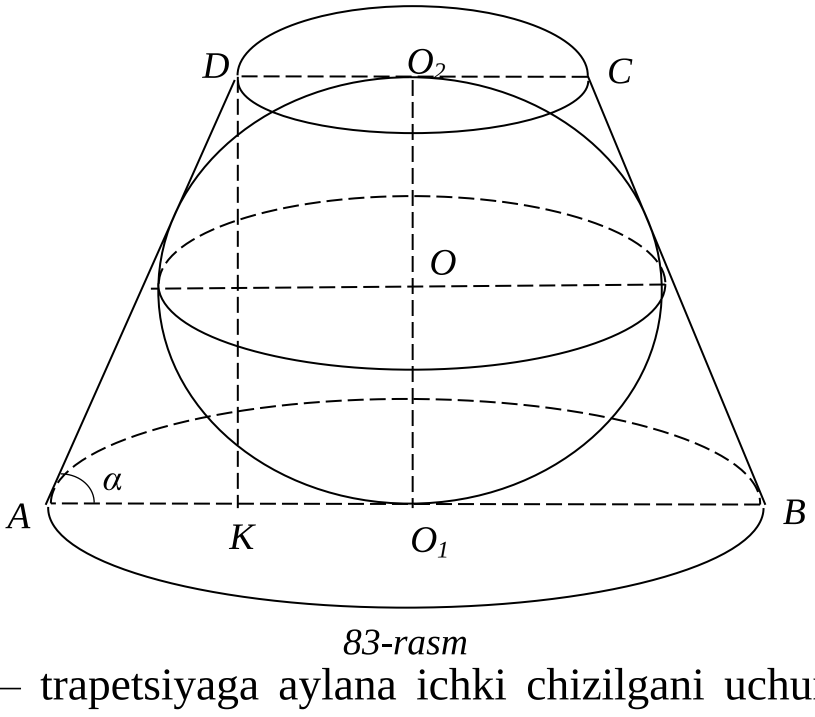
26 - masala. R radiusli shar kesik konusga ichki chizilgan. Konus yasovchisining ostki asos tekisligiga og‘ish burchagi α ga teng. Kesik konus asoslarining radiuslarini va yasovchisini toping (83- rasm).

Berilgan: $ABCD$ — kesik konus, sh(O, R) shar unga ichki chizilgan, $\angle DAO_1 = \alpha$.

Topish kerak: $AO_1 = r_1 -?,\ DO_2 = r_2 -?,\ AD = l -?$,

Yechish: sh(O, R) shar $ABCD$ kesik konusga ichki chizilgani uchun, uning balandligi, $O_1O_2 = H = 2R$, $DK \perp AO_1$, demak, $DK = O_1O_2 = 2R$.

ΔAKD dan: $\frac{DK}{AD} = sin\alpha \Rightarrow l = \frac{2R}{sin\alpha}$, $\frac{AK}{DK} = ctg\alpha$, $AK = 2Rctg\alpha$.



83-rasm

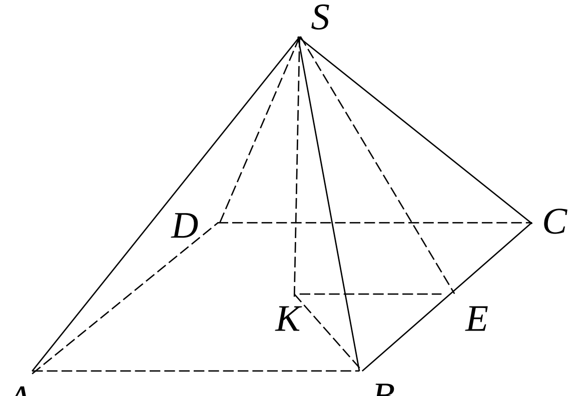
$ABCD$ — trapetsiyaga aylana ichki chizilgani uchun, qarama-qarshi tomonlar yig‘indisi teng: $AB + CD = AB + BC$, bu yerda, $AB = 2r_1 = 2(r_2 + AK) = 2(r_2 + 2Rctg\alpha)$; $CD = 2r_2$; $AD = BC = l$

Demak, $2(r_2 + 2Rctg\alpha) + 2r_2 = \frac{4R}{sin\alpha}$ yoki $r_2 + 2Rctg\alpha = \frac{R}{sin\alpha} \Rightarrow$

$r_2 = R\left(\frac{1}{sin\alpha} - ctg\alpha\right) = \frac{R(1-cos\alpha)}{sin\alpha} = R \cdot \frac{2sin^2\frac{\alpha}{2}}{2sin\frac{\alpha}{2}cos\frac{\alpha}{2}} = R \cdot tg\,\frac{\alpha}{2}$.

Bundan, $r_1 = \frac{R}{g\frac{\alpha}{2}} = R \cdot ctg\,\frac{\alpha}{2}$.

Javob: $R \cdot ctg\,\frac{\alpha}{2}$, $R \cdot tg\,\frac{\alpha}{2}$, $\frac{2R}{sin\alpha}$.



84- rasm

27- masala. To‘rtburchakli muntazam piramida asosining tomoni a ga, uchidagi tekis burchagi esa α ga teng. Ichki chizilgan va tashqi chizilgan sharlarning radiuslarini toping (84-rasm).

Berilgan: $SABCD$ — muntazam piramida, $ABCD$ — kvadrat, $AB = a, \angle ACB = \alpha.$

Topish kerak: sh(O, R) tashqi chizilgan shar $R-$?, sh(O, r) ichki chizilgan shar $r-$?

Yechish: $SE \perp BC$ o‘tkazamiz. U holda $\angle BSE = \frac{\alpha}{2}$; $BE = CE = \frac{a}{2}$.

ΔBES dan: a) $SE = BE \cdot ctg\,\frac{\alpha}{2} = \frac{a}{2}ctg\,\frac{\alpha}{2}$,

b) $SB = BE \cdot cosec\,\frac{\alpha}{2} = \frac{a}{2sin\frac{\alpha}{2}}$.

ΔSKB da $\angle SBK$ qirra $BK = \frac{BD}{2} = \frac{\sqrt{2}a}{2}$ bilan $(ABCD)$ asos orasidagi burchak φ: $cos\varphi = \frac{BK}{SB} = \frac{\frac{\sqrt{2}a}{2}}{\frac{a}{2 \cdot sin\frac{\alpha}{2}}} = \sqrt{2} \cdot sin\,\frac{\alpha}{2}$;

$\varphi = \arccos(\sqrt{2} \cdot sin\frac{\alpha}{2})$.

43- masalaga ko‘ra, $R = \frac{a}{2 \cdot sin2\varphi \cdot sin45°} = \frac{a}{\sqrt{2} \cdot sin2\varphi}$.

Endi $sin2\varphi$ ni α orqali ifodalaymiz:

$$sin2\varphi = sin2\left(\arccos\left(\sqrt{2} \cdot sin\frac{\alpha}{2}\right)\right) =$$

$$= 2sin\left(\arccos\left(\sqrt{2} \cdot sin\frac{\alpha}{2}\right)\right) \cdot cos\left(\arccos\left(\sqrt{2} \cdot sin\frac{\alpha}{2}\right)\right) =$$

$$= 2\sqrt{1 - cos^2\left(\arccos\left(\sqrt{2} \cdot sin\frac{\alpha}{2}\right)\right)} \cdot \sqrt{2} \cdot sin\frac{\alpha}{2} =$$

$$= 2\sqrt{2} \cdot \sqrt{1 - sin^2\frac{\alpha}{2}} \cdot \ sin\frac{\alpha}{2} =$$

$$= 2\sqrt{2} \cdot \sqrt{cos^2\frac{\alpha}{2} + sin^2\frac{\alpha}{2} - 2sin^2\frac{\alpha}{2}} \cdot sin\frac{\alpha}{2} =$$

$= 2\sqrt{2} \cdot \sqrt{cos\alpha} \cdot sin\frac{\alpha}{2}$.

Shunday qilib, $R = \frac{a}{4sin^2\frac{\alpha}{2}\cdot\sqrt{cos\alpha}}$.

Xuddi shu yo‘l bilan va 42-masala yordamida $r = \frac{a}{2}\sqrt{\frac{1-tg\frac{\alpha}{2}}{1+tg\frac{\alpha}{2}}}$ topiladi.

MUSTAQIL ISHLASH UCHUN

1. Silindr asosining radiusi 2 m, balanligi 3 m. O‘q kesimining diagonalini toping.

2. Silindrning balandligi 8 dm, asosining radiusi 5 dm. Silindr tekislik bilan shunday kesilganki, kesimda kvadrat hosil qilingan. Bu kesimdan o‘qqacha masofani toping.

3. Konus asosining radiusi 3 m, balandligi 4 m. Yasovchisini toping.

4. Konus asosining radiusi R va balandligi h berilgan. Konusga ichki chizilgan kubning qirrasini toping.

5. Kesik konus asoslarining radiuslari 3 m va 6 m, balandligi 4 m. Yasovchisini toping.

6. Kesik konus asoslarining radiuslari R va r, yasov-chisi asosga 45° burchak ostiga og‘gan. Balandligini toping.

7. Kesik konus asoslarining radiuslari 3 dm va 7 dm. Yasovchisi 5 dm. O‘q kesimning yuzini toping.

8. Sharning radiusi R. Radiusning uchidan unga 60° li burchak ostida tekislik o‘tkazilgan. Kesimning yuzini toping.

9. Yer sharining radiusi R. Agar parallelning kengligi 60° bo‘lsa, uning uzunigi qanday.

10. N shahri shimoliy kenglikning 60° ida joylashgan. Yerning o‘z o‘qi atrofida aylanishi natijasida bu shahar 1 soatda qanday masofani o‘tadi?

11. Sharning diametri 25 sm. Uning sirtida A punktda va hamma nuqtalari to‘g‘ri chiziq bo‘yicha hisoblaganda A punktdan 15 sm masofada yotgan aylana berilgan. Shu aylananing radiusini toping.

12. Uchburchakning tomonlari 13 sm, 14 sm, 15 sm. U uchburchak tekisligidan uchburchakning hamma tomonlariga urinadigan sharning markazigacha masofani toping. Shaming radiusi 5 sm.

13. Rombning diagonallari 15 sm va 20 sm. Shar sirti uning hamma tomonlariga urinadi. Shar markazidan romb tekisligigacha masofani toping.

14. Sharlarning radiuslari 25 dm va 29 dm, ularning markazlari orasidagi masofa esa 36 dm ga teng. Sharlaming sirtlari kesishadigan chiziqning uzunligini toping.

15. $\boldsymbol{n}$ burchakli muntazam piramida asosining tomoni a, asosidagi ikki yoqli burchagi φ ga teng. Piramidaga ichki chizilgan sharning radiusini toping.

16. R radiusli sharga S uchidagi yassi burchagi α ga teng bo‘lgan uchburchakli muntazam piramida ichki chizilgan. Piramidaning balandligini toping.

17. Sharlarning radiuslari 25 dm va 29 dm, ularning markazlari orasidagi masofa esa 36 dm ga teng. Sharlaming sirtlari kesishadigan chiziqning uzunligini toping.

18. Tomoni a bo‘lgan kubga tashqi chizilgan sharning radiusini toping.

19. $\boldsymbol{n}$ burchakli muntazam piramida asosining tomoni a ga teng, yon qirrasi asos tekisligiga a burchak ostida og‘gan bo‘lsa, piramidaga tashqi chizilgan sharning radiusini toping

VII. JISMLARNING HAJMLARI

JISMLARNING HAJMLARINI HISOBLASH FORMULALARI

1. Prizma: $V = SH$, bunda S — asos yuzi, H — balandlik.
2. Parallelepiped: $V_{p-d} = SH$.
3. Piramida: $V_{pr} = \frac{1}{3}SH$.

3'. Kesik piramida: $V_{k\,pr} = \frac{H}{3}(S_1 + S_2 + \sqrt{S_1 S_2});\quad S_1,\ S_2$ — asoslari yuzlari.

4. Silindr: $V_s = \pi R^2 H$, R — asos balandligi, H — balandligi.

5. Konus: $V_k = \frac{1}{3}\pi R^2 H$.

6. Kesik konus: $V_{k\,k} = \frac{\pi}{3} H(R^2 + Rr + r^2)$.

7. Shar: $V_{sh} = \frac{1}{3}\pi R^3$.

8. Shar segmenti: $V_{sh\,seg} = \pi H^2 (R - \frac{H}{3})$, H — segment balandligi, R — shar radiusi

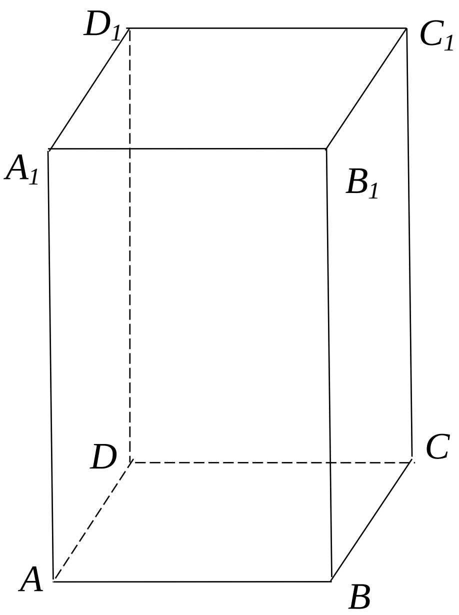
9. Shar sektori: $V_{sek} = \frac{2}{3}\pi R^2 H$, H — tegishli shar segmenti balandligi.

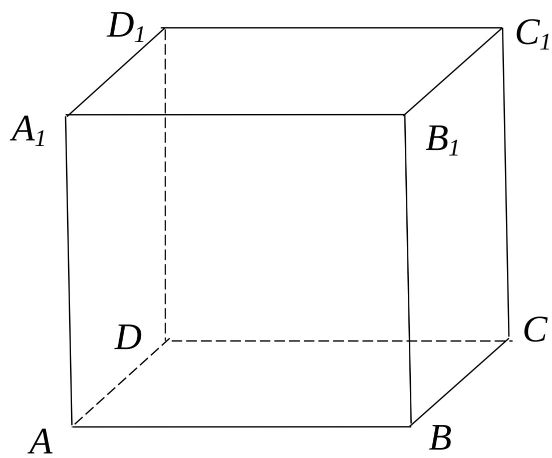
10. Aylanish jismi hajmlarini topish formulasi:
$V = \pi \int_a^b f^2(x)dx,\quad a < b.\quad y = f(x),\quad x = a,\quad x = b,\quad y = 0$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning Ox o‘qi atrofida aylani-shidan hosil bo‘lgan jism hajmi.

1- masala. To‘g‘ri burchakli parallelepipedning o‘lchamlari 15 m, 36 m va 50 m. Unga tengdosh kubning qirrasini toping (85- rasm).

Berilgan: $ABCDA_1B_1C_1D_1$ — to‘g‘ri parallelepiped, $AB = 15$ m, $BC = 36$ m, $AA_1 = h = 50$ m, $ABCD \perp AA_1$.

Topish kerak: $a_{t\,k}$—?





85- rasm

Yechish: $V_{par} = S_{asos} \cdot AA_1 = 15 \cdot 36 \cdot 50 = 27000$ m^2
Unga tengdosh kubning hajmi $V_{kub} = a^3 = 27000$.
Bundan $a^3 = 27000$, $a = \sqrt[3]{27000} = 30$ m.

Javob: 30 m.

2- masala. Og‘ma parallelepipedning asosi – tomoni 1 m ga teng bo‘lgan kvadratdan iborat. Yon qirralaridan biri 2 m ga teng va asosining o‘ziga yopishgan har bir tomoni bilan 60° li burchak tashkil etadi. Parallelepipedning hajmini toping (86- rasm).

Berilgan: $ABCDA_1B_1C_1D_1$ — og‘ma parallelepiped, $AB = 1$ m, $A_1B_1 = 2$ m, $\angle AA_1O = 60°$.

Topish kerak: $V-$?

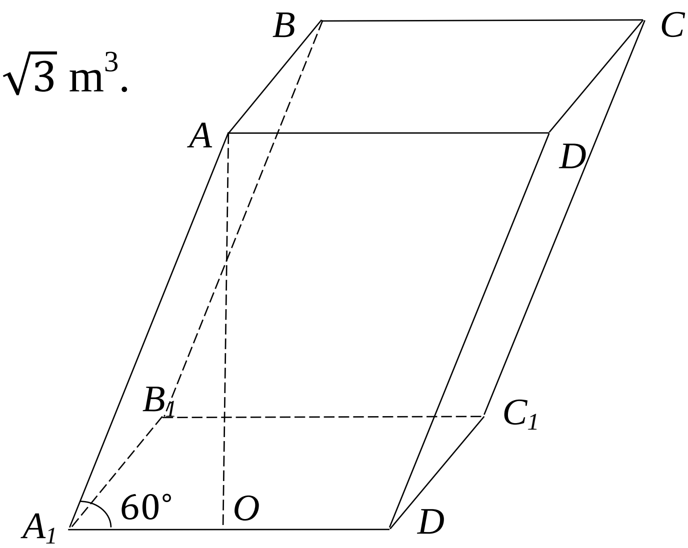
Yechish: ΔAOA_1 — to‘g‘ri burchakli uchburchak.

$sin60° = \frac{AO}{AA_1} = \frac{AO}{2}, \frac{\sqrt{3}}{2} = \frac{AO}{2}, AO = \sqrt{3} = h$.

$S_{asos} = a^2 = 1^2 = 1$ m^2.

$V = S_{asos} \cdot h = 1 \cdot \sqrt{3} = \sqrt{3}$ m^3.

Javob: $\sqrt{3}$ m^3.



86-rasm

3- masala. Ko‘ndalang kesimi — asosi 1,4 m, balandligi 1,2 m bo‘lgan teng yonli uchburchak shaklida bo‘lgan suv chiqaruvchi trubaning suv o‘tkaza olish qobiliyatini (1 soatda kub metrlar bilan) hisoblang. Suvning oqish tezligi 2 m/s (87- rasm).

Berilgan: $AB = 1{,}4$, $CD = 1{,}2$; suvning oqish tezligi $\upsilon = 2$ m/s.

Topish kerak: $\eta-$?, η— suv o‘tkazish qobiliyati (m/s).

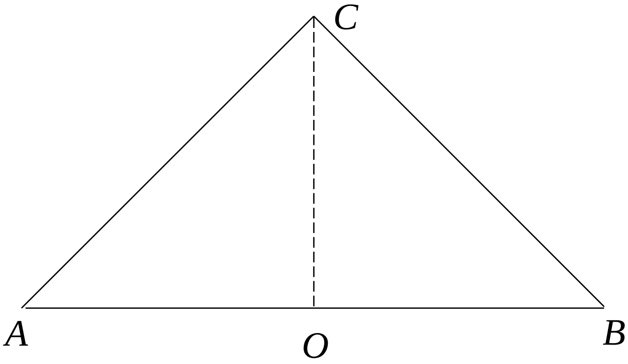
Yechish: Asosi ABC uchburchak bo‘lgan 1 m li truba olamiz. Hosil bo‘lgan uchburchakli prizma hajmi $V = S_{\Delta ABC} \cdot 1$ m^3,

$S_{\Delta ABC} = \frac{1}{2} AB \cdot CD = 0{,}84$ m^2, $V = 0{,}84$ m^3.

$\upsilon = 2$ m/s bo‘lganligidan, 1 m li masofani suv 0,5 s da bosib o‘tadi. U holda, $\eta = \frac{V}{0,5s} = 1,68\ \frac{\mathrm{m}^3}{\mathrm{sek}} = 1,68 \frac{\mathrm{m}^3}{\frac{1}{3600}\mathrm{soat}} = 1,68 \cdot 3600 =$

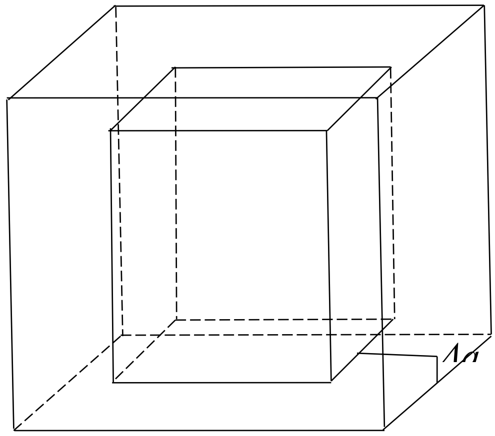
$= 1,68 \cdot 3600\ \frac{\mathrm{m}^3}{\mathrm{soat}} = 6048\ \frac{\mathrm{m}^3}{\mathrm{soat}}$.

Javob: $6048\ \frac{\mathrm{m}^3}{\mathrm{soat}}$.



87-rasm

4- masala. Metalldan yasalgan kubning tashqi qirrasi 10,2 sm va massasi 514,15 g. Devorlarining qalinligi 0,1 sm ga teng. Kub yasalgan metallning zichligini toping (88- rasm).


88-rasm

Berilgan: $a = 10,2$ sm, $m = 514,15$ g, $\Delta a = 0,01$ sm.

Topish kerak: $\rho -$?

Yechish: Kubning tashqi qirrasini a deb belgilab, uning ichiga chizilgan kub devorlar qalinligi 0,1 sm ga farq qilgan kubning qirrasini b deb olsak, $b = a - 2\Delta a = 10$ sm, $V = a^3 - b^3 =$ 61,208 sm^3, u holda $\rho = \frac{m}{V} = \frac{514,15}{60,208} = 8,4$ g/sm^3.

Javob: 8,4 g/sm^3.

5- masala. 25×12×6,5 sm o‘lchamdagi g‘ishtning massasi 3,5 kg. Uning zichligini toping (89- rasm).

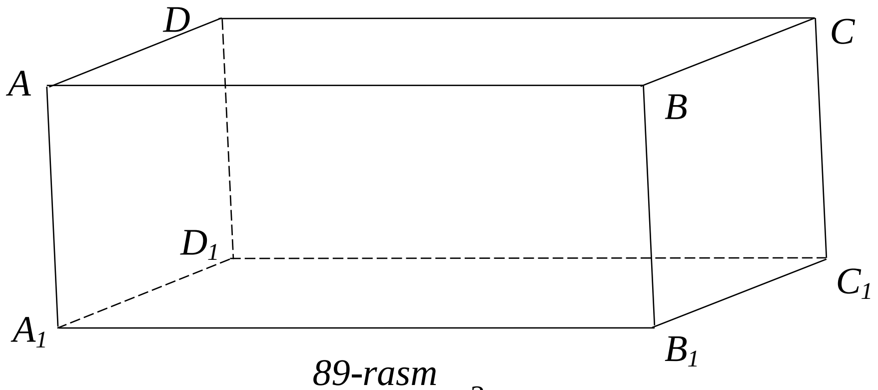
Berilgan: $ABCDA_1B_1C_1D_1$ to‘g‘ri burchakli parallelepiped, $a = AB = 12$ sm, $b = AD = 25$ sm, $c = AA_1 = 6,5$ sm, $m = 3,51$ kg.

Topish kerak: $\rho -$?

Yechish: $V = abc = 25 \cdot 12 \cdot 6,5 = 1950$ sm^3.

$\rho = \frac{m}{V} = \frac{3{,}51}{1950} = 0{,}0013$ kg/sm^3 $= 0{,}0013 \cdot 1000$ g/sm^3 $= 1{,}8$ g/sm^3 .

Javob: 1,8 g/sm^3.



89-rasm

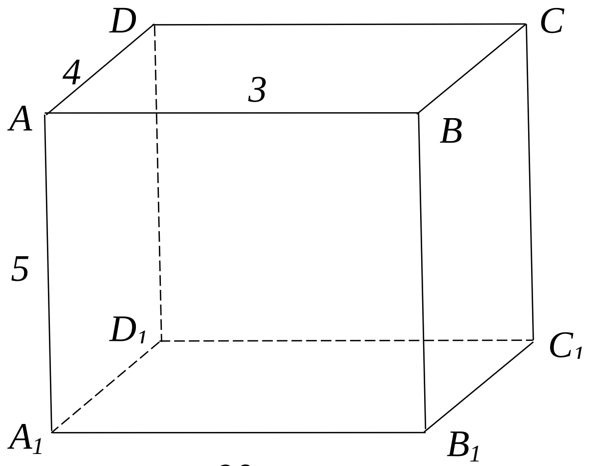
6- masala. Suv solinadigan 10 m^3 sig‘imli idishni uning tubi vazifasini bajaradigan 2,5×1,75 m o‘lchovli maydonchaga o‘rnatish talab qilinadi. Idishning balandligini toping.

Berilgan: $a = AB = 2{,}5$ m, $b = BC = 1{,}75$ m, $V = 10$ sm^3.

Topish kerak: $AA_1 = H-?$

Yechish: $V = abH$. $\quad H = \frac{V}{ab} = \frac{10}{2{,}5 \cdot 1{,}75} = 2{,}29$ m.

Javob: 2,29 m.



90-rasm

8- masala. To‘g‘ri burchakli brusokning o‘lchovlari 3 sm, 4 sm, 5 sm. Agar uning har bir qirrasini x sm orttirsak, sirti 54 sm^2 ga ortadi. Uning hajmi qancha ortadi (90- rasm)?

Berilgan: $ABCDA_1B_1C_1D_1$ — parallelepiped, $a = AB = 3$ sm, $b = AD = 4$ sm, $c = AA_1 = 5$ sm, $a_1 = 3 + x$, $b_1 = 4 + x$, $c_1 = 5 + x$, $S_x = S + 54$ sm^2.

Topish kerak: $\frac{V_x}{V} -?$

Yechish: $S = S_{yon\ s} = 2(3 \cdot 4 + 3 \cdot 5 + 4 \cdot 5) = 94$ sm^2,

$S_x = 2[(3 + x) \cdot (4 + x) + (3 + x) \cdot (5 + x) + (4 + x) \cdot (5 + x)] =$
$= 2(47 + 24x + 3x^2) = 94 + 48x + 6x^2$, $\quad S_x = S + 54$ tenglikka ko‘ra $94 + 48x + 6x^2 = 94 + 54$, $6x^2 + 48x - 54 = 0$,

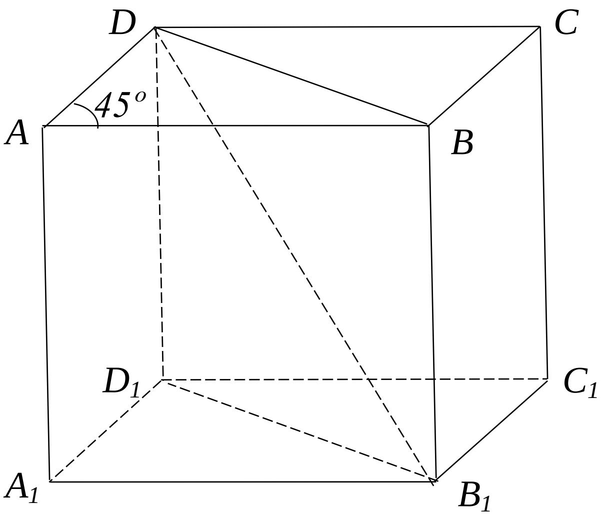
$x^2 + 8x - 9 = 0$, $x_{1,2} = -4 \pm \sqrt{16 + 9} = -4 \pm 5$, $x_1 = -9$,
$x_2 = 1$.

$x = -9$ sm bo‘lishi mumkin emas. Demak, $x = 1$ sm, u holda

$a_1 = 4$, $b_1 = 5$, $c_1 = 6$.

$V = a_1 b_1 c_1 = 4 \cdot 5 \cdot 6 = 120$ sm^3, $V = abc = 3 \cdot 4 \cdot 5 = 60$ sm^3.

Javob: $\frac{V_x}{V} = 2$ marta ortadi.



91-rasm

9- masala. To‘g‘ri parallelepipedda asosining $2\sqrt{2}$ sm li va 5 sm li tomonlari orasidagi burchak 45° ga teng. Parallelepipedning kichik diagonali 7 sm ga teng. Uning hajmini toping (91- rasm).

Berilgan: $a = AB = 5$ sm, $b = AD = 2\sqrt{2}$ sm, $DB_1 = d_1 = 7$ sm, $\angle DAB = 45°$.

Topish kerak: $V-?$

Yechish: ΔABC dan kosinuslar teoremasiga ko‘ra:

$$BD^2 = AB^2 + AD^2 - 2AB \cdot AD\cos 45° = 25 + 8 - 2 \cdot 5 \cdot 2\sqrt{2} \cdot \frac{\sqrt{2}}{2} =$$

$= 33 - 20 = 13$, $BD = \sqrt{13}$.

$\angle BDD_1$ dan: $BB_1 = H = \sqrt{B_1D^2 - BD^2} = \sqrt{49 - 13} = \sqrt{36} = 6$ sm,
$S_{asos} = ab\sin\varphi = 5 \cdot 2\sqrt{2} \cdot \sin 45° = 10$ sm^2, $V = S_{asos} \cdot H = 60$ sm^3.

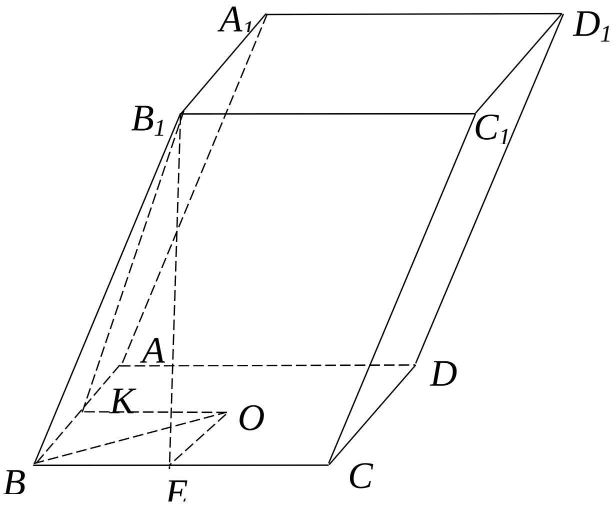
Javob: 60 sm^3.

10- masala. Og‘ma parallelepipedning asosi kvadrat bo‘lib, tomoni l m ga teng. Yon qirralaridan biri 2 m ga teng va asosning o‘ziga yopishgan har bir tomoni bilan 60° li burchak tashkil etadi. Parallelepipedning hajmini toping (92- rasm).

Berilgan: $ABCDA_1B_1C_1D_1$ — og‘ma parallelepiped,
$ABCD$ — kvadrat, $AB = 1$ m, $BB_1 = 2$ m,
$(BB_1 \hat{} AB) = (BB_1 \hat{} BC) = 60°$.

Topish kerak: $V_{p-d}-?$

Yechish: $BK \perp AB$ va $B_1E \perp BC$ ni o‘tkazamiz.
ΔB_1BK dan: $B_1K = BB_1 \sin 60°$.



92-rasm

$B_1K = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3} \Rightarrow B_1K = B_1E = \sqrt{3}$.

Xuddi shuningdek, $BK = BB_1cos60° = 1$ m.

Demak, K va E nuqtalar A va C nuqtalarga mos tushar ekan, chunki $AB = 1$ m edi. U holda, B_1 nuqtadan $(ABCD)$ asosga o‘tkazilgan perpendikular D nuqtaga tushadi. $B_1D \perp (ABCD)$, $H = B_1D -?$ $(ABCD)$ kvadratligidan, $BD = \sqrt{2}$ m.

ΔB_1DB dan: $B_1D = \sqrt{BB_1{}^2 - BD^2} = \sqrt{4-2} = \sqrt{2}$ m,

$S_{p-d} = S_{asos} \cdot H = 1 \cdot \sqrt{2} = \sqrt{2}$ m^3.

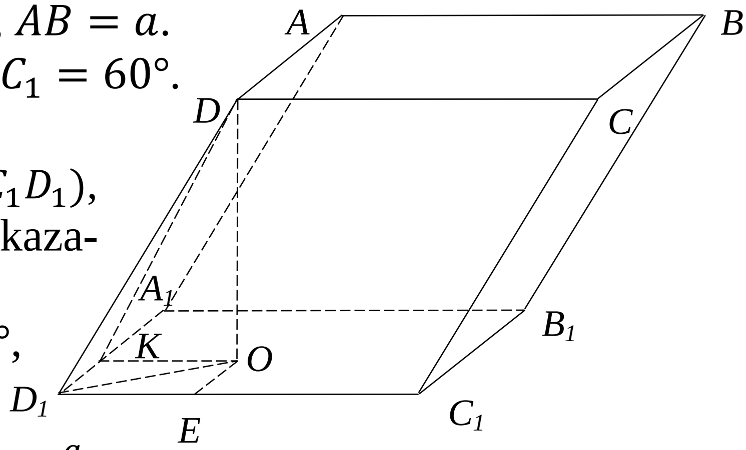
11- masala. Parallelepipedning yoqlari tomoni Q, o‘tkir burchagi 60° bo‘lgan teng romblardan iborat. Parallelepipedning hajmini toping (93- rasm).

Berilgan: $ABCDA_1B_1C_1D_1$ — og‘ma parallelepiped, $ABCD$, ABB_1A_1, ADD_1A_1 — romb, $AB = a$.
$\angle ADC = \angle DD_1A_1 = \angle DD_1C_1 = 60°$.

Topish kerak: $V_{p-d} -?$

Yechish: $B_1D \perp (A_1B_1C_1D_1)$, $DK \perp A_1D_1$, $DE \perp C_1D_1$ o‘tkazamiz.

ΔDD_1K dan $\angle DD_1K = 60°$, $D_1Kcos60° = \frac{a}{2}$.



93-rasm

Xuddi shuningdek, $D_1E = \frac{a}{2}$.

Endi D_1EOK to‘rtburchakni qaraymiz.

$\angle DD_1E = 60°$, $\Delta D_1OK = \Delta D_1OE$, chunki $D_1K = D_1E$, DO —

umumiy, uchburchaklar to‘g‘ri burchakli. Demak, D_1O bissektrisa, ya’ni $\angle KD_1O = 30°$.

ΔKD_1O dan: $D_1O = \frac{HD_1}{cos60°} = \frac{\frac{a}{2}}{\frac{\sqrt{3}}{2}} = \frac{a}{\sqrt{3}}$.

Endi, $DO \perp (A_1B_1C_1D_1)$ ekanligidan $DO = H-?$

ΔDOD_1 dan: $H = DO = \sqrt{DD_1{}^2 - D_1O^2} = \sqrt{a^2 - \frac{a^2}{3}} = a\sqrt{\frac{2}{3}}$;

$V_{p-d} = S_{asos} \cdot H = a \cdot a \cdot sin60° \cdot a\sqrt{\frac{2}{3}} = a^3 \cdot \frac{\sqrt{3}}{2} \cdot \sqrt{\frac{2}{3}} = \frac{a^3}{\sqrt{2}}$.

12- masala. Tomoni 3,2 sm va qalinligi 0,7 sm bo‘lgan muntazam sakkizburchak shaklidagi yog‘och plitkaning massasi 17,3 g. Yog‘ochning zichligini toping (94- rasm).

Berilgan: $ABCDEFMN$ — muntazam sakkizburchak, $AB = 3{,}2$;
$AA_1 = h = 0{,}7$ sm, $m = 17{,}3$ g.

Topish kerak: $\rho-?$

Yechish: $\rho = \frac{m}{V}$ formulaga ko‘ra, V ni topamiz:

$V = S_{asos} \cdot H = 0{,}7 \cdot S_{asos}$.

BO — $(ABCDEFMN)$ sakkizburchakka tashqi chizilgan aylana radiusidir. $R = \frac{a}{2 \cdot sin\frac{180°}{n}}$ formulaga ko‘ra, $BO = R = \frac{a}{2 \cdot sin\frac{180°}{n}} = \frac{a}{2 \cdot sin\frac{\pi}{8}}$.

$sin\frac{\pi}{8}$ ni jadval yordamisiz yozamiz.

$$sin\frac{\pi}{8} = \sqrt{\frac{1-cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}.$$

Sakkizburchakning har bir burchagida,

$$\angle A = \angle B = \cdots = \frac{180°(8-2)}{8} = \frac{6\pi}{8} = \frac{3\pi}{4}$$

BO — tashqi chizilgan aylana radiusi bo‘lganidan, $\angle ABO =$
$= \angle OBC = \frac{\angle B}{2} = \frac{3\pi}{4}$.

Xuddi shuningdek, $\angle BCO = \frac{3\pi}{4}$. U holda, $\angle BOC = \pi - 2 \cdot \frac{3\pi}{8} =$
$= \pi - \frac{3\pi}{4} = \frac{\pi}{4}$.

Demak,

$$S_{\Delta BOC} = \frac{1}{2} \cdot BO \cdot CO \cdot sin\angle BOC = \frac{1}{2} \cdot \frac{a}{\sqrt{2-\sqrt{2}}} \cdot \frac{a}{\sqrt{2-\sqrt{2}}} \cdot sin45° =$$

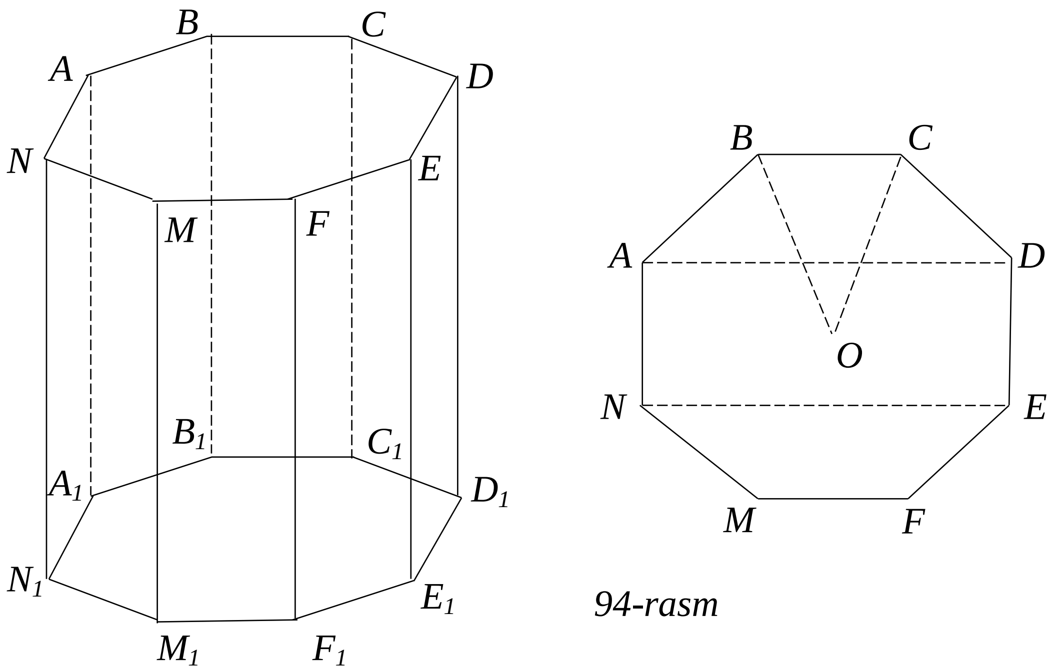
$$= \frac{1}{2} \cdot \frac{a^2}{2-\sqrt{2}} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}a^2}{4(2-\sqrt{2})}.$$

$(ABCDEFMN)$ sakkizburchak ΔBOC ga o‘xshash 8 ta uchburchakdan iborat.

Demak, $S_{asos} = 8 \cdot S_{\Delta BOC} = \frac{2\sqrt{2}a^2}{2-\sqrt{2}}$.

Shunday qilib, $V_{p-d} = 0{,}7 \cdot \frac{2\sqrt{2}a^2}{2-\sqrt{2}} \approx \frac{0{,}7 \cdot 2{,}14 \cdot 10{,}24}{2-1{,}4} \approx 30{,}6$ sm^3.

Demak, $\rho = \frac{m}{V} = \frac{17{,}3}{30{,}6} = 0{,}56$ g/sm^3.



94-rasm

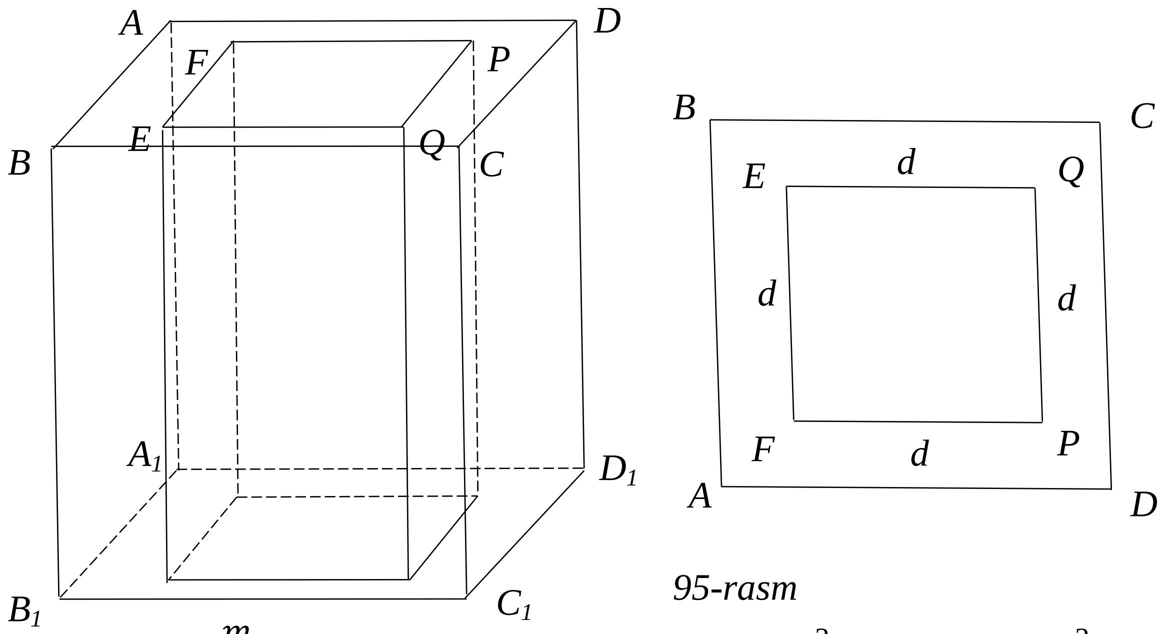
13- masala. Cho‘yan truba kvadrat shaklida bo‘lib, uning tashqi tomonining uzunligi 25 sm, devorlarining qalinligi 3 sm 1 metr uzunlikdagi trubaning massasi qancha? Cho‘yanning zichligi 7,3 g/sm^3 (95- rasm).

Berilgan: $ABCD, EFPQ$ — kvadrat, $AB = 25$ sm,
$H = AA_1 = 1$ m$= 100$ sm, $d = 3$ sm, $\rho = 7{,}3$ g/sm^3.

Topish kerak: $m-?$

Yechish: $EF = AB - 2d = 25 - 6 = 19$ sm, $V_1 = S \cdot H =$
$= 625 \times 100 = 62500$ sm^3, $V_2 = S_{EFPQ} \cdot H = 361 \cdot 100 = 36100$ sm^3.

$V = V_1 - V_2 = 62500 - 36100 = 26400$ sm^3.



95-rasm

$\rho = \frac{m}{V} \Rightarrow m = \rho V$, $\quad m = 7{,}3$ g/sm^3 $\cdot\, 26400$ sm^3 $= 192720$ g$==$ $192{,}7$ kg.

Javob: 192,7 kg.

14- masala. Uchburchakli muntazam prizma asosining tomoni Q ga teng, yon sirti asoslari yuzlarining yig'indisiga teng. Uning hajmini toping (96- rasm).

Berilgan: ΔABC — muntazam uchburchak, $AB = BC = SA = Q$, $S_{yon} = 2S_{asos}$.

Topish kerak: $V-?$

Yechish: ΔABC — muntazam, $S_{\Delta ABC} = \frac{a^2}{4}\sqrt{3}$ va H — balandlikka ko'ra, masala shartidan, $3aH = 2\frac{a^2}{4}\sqrt{3}$, $\quad H = \frac{a}{6}\sqrt{3}$.

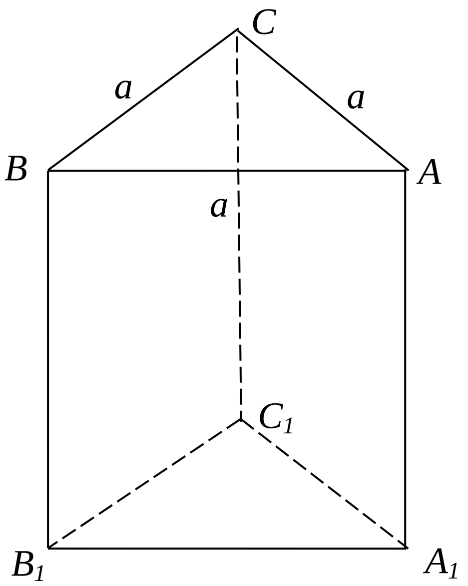
U holda, $V = S_{asos} \cdot H = \frac{a^2}{4}\sqrt{3} \cdot \frac{a}{6}\sqrt{3} = \frac{3a^3}{24} = \frac{a^3}{8}$.

Javob: $\frac{a^3}{8}$.

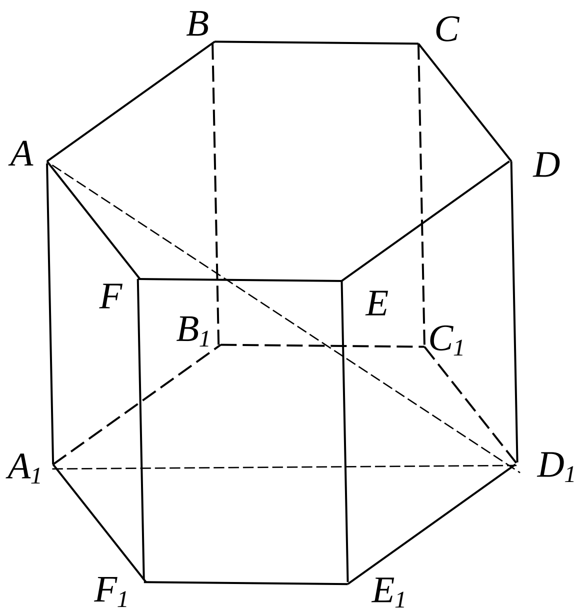
15- masala. Oltiburchakli muntazam prizmada eng katta diagonal kesimning yuzi 4 sm^2, ikkita qarama-qarshi yon qirralari orasidagi masofa 2 m ga teng. Prizmaning hajmini toping (97- rasm).

Berilgan: $ABCDEF$ — muntazam oltiburchak, $S_{ADD_1A_1} = 4$ m^2, $AD = 2$ m.

Topish kerak: $V-?$



96-rasm



97- rasm

Yechish: ADD_1A_1 — to‘g‘ri burchakli to‘rtburchak, demak, $S_{ADD_1A_1} = AA_1 \cdot AD$, bundan, $H = 2AA_1 \Rightarrow AA_1 = 2$ m. Demak, $H = AA_1 = 2$ m. Endi oltiburchakning yuzini topamiz. $ABCDEF$ — muntazam oltibutchak ekanligidan, $\angle A = \angle B = \cdots = \angle A = 120°$, bundan, $\angle BAK = 60°$. Faraz qilaylik, $AB = BC = \cdots = a$ bo‘lsin. U holda ΔABK dan: $AK = AB \cdot \angle BAK = a \cdot cos60° = \frac{a}{2}$.

Xuddi shuningdek, $DN = \frac{a}{2}$, u holda, $AD = AK + NK + DN =$ $= \frac{a}{2} + a + \frac{a}{2} = 2a = 2 \Rightarrow$ a = 1 m.

$S_{ABCD} = \frac{AD+BC}{2} \cdot BK = \frac{2+1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$ m.

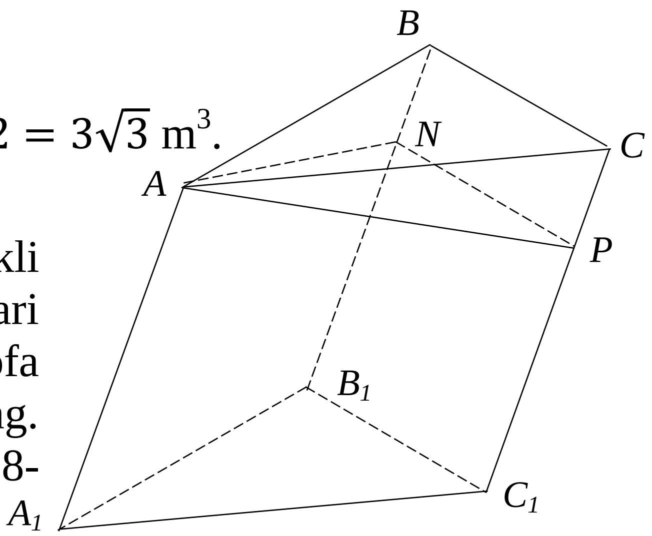
(Biz bu yerda $BK = AB \cdot sin60°$ dan foydalandik.)

$S_{asos} = 2S_{tr} = \frac{3\sqrt{3}}{2}$.

Demak, $V = S_{asos} \cdot H = \frac{3\sqrt{3}}{2} \cdot 2 = 3\sqrt{3}$ m^3.

Javob: $3\sqrt{3}$ m^3.

16- masala. Uchburchakli og‘ma prizmaning yon qirralari 15 m ga, ular orasidagi masofa esa 26 m, 25 m va 17 m ga teng. Prizmaning hajmini toping (98-rasm).



98-rasm

Berilgan: $ABCA_1B_1C_1$ og‘ma prizma, $AA_1 = 15$ m, $\rho(AA_1, BB_1) = 17$ m, $\rho(AA_1, CC_1) = 26$ m, $\rho(BB_1, CC_1) = 25$ m.

Topish kerak: $V-?$

Yechish: A nuqtadan CC_1 va BB_1 tomonlarga AD va AN perpendikular o‘tka-zamiz. Hosil bo‘lgan APN uchburchak tomonlariga AA_1, BB_1 va CC_1 qirralar perpendikulardir. Demak, APN uchburchakni prizmaning asosi deb, faraz qilsak, $AA_1 = 15$ m uning baland-ligi bo‘ladi. Bundan:

$V = S_{\Delta APN} \cdot AA_1$.

Yasashga ko‘ra $\rho(AA_1, BB_1) = AN = 17$ m,
$\rho(AA_1, CC_1) = AP = 26$ m, $\rho(BB_1, CC_1) = PN = 25$ m.

ΔAPN uchun, $p = \frac{a+b+c}{2} = 35$, $S_{\Delta APN} = \sqrt{34 \cdot 17 \cdot 8 \cdot 9} =$
$= \sqrt{2 \cdot 17^2 \cdot 2^2 \cdot 3^2} = 204$ m^2.

(1) ga ko‘ra, $V = 204 \cdot 15 = 3060$ m^3.

Javob: 3060 m^3.

17- masala. Ko‘ndalang kesimi — asosi 1,4 m, balandligi 1,2 m bo‘lgan teng yonli uchburchak shaklida bo‘lgan suv chiqaruvchi trubaning suv o‘tkaza olish qobiliyatini (1 soatda kub metrlar bilan) hisoblang. Suvning oqish tezligi 2 m/s (99- rasm).

Berilgan: $AB = 1{,}4$, $CD = 1{,}2$; suvning oqish tezligi $\upsilon = 2$ m/s.

Topish kerak: $\eta-?$, η— suv o‘tkazish qobiliyati (m/s).

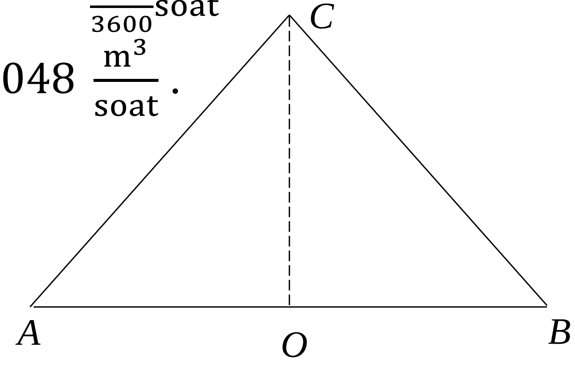
Yechish: Asosi ABC uchbur-chak bo‘lgan 1 m li truba olamiz. Hosil bo‘lgan uchburchakli prizma hajmi $V = S_{\Delta ABC} \cdot 1$ m^3,

$S_{\Delta ABC} = \frac{1}{2} AB \cdot CD = 0{,}84$ m^2, $V = 0{,}84$ m^3.

$\upsilon = 2$ m/s bo‘lganligidan, 1 m li masofani suv 0,5 s da bosib o‘tadi. U holda,

$$\eta = \frac{V}{0{,}5s} = 1{,}68\ \frac{\mathrm{m}^3}{\mathrm{sek}} = 1{,}68 \frac{\mathrm{m}^3}{\frac{1}{3600}\mathrm{soat}} = 1{,}68 \cdot 3600 =$$
$$= 1{,}68 \cdot 3600\ \frac{\mathrm{m}^3}{\mathrm{soat}} = 6048\ \frac{\mathrm{m}^3}{\mathrm{soat}}.$$

Javob: $6048\ \frac{\mathrm{m}^3}{\mathrm{soat}}$.



99-rasm

18- masala. Temir yo‘l ko‘tarmasining kesimi trapetsiya shaklida bo‘lib, uning pastki asosi 14 m, yuqori asosi 8 m, balandligi 3,2 m. 1 km ko‘tarmaga qancha kub metr tuproq to‘g‘ri kelishini hisoblang (100- rasm).

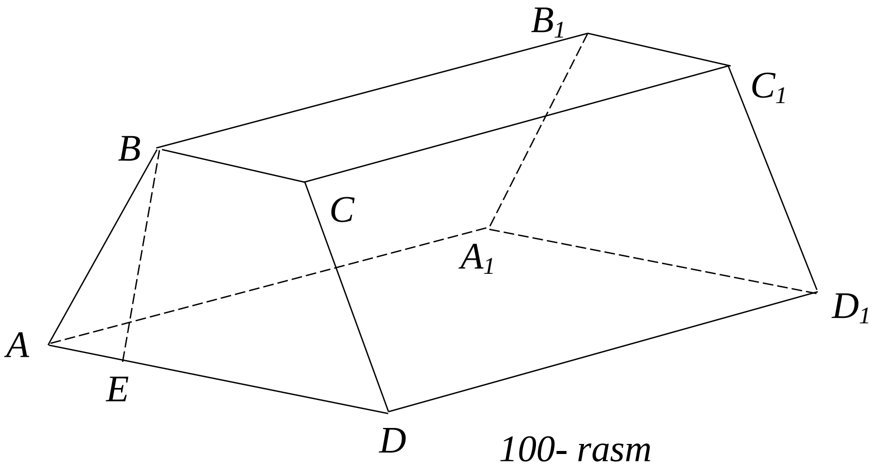
Berilgan: $ABCD$ — teng yonli trapetsiya, $AD = 14$ m, $BC = 8$ m, $BE \perp AD$, $BE = 3{,}2$ m, $AA_1 = H = 1$ km = 1000 m.

Topish kerak: $V-$?

Yechish: $V = S \cdot H$ formulaga ko‘ra, H = 1000 m.

$S = S_{tr} = \frac{AD+BC}{2} \cdot BE = 11 \cdot 35{,}2$ m^2, $V = 35200$ m^3.

Javob: $V = 35200$ m^3.



100- rasm

19- masala. Kesik konus asoslarining radiuslari R va r, yasovchisi asos tekisligiga 45° burchak ostida og‘gan. Hajmini toping.

Berilgan: $AO_1 = r,\ BO = R, \angle ABO = 45°$.

Topish kerak: $V_{k\,k}-$?

Yechish: $BE = BO - AO_1 = R - r$.

ΔABE dan: $AE = BE \cdot tg\angle ABO = (R - r)tg45° = R - r$, $AE = OO_1 = H$.

Demak $V_{k\,k} = \frac{\pi H}{3}(R^2 + Rr + r^2) = \frac{\pi(R^3-r^3)}{3}$, Agar $R > r$ bo‘lsa, $V_{k\,k} = \frac{\pi(R^3-r^3)}{3}$, agar $R < r$ bo‘lsa, $V_{k\,k} = \frac{\pi(r^3-R^3)}{3}$

Demak, $\frac{\pi(R^3-r^3)}{3}$.

20- masala. Suyuqlik balandligi 0,18 m va asosining diametri 0,24 m bo‘lgan konus shaklidagi idishdan olinib, asosining diametri 0,1 m ga teng silindr idishga quyildi. Bu idishdagi suyuqlikning balandligini aniqlang.

Berilgan: $H_k = 0{,}18$ m, $D_k = 0{,}24$ m, $D_s = 0{,}1$ m, $V_k = V_s$

Topish kerak: $H_s-?$

Yechish: $V_k = \frac{1}{3}\pi {R_k}^2 H_k = \frac{1}{12}\pi {D_k}^2 H_k$, $V_s = \pi {R_s}^2 H_s = \frac{1}{4}\pi {D_s}^2 H_s$.

$V_k = V_s \Rightarrow H_s = \frac{4{D_k}^2 H_k}{12{D_s}^2} = \frac{{D_k}^2 H_k}{3{D_s}^2} = \frac{(0{,}24)^2\cdot 0{,}18}{3\cdot(0{,}1)^2} = 0{,}35.$

Javob: $\approx 0{,}35$ m.

21- masala. Katetlari a va b bo‘lgan to‘g‘ri burchakli uchburchak gipotenuzasi atrofida aylanmoqda. Hosil qilingan jismning hajmini toping (101- rasm).

Berilgan: ΔABC — to ‘g‘ri burchakli, $\angle C = 90°$, $AC = a$, $BC = b$, ABC uchburchak AB tomon atrofida aylanadi.

Topish kerak: $V-?$, (V — aylanish jismining hajmi).

Yechish: ABC uchburchak AB atrofida aylanib, ikkita konus hosil qiladi: l) $V_k(ACC_1)$; 2) $V_k(BCC_1)$, u holda

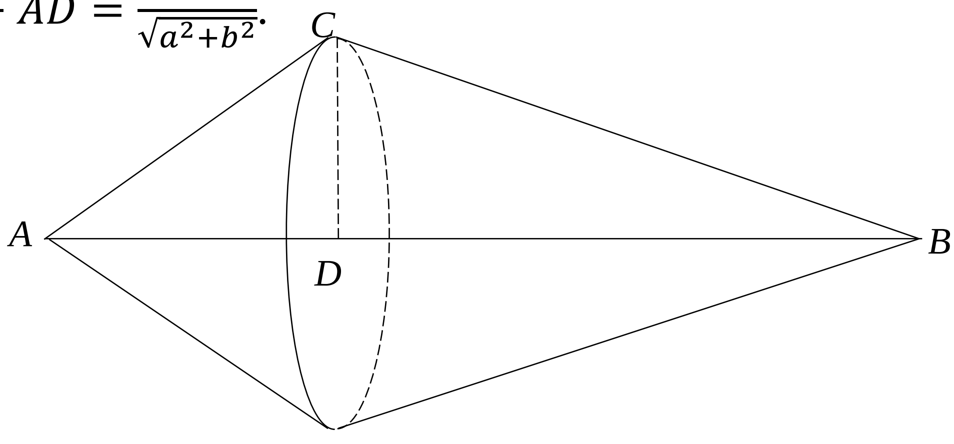
$V = V_k(ACC_1) + V_k(BCC_1)-?$

ΔABC dan: $AB = \sqrt{a^2+b^2}$, $S_{\Delta ABC} = \frac{1}{2}\cdot AC\cdot BC = \frac{1}{2}\cdot AB\cdot CD \Rightarrow$

$\Rightarrow CD = \frac{AC\cdot BC}{AB} = \frac{ab}{\sqrt{a^2+b^2}}$.

ΔACD dan: $AD = \sqrt{AC^2 - CD^2} = \sqrt{a^2 - \frac{a^2b^2}{a^2+b^2}} = \frac{a^2}{\sqrt{a^2+b^2}}$, u holda

$BD = AB - AD = \frac{b^2}{\sqrt{a^2+b^2}}$.



101- rasm

Shunday qilib, $V_k(ACC_1)$ konus uchun, $R = \frac{ab}{\sqrt{a^2+b^2}}$, $H = \frac{a^2}{\sqrt{a^2+b^2}}$.

$V_k(BCC_1)$ konus uchun, $R = \frac{ab}{\sqrt{a^2+b^2}}$, $H = \frac{b^2}{\sqrt{a^2+b^2}}$.

$$V = \frac{1}{3} \cdot \pi \cdot \frac{a^2 b^2}{a^2+b^2} \cdot \left(\frac{a^2}{\sqrt{a^2+b^2}} + \frac{b^2}{\sqrt{a^2+b^2}}\right) = \frac{\pi}{3} \cdot \frac{a^2 b^2}{\sqrt{a^2+b^2}}.$$

MUSTAQIL ISHLASH UCHUN

1. Qirralari 3 sm, 4 sm, 5 sm bo‘lgan jezdan yasalgan uchta kubdan bitta kub quyilgan. Bu kub qirrasining uzunligini toping.

2. Kubning har bir qirrasi 1 m orttirilsa, uning hajmi 125 marta ortadi. Qirrasini toping.

3. To‘g‘ri parallelepipedning asosi yuzi 1 m^2 bo‘lgan rombdan iborat. Diagonal kesimlarining yuzlari 3 m^2 va 6 m^2 Parallelepipedning hajmini toping

4. Avvalgi masalani umumiy holda, ya’ni rombning yuzi Q, diagonal kesimlaming yuzlari M, N deb faraz qilib yeching.

5. 1) uchburchakli; 2) to‘rtburchakli; 3) oltiburchakli muntazam prizma asosining a tomoni va yon qirrasi bo‘yicha hajmini toping.

6. To‘rtburchakli muntazam prizmaning diagonali 3,5 sm, yon yog‘ining diagonali 2,5 sm ga teng. Prizmaning hajmini toping.

7. Uchburchakli to‘g‘ri prizma asosining tomonlari 4 sm, 5sm, 7 sm g, yon qirrasi esa asosining katta balandligiga teng. Prizmaning hajmini toping.

8. Prizmaning asosi uchburchak bo‘lib, uning bir tomoni 2 sm ga teng, qolgan ikki tomoni 3 sm dan. Yon qirrasi 4 sm ga teng va asos tekisligi bilan 45° li burchak tashkil etadi. Unga tengdosh kubning qirrasini toping.

9. Og‘ma prizmaning asosi tomoni a ga teng bolgan teng tomonli uchburchak, yon yoqlaridan biri asosga perpendikular va kichik diagonali c ga teng rombdan iborat. Prizmaning hajmini toping.

10. Og‘ma parallelepipedning har bir qirrasi 1 sm dan. Parallelepipedning uchlaridan biridagi uchala yassi burchak o‘tkir bo‘lib, har biri 2α dan. Parallelepipedning hajmini toping.

11. Uchburchakli piramidaning yon qirralari o‘zaro perpendikular bo‘lib, har biri b ga teng. Piramidaning hajmini toping.

12. Oktaedrning a qirrasi bo‘yicha hajmini toping.

13. Piramidaning asosi tomonlari 9 m va 12 m bo‘lgan to‘g‘ri to‘rtburchak, hamma yon qirralari 21,5 m ga teng. Piramidaning hajmini toping.

14. Piramidaning asosi tomonlari 6 sm, 6 sm va 8 sm bo‘lgan teng yonli uchburchak. Hamma yon qirralari 9 sm ga teng. Piramidaning hajmini toping.

15. Uchburchakli piramidaning bitta yon qirrasi 4 sm ga teng, qolganlarining har biri 3 sm dan. Piramidaning hajmini toping.

16. Har bir qirrasi a ga teng bo‘lgan muntazam oltiburchakli prizmaga ichki chizilgan silindrning hajmini toping.

17. Devorining qalinligi 4 mm bo‘lgan qo‘rg‘oshin trubaning (qo‘rg‘oshinning zichligi 11,4 g/sm^3) ichki diametri 13 mm. Shunday 25 m li trubaning massasi (og‘irligi)ni toping.

18. Asoslarining radiuslari 4 sm va 22 sm bo‘lgan kesik konusni shunday balandlikdagi unga tengdosh silindrga aylantirish talab qilinadi. Bu silindr asosining radiusi qanchaga teng bo‘ladi.

19. Asoslarining berilgan R, r radiuslari bo‘yicha kesik konus bilan butun konus hajmlarining nisbatini aniqlang.

20. Massasi 1 kg bo‘lgan qo‘rg‘oshin bo‘lagi bor. Bu bo‘lakdan diametri 1 sm bo‘lgan sharchalardan nechta quyish mumkin (Qo‘rg‘oshinning zichligi 11,4 g/sm^3).

VIII. JISMLAR SIRTLARINING YUZLARI

JISM SIRTLARINING YUZLARI FORMULAUARI

1. Silindr s(R, H);

a) $S_{yon\,s} = 2\pi RH$.

b) $S_{t\,s} = 2\pi R(H + R)$.

2. Konus $K(R, H)$;

a) $S_{yon\,s} = \pi Rl$.

b) $S_{t\,s} = 2\pi R(l + R)$, bu yerda l — yasovchi:

3. Kesik konus $K(R, r, H, l)$, bu yerda l — yasovchi:

a) $S_{yon\,s} = \pi l(R + r)$.

b) $S_{t\,s} = \pi l(R + r) + \pi(R^2 + r^2)$.

4. Shar sh(O, R);

$S_{shar} = 4R^2$.

5. Sferik segment

$S = 2\pi RH$, H — segment balandligi.

1- masala. Ikki shar sirtlarining nisbati $m\colon n$ ga teng. Ular hajmlarining nisbatini toping.

Berilgan: $\mathrm{sh}(O)$, $\mathrm{sh}(O_1)$ sharlar, $S_1\colon S_2 = m\colon n$

Topish kerak: $V_1\colon V_2 - ?$

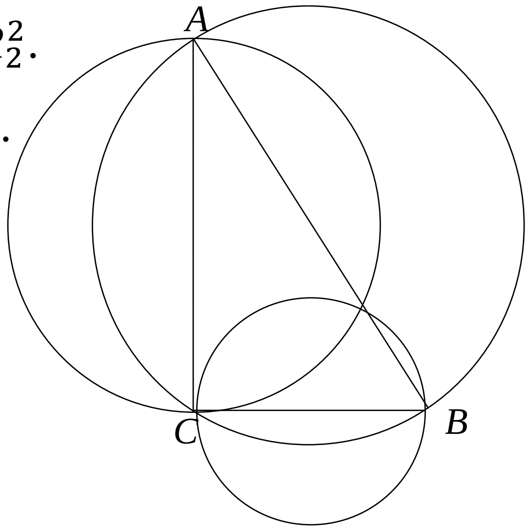
Yechish: $S_1 = 4\pi R_1^2$; $S_2 = 4\pi R_2^2$.

$$\frac{S_1}{S_2} = \frac{4\pi R_1^2}{4\pi R_2^2} = \left(\frac{R_1}{R_2}\right)^2 = \frac{m}{n} \Rightarrow \frac{R_1}{R_2} = \sqrt{\frac{m}{n}}.$$

Demak,

$$\frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \left(\frac{R_1}{R_2}\right)^3 = \frac{m}{n}\sqrt{\frac{m}{n}}.$$

Javob: $\frac{m}{n}\sqrt{\frac{m}{n}}$.



102- rasm

2- masala. Gipotenuza va katetlar uchta sharning diametrlaridir. Ularning sirtlari orasida qanday bog‘lanish bor (102- rasm)?

Berilgan: ΔABC — to‘g‘ri burchakli, $AB = 2R = c$, $AC = 2r_1 = a$, $BC = 2r_2 = b$.

Topish kerak: $S = f(S_1, S_2) - ?$

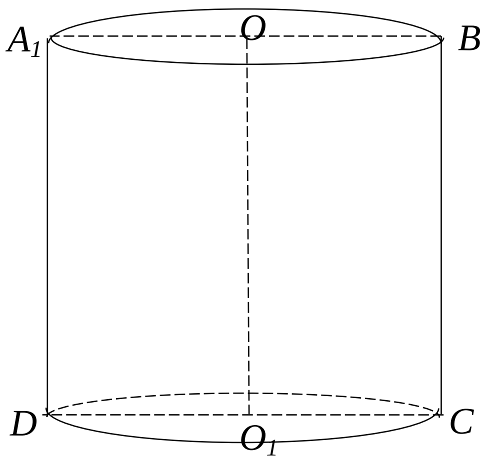
Yechish: Faraz qilaylik, $AB = 2R$ bo‘lsin.

ΔABC da, $AC = a$, $BC = b$ bo‘lsin. Pifagor teoremasiga ko‘ra: $a^2 + b^2 = 4R^2$.

Agar AC tomonni diametr qilib shar chizsak, uning radiusi $r_1 = \frac{a}{2}$, xuddi shuningdek, diametri BC bo‘lgan shar radiusi $r_2 = \frac{b}{2}$ bo‘ladi. Endi ularning sirtlarini topamiz: $S_1 = 4\pi r_1^2 = \pi a^2$, $S_2 = 4\pi r_2^2 = \pi b^2$ va hadma-had qo‘shsak $S_1 + S_2 = \pi a^2 + \pi b^2 = \pi(a^2 + b^2) = 4\pi R^2 = S$. $S_1 + S_2 = S$ bo‘ladi.

Demak, katta sirt qolgan ikki sirt yig‘indisiga teng.

3- masala. Diametri 65 sm bo‘lgan silindr trubaning balandligi 18 sm. Agar parchinlashga hamma materialning 10%i ketsa, bunday trubani tayyorlash uchun qancha tunuka kerak (103-rasm)?



103-rasm

Berilgan: $d = AB = 65$ sm, $H = AD = 18$ sm parchinlashga 10% ketadi.

Topish kerak: $\bar{S}$ ketgan material.

Yechish: Silindrning yon sirti $S_{yon\,s} = 2\pi RH = 3{,}14 \cdot 65 \cdot 18 =$
$= 3{,}14 \cdot 0{,}05 \cdot 18 = 96{,}7$ m^2, S -100%.
$S_{yon\,s} - 90\% \Rightarrow \bar{S} = \frac{S_{yon\,s} \cdot 100}{90} = \frac{967}{9} = 107{,}4$.

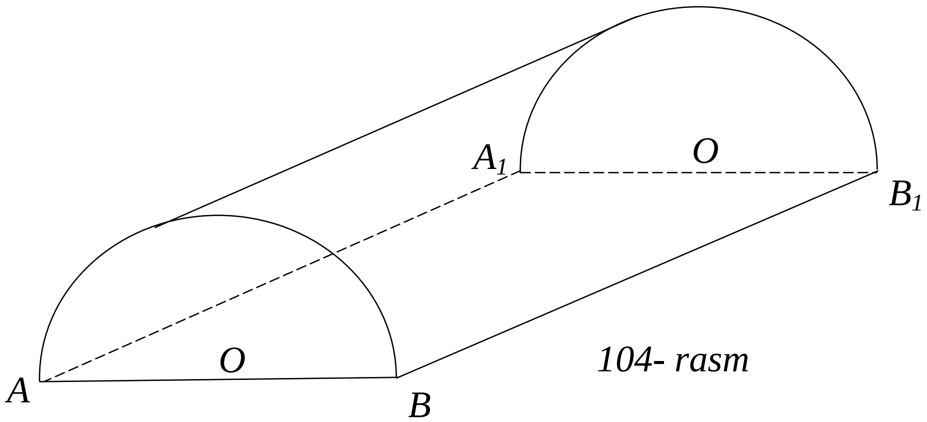
Javob: 107,4 m^2.

4-masala. Yerto‘ladagi yarim silindrik gumbazning uzunligi 6 m va diametri 5,8 m. Yerto‘laning to‘la sirtini toping (104- rasm).

Berilgan: $AA_1 = 6$ m, $AB = 5{,}8$ m.

Topish kerak: $\bar{S}_{t\,s}$$-$?

Yechish: $AB = 2R = 5{,}8$ m $\Rightarrow R = 2{,}9$ m. $AA_1 = H = 6$ m.

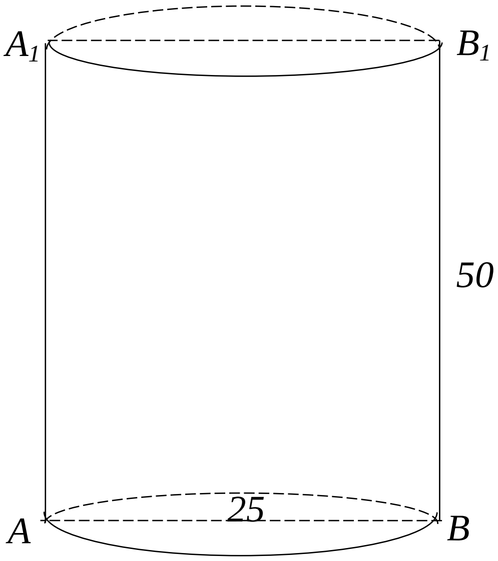


104- rasm

$$\frac{1}{2}S_{t\,s} = \frac{1}{2}(2\pi RH + 2\pi R^2) = \pi RH + \pi R^2 = \pi R(H + R) =$$
$= 3{,}14 \cdot 2{,}9(6 + 2{,}9) = 3{,}14 \cdot 2{,}9 \cdot 8{,}9 \approx 81$. U holda

$\bar{S}_{t\,s} = 81 + 65{,}8 \approx 116$.

Javob: 116 m^2.



105- rasm

5- masala. Dumaloq metal listdan diametri 25 sm va balandligi 50 sm bo‘lgan silindr stakan shtamplangan. Shtamplashda listning yuzi o‘zgarmagan deb faraz qilib, listning diametrini toping (105- rasm).

Berilgan: $D_s = 25$ sm, $H_s = 50$ sm, $D(R)$— R radiusli doira, $S(D) = S_{t\,s}$.

Topish kerak: $2R-?$

Yechish: Stakan bo‘lgani uchun, uning bitta asosi bor.

$$S_{t\,s} = 2\pi RH + 2\pi R^2 = \pi\, D_s H + \frac{\pi {D_s}^2}{4} = D_s\left(H + \frac{D_s}{4}\right)\pi =$$

$$= 25\left(50 + +\frac{25}{4}\right)\pi = \frac{25 \cdot 225}{4}\pi$$

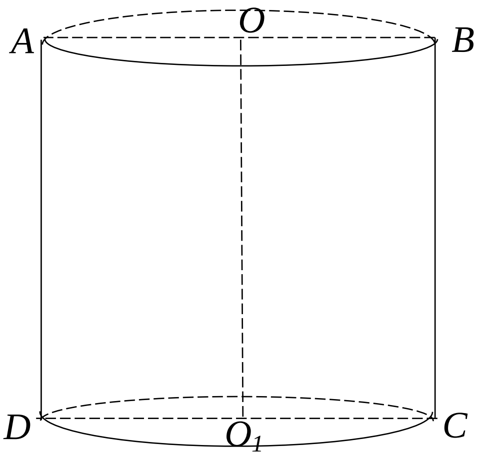
$$S(D) = \pi R^2, \pi R^2 = \frac{25\cdot225}{4}\pi \Rightarrow \mathrm{R} = \sqrt{\frac{25\cdot225}{4}} = \frac{5\cdot15}{2} = \frac{75}{2}, 2R = 75.$$

Javob: 75 sm.

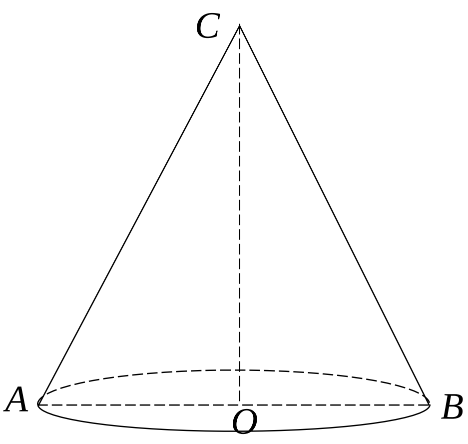
6- masala. Silindr asosining yuzi Q, o‘q kesimining yuzi M. Silindrning to‘la sirtini toping (106- rasm).

Berilgan: $S_{ABCD} = M,\ S_{asos} = Q.$

Topish kerak: $S_{t\,s}-?$



106-rasm



107- rasm

Yechish: $S_{asos} = Q \Rightarrow \pi R^2 = Q$, $R = \sqrt{\frac{Q}{\pi}}$, $S_{ABCD} = 2RH \Rightarrow 2RH = M$, $H = \frac{M}{2R} = \frac{M}{2}\sqrt{\frac{\pi}{Q}}$.

Demak, $S_{t\,s} = 2\pi R(H + R) = 2\pi\sqrt{\frac{Q}{\pi}}\left[\frac{M}{2}\sqrt{\frac{\pi}{Q}} + \sqrt{\frac{Q}{\pi}}\right] = 2\pi\sqrt{\frac{Q}{\pi}} \cdot \frac{M\pi+2Q}{2\sqrt{Q\pi}} =$ $= \sqrt{Q} \cdot \frac{M\pi+2Q}{\sqrt{Q}} = M\pi + 2Q.$

Javob: $M\pi + 2Q$.

7- masala. Balandligi 3,5 m, asosining diametri 4 m bo‘lgan konussimon palatka qalin mato bilan yopilgan. Palatkaga necha kvadrat metr qalin mato ketgan (107- rasm)?

Berilgan: $AB = D = 4$ m, $CO = H = 3{,}5$ m.

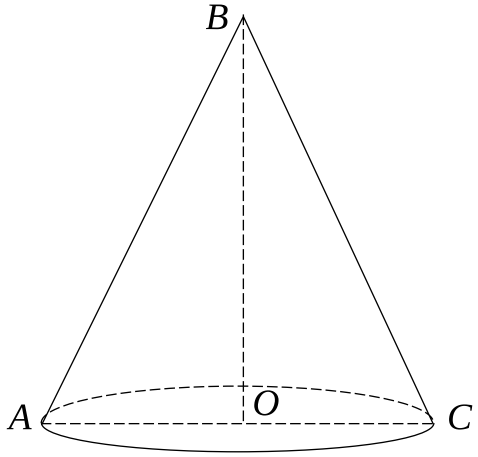
Topish kerak: $S_{yon\,s}-?$

Yechish: $S_{yon\,s} = 2\pi Rl$, bunda $l-?$

ΔAOC dan: $l = \sqrt{AO^2 + CO^2} = \sqrt{2^2 + 3{,}5^2} = 4$ m.

$R = \frac{D}{2} = 2$ m $\Rightarrow S_{yon\,s} = 3{,}14 \cdot 2 \cdot 4 \approx 25{,}3$.

Javob: $\approx 25{,}3$ m^2.



108- rasm

8- masala. Silos minorasining tomi konus shaklida. Tomning balandligi 2 m, minoraning diametri 6 m. Tomning sirtini toping (108- rasm).

Berilgan: $2R = 6$ m, $H = 4$ m.

Topish kerak: $S_{yon\,s}-?$

Yechish: $S_{yon\,s} = 2\pi Rl$ formulaga ko‘ra topiladi. $l = AB-?$

ΔAOB dan $AO = R = 3$, $BO = H = 4$.

Demak, $l = \sqrt{3^2 + 4^2} = 5$

Shunday qilib, $S_{yon\,s} = \pi \cdot 3 \cdot 5 = 15\pi$ m^2.

Javob: 15π m^2.

9- masala. Teng yonli konus-ning (kesimi muntazam uchburchak) yon sirti bilan to‘la sirtining nisbatini toping (109- rasm).

Berilgan: ΔABC — muntazam.

Topish kerak: $S_{yon\,s} : S_{t\,s}-?$

Yechish: Faraz qilaylik, $AC = 2R$ bo‘lsin. ΔABC muntazam ekanligidan, $AB = l = 2R$, $S_{yon\,s} = 2\pi Rl = \pi R \cdot 2R = 2\pi R^2$,

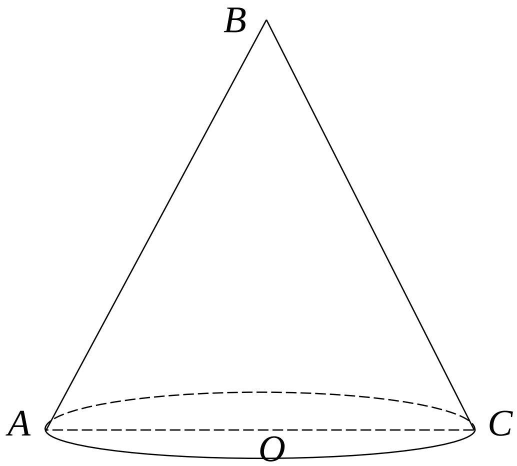
$S_{t\,s} = S_{yon\,s} + S_{asos} = 2\pi R^2 + \pi R^2 = 3\pi R^2$.

Demak, $S_{yon\,s} : S_{t\,s} = 2\pi R^2 : 3\pi R^2 = 2:3$

Javob: $2:3$ kabi.

Bundan, bu silindrning to‘la sirti S_j jism sirtiga teng.

Demak, $S_j = S_{t\,s} = 2\pi RH + 2\pi R^2 = 2\pi a \cdot a + 2\pi a^2 = 4\pi a^2$.



109- rasm

10- masala. Sharning radiusi 15 sm shar markazidan 25 sm naridagi nuqtadan shu shar sirtining ko‘rinib turgan qismining yuzini toping (110- rasm).

Berilgan: $OC = R = 15$ sm, $OA = 25$ sm.

Topish kerak: S_{seg}—? (S_{seg}— sferik segment sirti).

Yechish: AB chiziq sharga urinmadir. Shu sababli, $OB \perp AB \Rightarrow \Delta OBA$ — to‘g‘ri burchakli. Bu uchburchakning BO_1 balandligini topamiz.

$S_{\Delta OAB} = \frac{1}{2} \cdot BO_1 \cdot AO = \frac{1}{2} \cdot OB \cdot AB \Rightarrow BO_1 = \frac{OB \cdot AB}{OA}$, bu yerda, $OB = R = 15$, $OA = 25$.

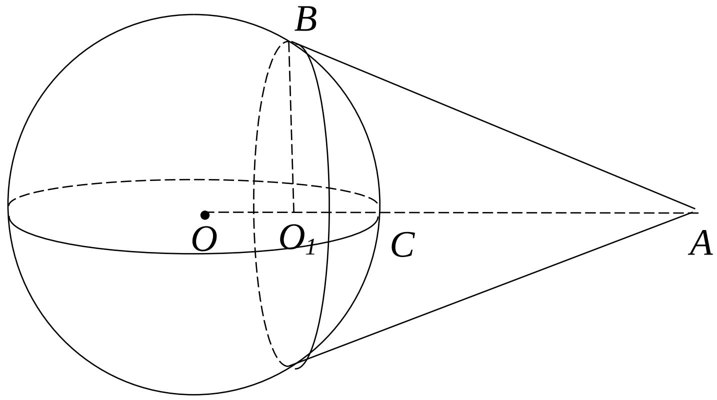
$AB = \sqrt{AD^2 - OB^2} = \sqrt{25^2 - 15^2} = \sqrt{625 - 225} = 20$.

Demak, $BO_1 = \frac{15 \cdot 20}{25} = 12$.

ΔBOO_1 dan: $BO_1 = \sqrt{AD^2 - OB^2} = \sqrt{15^2 - 12^2} = \sqrt{225 - 144} = 9$, bundan, $CO_1 = H = OC - OO_1 = 15 - 9 = 6$ sm. Bu yerda H — sferik segment balandligidir. Shunday qilib, formulaga ko‘ra:

$S_{seg} = 2\pi \cdot 15 \cdot 6 = 180\pi$ sm^2.

Javob: 180π sm^2.



110- rasm

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MUNDARIJA

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STEREOMETRIK MASALALARNI YECHISH

Umumiy o‘rta ta’lim maktablari va akademik litseylar uchun o‘quv qo‘llanma