Fundamentals of Road Design

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W. Kühn
Germany

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Special thanks go to my wife, Heike Kühn, who often had to spend her free time alone while work was progressing on the book. As a result, I would like to dedicate the book to her.

About the Author

Wolfgang Kühn studied transportation engineering at the "Friedrich List" University of Transportation and Traffic Sciences (HfV) in Dresden and completed his degree as a "Diplomingenieur" (graduate engineer) in 1977. After obtaining a qualification to teach at a university, he obtained his doctorate (Dr.-Ing) in 1981. He was a design engineer, estimator and construction manager at various planning offices and construction companies at different levels during the period 1981 – 1990. He founded the DELTA-PLAN GmbH planning office in 1990 and he has been successfully managing this as the senior partner for 22 years. He completed his university teaching qualification by obtaining his Dr.-Ing. Habilitatus degree at the Technical University of Dresden in 2002. He was university professor at the University of Leipzig from 2004 until 2008 and he has been professor at the Institute of Transportation Systems Engineering at the University of Applied Sciences Zwickau since 1 March 2008.

During his many years of research work in the field of road design, he has produced 4 dissertations, 70 publications in professional journals and given 98 lectures at specialist conferences. He has also provided support for a number of students writing their degree dissertations or taking their master's degree or doctoral theses. He has played a significant role in further developing the body of rules and regulations as head of the "Visualization" working group and a member of the "Designing new roads" working group at the German Road and Transportation Research Association (FGSV). He was awarded the Friedrich List Prize in 2002 for his academic achievements in the field of developing new kinds of models and procedures for road design. Due to his institutional activities in transportation research he was appointed members of both the Geometric Design Committee and the Visualization in Transportation Committee of the TRB (Transportation Research Board). In 2011 Wolfgang received an outstanding paper award at the 4th International Geometric Design Symposium in Valencia (Spain).

Contents

Preface

This reference book developed as a result of Professor Dr Kühn's teaching at the Universities of Leipzig, Magdeburg and Zwickau, his long-standing practical work as a design engineer at various planning offices and his detailed research work in the German Road and Transportation Research Association (FGSV) and international working groups.

This reference book communicates the basic theoretical knowledge and the practical requirements and experience for designing, mapping, calculating and checking roads and the planning process overall – and at the same time reveals important development trends.

The reference book is guided by the current rules and regulations in Germany, it universalizes this knowledge and also integrates important current research results in road design processes.

In order to restrict the scope of the book, it only deals with the theoretical principles and knowledge and the practical experience for designing rural roads, i.e. the special features of urban roads are not mentioned here. This clear separation primarily results from the different principles and rules and regulations.

The terms, definitions, abbreviations and formula symbols are based on German usage, but can be transferred to an international framework without any difficulties.

The work is particularly designed to be a course book for students of road design. As a reference book, it supports the ongoing training process for road transportation engineers in planning offices and public bodies. Any specialist working in the transportation sector can use it as a reference book.

Wolfgang Kühn, 2013

Chapter 1

Introduction

1.1 Overview

This book addresses the fundamental concepts of road design based on the procedure adopted in Germany and this can also be adopted by other countries. Professor Wolfgang Kühn outlines the theoretical principles for the geometric design process for rural roads in 12 chapters with a list of exercises at the end of each chapter. The current German guidelines for rural highways (RAS) form the basis for this process. Chapter 1 provides a historical perspective of mobility and need for transportation. Chapters 2 to 10 deal with various important concepts associated with the basic principles of road design. Chapters 11 and 12 provide a description of some emerging technologies for road design like visualization, simulation, and some new concepts dealing with the three-dimensional road design.

1.2 General Matters

One of the essential requirements for the development of any industrial society is the need for functioning infrastructure, which enables people and goods to be transported smoothly using the means of transportation available: by road, railroad, air, or sea.

The mobility patterns of people and corporations in the end determine the share of road, railroad, air, or sea traffic in the overall transportation services. This ratio, which is also called the modal split, is different in various industrialized countries. Figure 1.1 shows the imbalance in the modal split for Germany and the EU-25.

As standards of living increase and social conditions become established, one major feature of a free society is people's longing for individual mobility. This development was demonstrated not least by the soaring increase in vehicle ownership in former East Germany after German reunification in 1990 (Table 1.1) and in other Eastern European countries after they had joined the EU.

The increase in the share of road traffic in overall transportation volumes in comparison with the other modes of transportation and the growth in individual

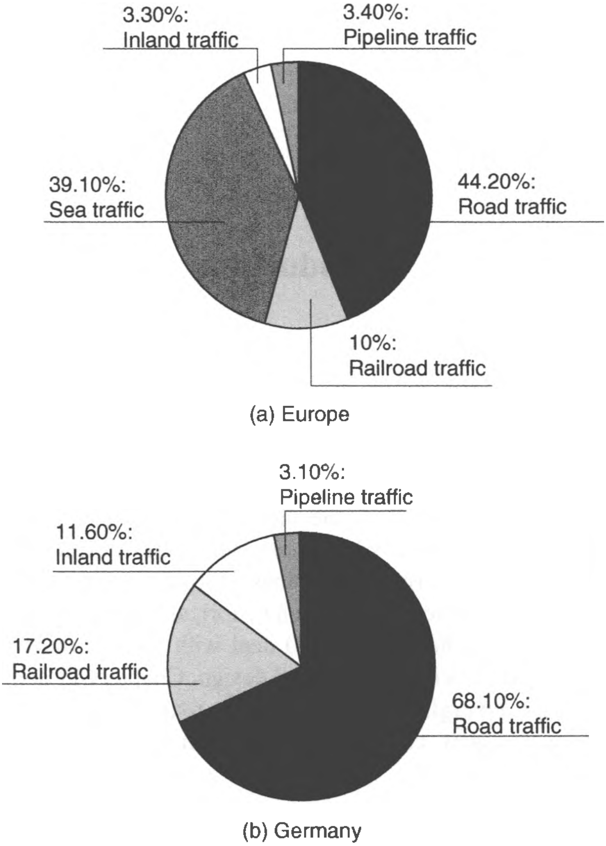

Figure 1.1: Comparison of modal split for Europe and Germany

transportation in comparison with public transportation are the major challenges for transportation policy in Europe. It will only be possible to increase public transportation over individual transportation and achieve a balanced distribution of transportation services with the individual modes of transportation if state restrictions are introduced.

The increasing dominance of road traffic will demand a new kind of overall approach in the future. In addition to upgrading an efficient road network on all classes of roads, traffic management and control systems (traffic telematics) are becoming increasingly important to achieve more uniform utilization levels.

The density of road networks in Europe is calculated by dividing the length of the road network by the territory covered; this provides a measure of the degree to which areas have been opened up to traffic to determine the need for expanding a network, particularly in industrial cities or recreation areas.

Table 1.1: Number of vehicles in Germany (in 000s)

Type of vehicle	1980	1990	2000	2011
Motorcycles	738	1,414	3,338	3,828
Cars	23,192	30,685	42,840	42,301
Buses	70	70	86	76
Trucks	1,277	1,389	2,527	2,441
Tractor trucks	1,640	1,756	1,920	1,991
Other vehicles	199	434	655	264
Total vehicles	**27,116**	**35,748**	**51,366**	**50,901**

While the density of the network can be determined separately for counties, regions, or federal states, it also serves as a comparative figure between individual European countries. Figure 1.2 shows the density of road networks in various European countries.

The EU Commission has called for a reduction in traffic deaths by 50% by the year 2020 to further increase traffic safety on road networks. Starting with this central demand, various safety programs have been developed in individual countries. The following major points are important for road design:

1. Comprehensive observation of the driver/vehicle/road relationship when planning, designing, building, and operating road traffic facilities;

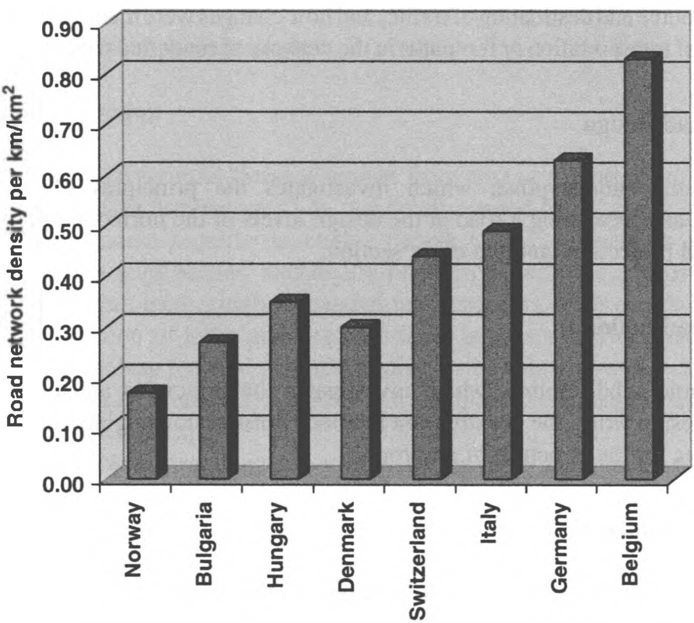

Figure 1.2: Road network density in European countries

2. Designing new roads safely as a result of greater standardization with recognition of the network function (self-explanatory roads);
3. Independent safety checks on a road traffic facility during the design and operations stages and introducing a safety management system for existing roads (safety auditing);
4. The increasing use of traffic telematics to achieve a better distribution of traffic on the road network overall;
5. Linking road traffic equipment and vehicles with the help of telecommunications equipment to form intelligent roads to increase efficiency, traffic safety, environmental friendliness, and profitability.

These major focuses must be taken into account during the academic work on road design and the further development of existing rules and regulations.

1.3 Subdisciplines

The subdisciplines of planning, designing, constructing, and operating a road traffic facility must be specified and attuned to each other to achieve a comprehensive academic approach.

1.3.1 Road planning

An academic subdiscipline, which investigates the principles of why the road was built (the source and destination of traffic) and how changes were made to the location by modes of transportation or footpaths in the network of roads and paths in any area.

1.3.2 Road design

An academic subdiscipline, which investigates the principles of directing, designing, and measuring a road at the design levels of the horizontal projection, the vertical projection, and the cross section.

1.3.3 Road building

An academic subdiscipline, which investigates the principles and correlations between design work, the construction method, construction engineering, and the calculations for the structure of any road.

1.3.4 Road operations

An academic subdiscipline, which investigates the principles and correlations between traffic control systems when a road is in use, while also observing the special requirements for maintaining a road and providing winter services.

This reference book essentially demonstrates the design principles and the design process for rural two-lane roads. The other subdisciplines will be covered by subsequent publications.

1.4 Rules and Regulations

Existing studies have demonstrated that the design of a road traffic facility in a rural area in Germany and many other countries takes place according to the "design speed" principle. As Germany has a very firm set of rules and regulations, this reference book directly refers to the valid parts of the rules and regulations for rural roads. The current "Guidelines for Road Facilities" (Richtlinie für die Anlage von Strassen, RAS) has the following parts:

- RAS-N: Network design
- RAS-L: Alignment design
- RAS-Q: Cross-section design
- RAS-K: Intersections design

These parts of the guidelines are currently being revised and brought together into a unified set of guidelines for rural roads. This marks a change from the current criteria of "design speed" to the new criteria of "design class."

Taking into account globalization, consideration should be given to unifying the guidelines in Europe and beyond in the long term.

1.5 Historical Developments

1.5.1 Introduction

The German word for road "Strasse" comes from the Latin expression "via strata" and roughly means "strewn path" or "artificially paved path."

Dirt tracks were required to enable people to live together in groups, go hunting, collect wild berries, and finally plant crops with the aim of producing food. Tracks were increasingly expanded into footpaths when people started the process of division of labor and began to trade in goods. Military requirements triggered a huge leap forward in road-building as the ruling powers needed a road network to rapidly transport weapons and equipment to conquer territory and expand central empires.

Archeological findings and old writings allow us to trace the development of road networks right up to the present time.

1.5.2 Antiquity

The Assyrians and Babylonians had a cohesive network of roads in the Near Eastern area. The first famous long-distance road was the 2,500 km long route, which the Assyrians used to link the cities of Susa and Sardes. The procession routes with the Ishtar Gate in Babylon were also known as cult sites. The royal roads in Persia served as routes for the army to administer the Persian Empire in an orderly manner. The extensive network of roads was designed to bypass settlements, even at that time.

The Greeks and the Romans in particular rendered outstanding services when it came to expanding road networks in Europe. During the Hellenist period, the authorities began to consciously plan roads in the colonial cities, i.e. they designed the road network in a geometric form taking into account settlement areas (e.g. in Milet).

The Romans were the people in the ancient world, who achieved most in terms of road building. A start was made on developing the Roman network of roads with the building of the "Via Appia" in 312 B.C. Rules of the road were established as early as 450 B.C. with the Laws of the Twelve Tables and they were expanded by Caesar in his "Lex Julia Municipalis." The Romans had created a significant network of roads by the year 200 A.D. It consisted of approx. 90,000 km of stone roads for heavy traffic and approx. 300,000 km of gravel roads for light traffic. This meant that they were able to move a legion from Jerusalem to Great Britain via Mainz in Jesus Christ's day is a perfectly acceptable expression.

The incense roads in Egypt and Mesopotamia, the caravan routes through the deserts, the procession route, and the transportation route from the Red Sea to the Nile were all well known in Africa. The unpaved caravan routes became major trade routes as spice roads from Arabia to Egypt and Syria.

The trade routes between China and the Mediterranean were well known in Asia. The silk roads provided an opportunity for the coveted cloth to be transported to the Roman Empire. But there was only an orderly network of roads within the Chinese Wall.

1.5.3 The Middle Ages

Road building came to a halt with the collapse of the Roman Empire. Trade relations developed in Eastern Europe between Scandinavia and Russia eastwards. Charlemagne was the person who brought the Roman roads back into the trade network. The different stony routes between the Baltic Sea and the Mediterranean were mainly dirt tracks. The authorities started to pave the most important roads in the cities with cobblestones in the 14th century. But more and more roads fell into disrepair because of the disunity of the German principalities.

The network of Inca roads was created in South America in the 14th and 15th centuries. The "Royal Coastal Road" and the "Royal Mountain Road" were linked by intersecting roads to form a network, which was approximately 10,000 km long. The route followed straight lines. Differences in height were overcome by using steps, as they were not yet familiar with the wheel. Marshlands and lakes were not circumvented, but were crossed by earthen dams.

1.5.4 The modern era

Karl VI and Joseph II built a network of roads across the Alps to link up trade in Silesia with the Mediterranean on overland routes in the 18th century. There were already 12,000 km of roads in Austria by about 1840.

The French road network was extended under Napoleon I in about 1800. The four Alpine routes are particularly famous.

The first European freeway in Europe was built in Italy from Milan to the lakes in Lombardy in 1923/24 and it was 130 km long.

In Germany, the development of the road network was severely hampered by the disunity of the small states. Chr. Von Lüder presented what was known as the "general route plan" in 1779 and his ideas reflect today's network of freeways. The rapid expansion of the road network in Germany only began when the German Customs Union was established.

1.6 Questions

(1) Explain why transportation is important for human population.
(2) While the historical perspective for transportation and mobility provided in this chapter was from Germany and Europe, develop a similar perspective for your country.

Chapter 2

Classification and Standardization

2.1 Classes of Roads

A network of roads is required to transport goods and carry people, and this network must take into consideration the interrelations between the various areas of life in society (places of residence, work, recreation, and supplies).

The classified road network in the Federal Republic of Germany has therefore been subdivided into the following road classes, depending on their function and which authority is responsible for them (with the obligation to build and maintain them):

1. Long-distance main roads
   - Freeways (A)
   - Main roads (B)
2. State roads (L)
3. District roads (K)
4. Local community roads (G)

Freeways (for which the German federal government is responsible) provide space for fast-moving traffic on separate lanes without any oncoming traffic. The intersections with other roads are designed to be at different levels and special ramps are used to drive on or off the highway.

Main roads (for which the German federal government is responsible) are long-distance routes, which are not freeways.

State roads (for which the federal state in question is responsible) serve non-local traffic in the federal state in question.

District roads (for which the district in question is responsible) serve local traffic within a district or between neighboring districts.

Local community roads carry local traffic in communities and parts of the community.

Each class of road has its own network. The networks of all five classes of roads make up the complete network of roads.

2.2 Road Categorization

2.2.1 Function of the road

Regardless of the road class and depending on its significance, the functional structure allows the road network to be designed in such a way that consistency is achieved between a road's function and the construction or operational standards.

The design of the road network makes a distinction between traffic functions (a connecting route, site development) and non-traffic functions (residence or functions, which result from the use of the surroundings in addition to site development). Traffic and non-traffic functions may overlap on individual sections of a road, so that a stretch of road has a mixture of different functions.

The main functions of a road are as follows:

1. Connecting or flow function: It allows efficient traffic movement between regions, cities, and parts of cities.
2. Site development or area distributor function: It allows reaching urban areas or real estate.
3. Residence or access function: It allows people or services to reach.

The connecting function of a road guarantees that goods and people can be transported quickly and cheaply, i.e. efficient connecting roads are absolutely essential to link business centers.

All roads have a site development function within urban areas. Site development roads guarantee that urban areas or adjacent real estate can be reached (residential, mixed, commercial, or industrial areas).

If a road has a connecting and site development function, safe facilities are usually needed for pedestrians and cyclists to cross it. The conflict between through traffic and people or cyclists crossing the road can only be reduced by imposing speed restrictions on motorized traffic.

Residential and communications functions are typical features of roads in urban areas. They are also built to develop an area and enable people to do their shopping, go for walks, play, and particularly provide access to public buildings (government agencies, schools, hospitals, stores, etc.).

While the connecting function is the most important and the site development function is restricted to agricultural use outside urban areas (rural areas), several functions normally intermingle with each other in urban areas (inner-city) and require a coordinated approach to road design.

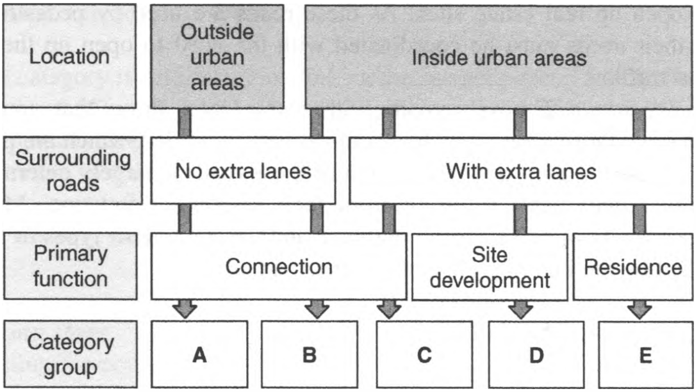

Figure 2.1: Category groups (RAS-N)

2.2.2 Category groups

Road sections according to RAS-N are divided into category groups A to E to place the location of a road in the network, taking into consideration different requirements for their use (Figure 2.1).

The following three criteria apply, depending on their function and significance:

1. Location with regard to urban areas (outside, inside)
2. Use to which surrounding roads are put (no extra lanes, extra lanes for pedestrians/cyclists, etc.)
3. Standard planning function (connecting, site development, residence)

Category group A covers roads without any extra lanes outside urban areas, which are primarily used to connect cities. The main quality requirements for designing these roads arise from their connecting function.

Category group B covers roads without any extra lanes near cities and within urban areas where their main function is to connect. Because of the lack of extra lanes, there is only a slight overlap between the connecting function and the site development function.

Category group C covers roads with extra lanes and roads where they could be built; they fulfill connecting, site development, and residential functions. The main quality requirements for designing these roads arise from the connecting function, but these requirements could be restricted by the extent and type of buildings. Depending on the scope of the site development and residential functions, measures to restrict drivers' speed should be taken into consideration on these roads.

Category group D is a summary of roads with extra lanes, those that could have extra lanes and those where they have not yet been provided – they mainly serve to open up real estate sites. As these roads are used by pedestrians and cyclists, their needs must be coordinated with the need to open up the area to motorized traffic.

Category group E covers roads with extra lanes, those that could have extra lanes and those where they have not yet been provided, which are primarily used for residential purposes. The design of these roads is largely determined by the quality requirements resulting from their residential function. Motorized traffic is normally of secondary importance and the mixture of types of traffic is exemplified by the building elements in the road area.

2.2.3 Connecting function level

The introduction to category groups clearly shows that roads in category groups A and B are more or less exclusively connecting roads and category group C also takes into account linking sources of traffic and destinations. From this it can be inferred that the division into category groups alone is not sufficient to describe the quality requirements for the connecting roads that form a network. National connections covering fairly large distances and between important cities should in principle have a higher level of construction work and traffic quality than less important connecting routes that cover smaller distances.

A differentiation in the functional assessment of roads is made in conjunction with the city and regional planning department using a system of central cities (main centers, medium centers, and less important centers). Central cities are local communities, which supply the population in the surrounding area above and beyond the needs of local residents.

A connecting function can be attributed to each road between the central cities (Table 2.1). Its varying significance for traffic should be taken into account here. Major connecting roads that cover huge distances will in principle be given a higher level of traffic quality and therefore better construction standards than less important connecting roads that cover small distances.

Table 2.1: Connecting function levels (RAS-N)

Traffic significance	Function level	Road category
Major road link (connecting two main centers)	I	A I
National/regional road link (connecting two medium and main centers)	II	A II
Road between communities (connecting basic centers)	III	A III
Road opening up sites (connecting communities)	IV	A IV
Less important road link (connecting real estate to communities)	V	A V
Local link (connecting real estates)	VI	A VI

2.2.4 Road category

The road category is determined by linking the category group and the connecting function level. The connecting function indicates the traffic importance in the network and the category group describes the demands made on the roads arising from their use in the surrounding area. The category of road is normally determined for interrelated road sections within the road network.

Five working stages are used to determine the road category (according to Figure 2.2):

1st working stage
Determining the connecting function (the highest function level is critical).

2nd working stage
Checking the section of road for its location in relation to urban areas (allocating rural roads to category group A).

3rd working stage
Checking the road sections within urban areas for their use in relation to the surrounding roads (allocating the sections without extra lanes to category group B).

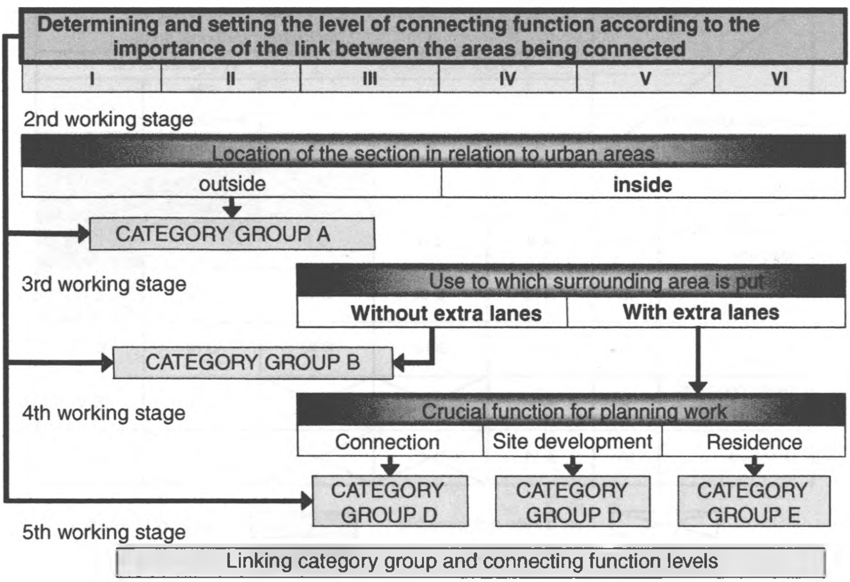

Figure 2.2: Working stages

4th working stage

Checking the road sections with extra lanes to see whether they meet the requirements emerging from their connecting, site development, and residence functions (determining the main functions).

5th working stage

Linking the category group and connecting function level to the road category for the section of road (Figure 2.3).

The connecting matrix takes into consideration all 30 theoretically possible road categories. As not all the combinations are sensible, some are excluded from use.

The desirable cruising speed for cars is the standard for linking functional road categories and the level of construction work that is appropriate for this to safeguard the city and regional planning goals. The speed takes into account both the transport policy and road design approaches (Table 2.2).

By using the road category, it is possible to determine the critical design and operating features of a section of road and use a different standard of construction work depending on the road's function (Tables 2.3 and 2.4). The design speed is the link for deriving the driving and geometrical parameters for the design elements on the horizontal and vertical projections.

Tables 2.5 and 2.6 illustrates the significant design approaches depending on the road function.

Category group / Connecting funktion level		Outside	Inside urban areas (including transitional areas)			
		Without extra lanes			With extra lanes	
		Connection			Site development	Residence
		A	B	C	D	E
Major road link	I	A I	B I	C I		
National/regional road link	II	A II	B II	C II	D II	
Road between communities	III	A III	B III	C III	D III	E III
Road opening up sites	IV	A IV	B IV	C IV	D IV	E IV
Less important road link	V	A V			D V	E V
Local link	VI	A VI				E VI

as a rute not possible
problematical
very problematical
not acceptable

Figure 2.3: Road categories (RAS-N)

Table 2.2: Desirable cruising speeds for cars (weekday traffic) according to RAS-N

Road functions				
	Category group/explanation	Road category/description		Target cruising speed [km/h] (workday traffic)
A	Roads with no extra lanes outside urban areas mainly with a connecting function	A I	Major road (long-distance roads)	70–100
		A II	National/regional road	60–90
		A III	Road linking communities	50–80
		A IV	Road opening up sites	40–60
		A V	Less important road	None
		A VI	Local link	None
B	Roads with no extra lanes in the vicinity and within built-up areas mainly with a connecting function	B II	Fast trunk road	50–70
		B III	Main road	40–60
		B IV	Main local distribution road	30–50
C	Roads with extra lanes inside urban areas mainly with a connecting function	C III	Main road	30–50
		C IV	Main local distribution road	30–40
D	Roads with extra lanes within urban areas	D IV	Distribution road	20–30
		D V	Residential road	None
E	Roads with extra lanes within urban areas mainly with a residential function	E V	Residential road	None
		E VI	Residential track	None

Table 2.3: Design and operating features category group A (RAS-L)

Road function			Design and operating features											
Category group	Road category		Type of traffic	Permissible speed (km/h)	Cross section	Intersections	Design speed (km/h)							
A Roads with no extra lanes outside urban areas mainly with a connecting function	A I	Long-distance road	Vehicles	–	Divided highways	(Different levels)	120	100						
			Vehicles	≤ 100(120)	Undivided highways	Different levels same level		100	90	(80)				
	A II	National/ regional road	Vehicles	–	Divided highways	Different levels (same level)		100	90	(80)				
			General	≤ 100	Undivided highways	Same level			90	80	(70)			
	A III	Road linking communities	General	≤ 100	Divided highways	(Different levels) same level			(90)	80	70			
				≤ 100	Undivided highways	Same level				80	70	60		
	A IV	Road opening up sites	General	≤ 100	Undivided highways	Same level					70	60	(50)	
	A V	Less important road	General	≤ 100	Undivided highways	Same level							(50)	None
	A VI	Business route	General	≤ 100	Undivided highways	Same level								None

Table 2.4: Design and operating features category group B-E (RAS-L)

B Roads with no extra lanes near a city and within urban areas mainly with a connecting function	**B I**	City highway		≤ 100	Divided highways	Different levels	100	90	80	(70)				
	B II	Fast trunk road		≤ 80	Divided highways	Different levels (same level)			80	70	(60)			
	B III	Main road	General	≤ 70	Divided highways	Same level				70	60	(50)		
			General	≤ 70	Undivided highways	Same level				70	60	(50)		
	B IV	Main local distribution road	General	≤ 60	Undivided highways	Same level					60	50		
C Roads with extra lanes inside urban areas mainly with a connecting function	**C III**	Main road	General	50	Divided highways	Same level				(70)	(60)	50	(40)	None
			General	50	Undivided highways	Same level					(60)	50	(40)	
	C IV	Main local distribution road	General	50	Undivided highways	Same level						50	(40)	
D Roads with extra lanes within urban areas mainly with a development function	**D IV**	Distribution road	General	≤ 50	Undivided highways	Same level								None
	D V	Residential road	General	≤ 50	Undivided highways	Same level								None
E Roads with extra lanes within urban areas mainly with a residential function	**E V**	Residential road	General	Step speed	Undivided highways	Same level								None
	E VI	Residential track	General	Step speed										None

Table 2.5: Design approaches category group A, B (RAS-L)

Road function		Design approaches						
Category group	Road category	Design principle	Determining v_{85}	Use of radial traction	Transition curve	Radii interlinked	Reach time	Overtaking visibility
A Roads with no extra lanes outside urban areas mainly with a critical connecting function	**A I** Long-distance road	Driving	Divided highways for $v_e \geq 100$ km/h $v_{85} = v_e + 10$ km/h $v_e < 100$ km/h for $v_{85} = v_e + 20$ km/h Undivided highways Case 1: depends on KU and B Case 2: depends on R and B	50% at max q 10% at min q	Necessary	Necessary	2.0s	Necessary (Undivided highway, two-lane roads)
	A II National/ regional road							
	A III Road linking communities							
	A IV Road opening up sites							
	A V Less important road							
	A VI Business route	Driving	None	None	Not necessary	Not necessary	N/A	Not necessary
B Roads with no extra lanes near a city and within urban areas mainly with a connecting function	**B I** City highway	Driving	v_{85} = zul v	50% at max q 10% at min q	Necessary	Not necessary	2.0s	Not necessary
	B II Fast trunk road							
	B III Main road							
	B IV Main local distribution road				Desirable			

zul v: permissible speed; KU: bendiness

Table 2.6: Design approaches category group C, D, E (RAS-L)

Road function			Design approaches						
Category group	Road category		Design principle	Determining v_{85}	Use of radial traction	Transition curve	Radii interlinked	Reach time	Overtaking visibility
C Roads with extra lanes inside urban areas with a critical connecting function	**C III**	Main road	Driving	$v_{85} = \text{zul } v$	None	Not necessary	Not necessary	N/A	Not necessary
	C IV	Main local distribution road							
D Roads with extra lanes within urban areas with a site development function	**D IV**	Distribution road	Driving	$v_{85} = \text{zul } v$	None	Not necessary	Not necessary	N/A	Not necessary
	D V	Residential road							
E Roads with extra lanes within urban areas mainly with a residential function	**E V**	Residential road	Driving	$v_{85} = \text{zul } v$	None	Not necessary	Not necessary	N/A	Not necessary
	E VI	Residential track							

2.3 Questions

(1) Which criteria are used to determine road classes?
(2) What are the tasks fulfilled by the public roads authority?
(3) What is the difference between a category group and a connecting function level?
(4) Explain the working stages to determine the category of a road.
(5) Describe the features of a rural road of road category A I.

Chapter 3

Driver – Vehicle – Road Interaction

3.1 Driving Area

The business of driving generally involves an interaction of three components: the driver, the vehicle and the road. It must be pointed out that the road itself is subject to other factors like weather conditions and its surroundings. Various design models have been used in literature to simplify the interaction of driver, vehicle and road in terms of a closed loop in order to illustrate the complex interaction within this system.

To define the driving area, it is generally necessary to differentiate between the three-dimensional, mathematical, objective area and the area that is observed by the driver's sense organs. The objective area does not depend on the driver, but the area perceived subjectively directly depends on the individual driver, because it is crucially affected by the driver's subjective qualities and abilities. The subjective area is called the driving area in road planning and it is described as a half-space that is restricted by the horizon. This half-space is composed of different discontinuous partial areas like the road itself, what is growing next to the road etc. Side areas are located on both sides of the road, which are part and parcel of the driving area. They contain a variety of information that is crucial for drivers as they cope with the driving task. So the design of the side areas should be taken into account right at the beginning of the planning work. Because drivers' perception of the side areas is subjective, it is difficult to derive objective, general rules for their design. There is hardly any literature on this subject apart from some suggestions about helping drivers find their way by using natural growth etc (Deiss, 1978).

Leutner (1974) and Biedermann (1984) and discovered that the driving area has an effect on driving behavior and documented this. Suggestions were made that the driving area can influence accidents, but no evidence was provided at the time apart from the role played by trees in accidents (Meewes and Eckstein, 1998).

3.2 Absorbing Information

3.2.1 Basic assumptions

A suitable form for describing the process of "driving along a road" is Durth's closed loop (1974). The input variables and process results represent the same system components as with a cyclic system. Using this feedback system, it is possible to keep variables within a certain tolerance range.

In Durth's closed loop, the main components driver (R), vehicle (RS) and road area (W) are supplemented by other external factors (Z), consisting of traffic and weather conditions (Figure 3.1). According to Durth, the system cannot be viewed as closed or settled, so no definite statements about specific driving processes can be derived from this closed loop.

Information is absorbed through the human senses and the largest proportion of the information is perceived through a person's eyes. According to Biedermann (1984), this is higher than 90%. In addition, a distinction is made between haptic[1], acoustic and proprioceptive[2] perception. But all these kinds of perception have an effect on drivers at the same time. The following estimated figures are specified in literature as the inflow capacities.

The information that is absorbed is processed in the brain and can trigger both conscious actions and unconscious reflexes. So it is clear that the design of the driving area can have a crucial effect on drivers' ability to perceive things and their driving behavior.

Drivers only view a small proportion of the volume of information that is available as relevant for traffic situations and this is processed to affect their driving behavior. According to Berger (1996), this particularly involves:

- the roadway,
- road users,
- road signs,
- obstacles.

3.2.2 Properties of the human eye

A distinction must be made between two areas that are visible for drivers when talking about visual perception:

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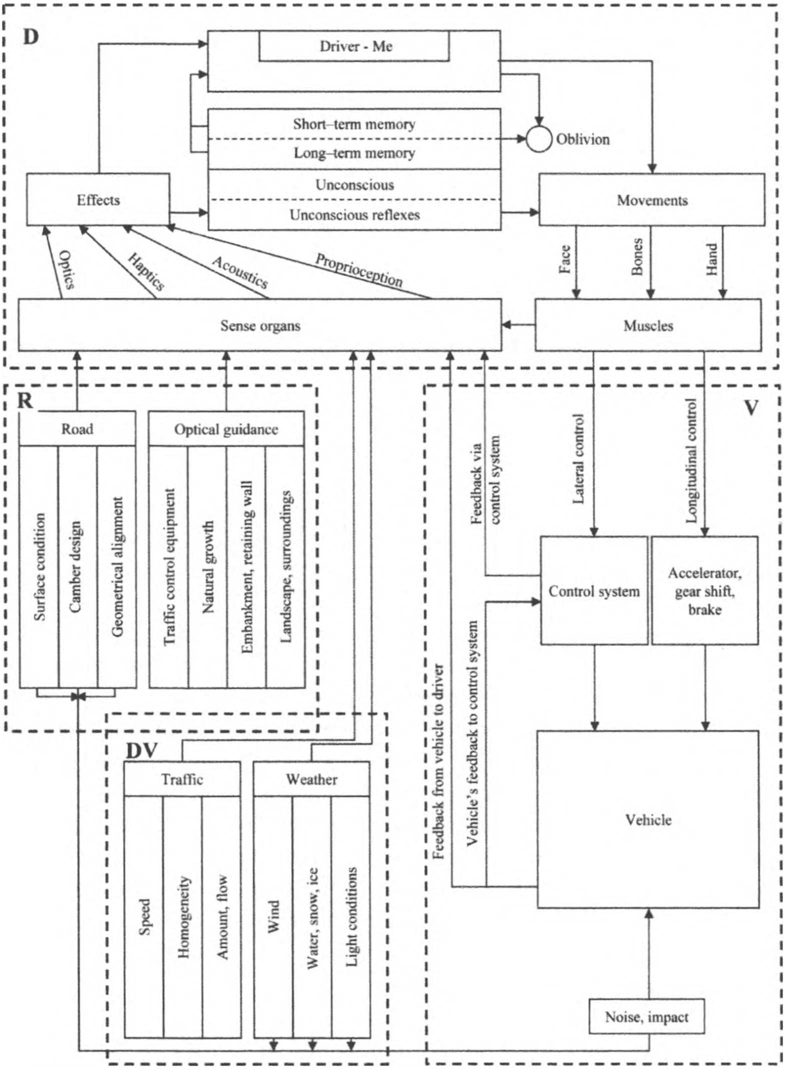
[1] Haptic perception encompasses all aspects of touch through hands.

[2] Proprioception deals with movements in the human body picked up by the muscles and balance system.



D	Driver
V	Vehicle
R	Reference variables
DV	Disturbance variables

Figure 3.1: Interaction of driver, vehicle, road according to Durth (1974)

Table 3.1: Estimated figures on the maximum flow of information according to Schmidt and Thews (1990)

Kind of perception	Inflow capacities [bit/s]	Psychological inflow capacities [bit/s]
Optical	10^7	40
Acoustic	10^5	30
Miscellaneous	10^6	7

Visual field:
The area where an object can be perceived without moving your eye or head is called the visual field according to Böhlke (1985). It is located within the range of 180°–220° horizontally and 130° vertically.

Range of vision:
According to Berger (1996), the partial area of the visual field where it is possible to see things in focus when moving your eyes but keeping your head still is called the range of vision. It is located within a range of 60° horizontally and 40° vertically.

The perception of the depth of an area is very important for judging distances and is possible as a result of stereoscopic vision. In this process two individual frames in a pair of eyes are brought together. Three-dimensional vision is restricted to approx. 6 m because of the small distance between one eye and the other (Weise and Durth, 1997). If this limit is exceeded, drivers can judge distances on account of the perspective image of objects on their retina with the help of their experience of the size of familiar objects and the concealment of objects and the distribution of light and shadow. The human eye can perceive moving objects like an oncoming vehicle at a distance of 600–800 m. But in the case of stationary objects, the distance is smaller, depending on the size of the object.

3.2.3 Classification of the driving area

That part of the range of vision, from which the driver absorbs relevant information for the driving task, is defined as the field of vision. Its size varies and is subjective, so it cannot be exactly quantified. According to Babkow (1975), the field of vision narrows as speed increases (Figure 3.2), but objects beyond the field of vision are detected during the driving process, if they are significant for the driver (Kayser and Sanders, 1989).

Discontinuous alignment can also lead to constriction in the field of vision. Information on the road ahead is of varying importance for decisions made by drivers depending on how far away the information is.

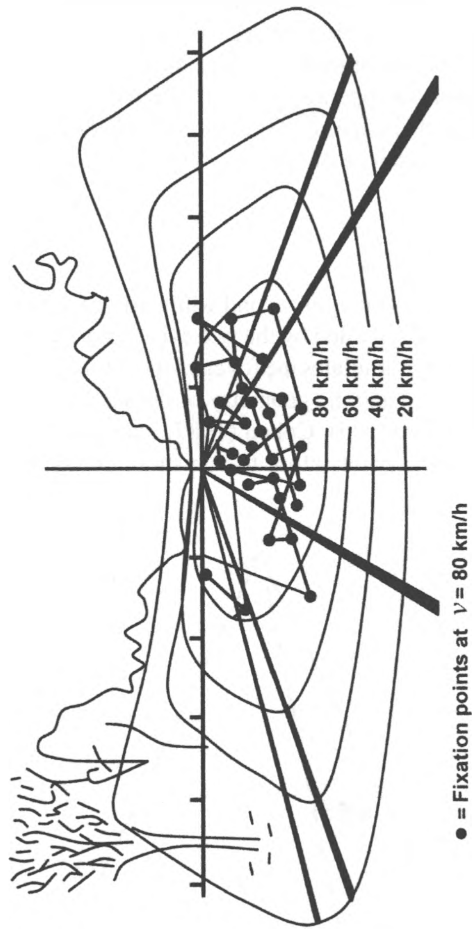


Figure 3.2: Field of vision constriction as speed increases according to Babkow (1975)

This is why Leutner (1974) defined three zones of perception:

Zone 1: Long-distance orientation and information (600 m–250 m)
The definition is based on the physiological ability of the human eye to recognize a moving object on a roadway at a distance of up to 600 – 800 m.

Zone 2: Readiness and decision zone (250 m–75 m)
The driver saves the information from the preceding road characteristics for a period of about 10 s. Leutner deduces from this that the driver would like to recognize and assess the road section which he will pass through in the next 10 s. This period of time corresponds to a distance of approx. 250 m.

Zone 3: Close orientation and operating zone (≤75 m)
Drivers reduce their speed about 3 s before they reach a bend. This corresponds to a distance of approx. 75 m.

3.2.4 Visual behavior

Drivers have about three fixations per second while driving ("fixation point" is the point in the visual field that is fixated by the two eyes in normal vision and for each eye is the point that directly stimulates the fovea of the retina). These fixation points depend on the driving speed and the geometry of the section of the road ahead.

Jogenson and Mortimer (1975), Cohen (1987) and Serafin (1993) discovered an accumulation of fixation points close to vanishing points on long straights in

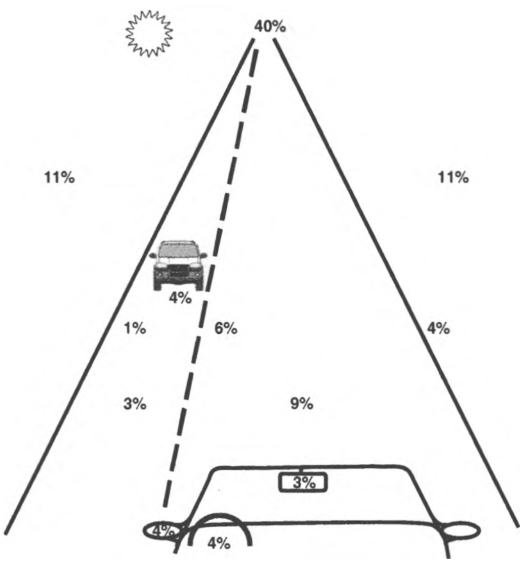


Figure 3.3: Fixations by drivers on a straight according to Serafin (1993)

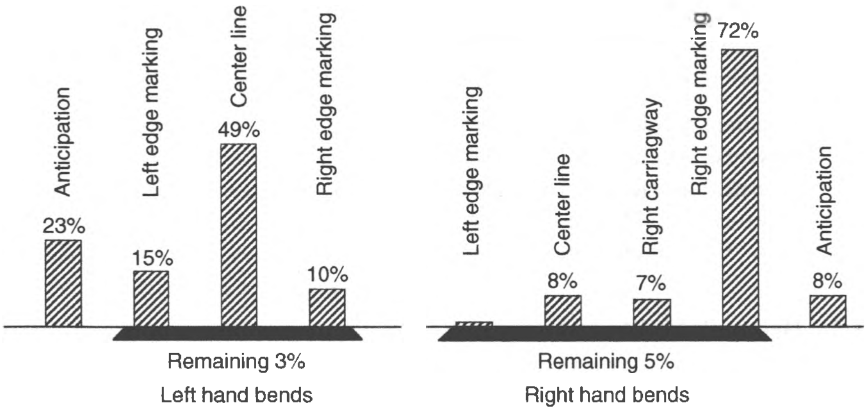


Figure 3.4: The distribution of fixation times for elements of the driving area in bends according to Friedinger (1982)

various experiments (Figure 3.3). For this, the time interval did not depend on speed and, on average, amounted to 6 s.

In contrast to long straights, Friedinger (1982) discovered that the accumulation of fixations lay on the appropriate driving line in bends. So the driver fixes the right edge of the road in right hand bends and the centre line in left hand bends (Figure 3.4).

The fixation distance is shorter in right hand bends than left hand bends. The point of greatest curvature is very important for the driver. This acts as the central point of vision in the central perspective image of the driving area on the retina, as is clear in Figure 3.5. On straights, this is the vanishing point.

3.2.5 Proprioception

Proprioception is the movement of the human body that is perceived by the muscles and balance system. This enables drivers to sense vehicle movements like acceleration or changes to it. A distinction is made between advance motion

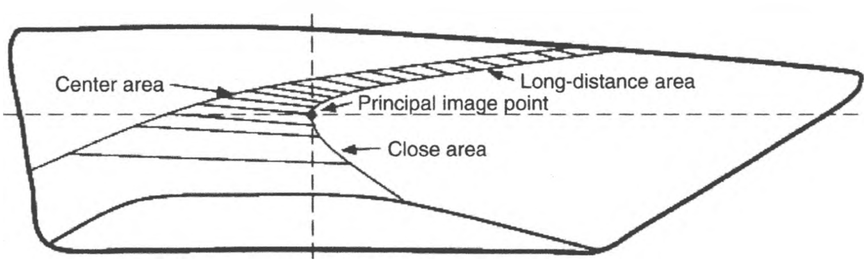


Figure 3.5: Main point of image when cornering (Appelt, 1998)

(longitudinal, transverse or upward movement) and rotational movement (pitching, yawing or rolling). If the following thresholds are exceeded in advance motion acceleration or changes to it, drivers find them disagreeable:

- Longitudinal acceleration: 3.0 m/s^2
- Transverse acceleration: 1.0 m/s^2
- Vertical acceleration: 1.0 m/s^2
- Horizontal change: 0.8 m/s^2
- Vertical change: 0.5 m/s^2

3.3 Drivers' Ability to React

A person's active behavior, which is triggered by a stimulus, is described as a reaction. The psycho-physiological reaction process can be broken down into two types:

- unconscious through conditioned reflexes and
- conscious by reactions triggered by controlled actions.

The first process type is used in situations that are often repeated. In unusual or surprising situations, actions are conscious and controlled. Especially in hazardous situations, the time taken for a deliberate action is crucial. The time process can be broken down into the following three stages:

- absorbing the information,
- processing the information and then making a decision,
- implementing the decision that has been made by carrying out a specific action.

The following Table 3.2 gives a summary of the average response times for drivers.

Table 3.2: Average response times for drivers according to Durth et al. (1982)

Procedure	Response time
Adapting their eyes (dark – bright)	3 s
Adapting their eyes (bright – dark)	5–300 s
Change of view	300 ms
Perception and recognition	300–400 ms
Peripheral perception and recognition	700 ms
Decision-making	200 ms
Moving their foot from the accelerator to the brake	100–300 ms
Total driver reaction time with a change of view	600 ms
Total driver reaction time without a change of view	900 ms

3.4 Questions

(1) Explain the difference between objective and subjective areas.
(2) What do you understand by the term "interaction" and what are its main components?
(3) Which types of perception are you familiar with?
(4) Explain the differences between the visual field and range of vision.
(5) What do you understand by fixation and where are the fixation points for a straight and a curve?
(6) Name and explain the perception zones.

Chapter 4

Vehicle Kinematics and Dynamics

4.1 Introduction

Carl Pirath (1934) suggested a very general definition of traffic:

"Traffic is the sum of the local changes of persons, goods and messages."

If we disregard the study of the transmission of messages – as this is now subject to other rules on account of enormous improvements in communications technologies – traffic always involves physical movement, which can be clearly described on account of the physical connections between distance, time, speed, and acceleration. Vehicle kinematics deals with the movements of an individual vehicle. Each movement triggers forces that have an effect on the vehicle. These forces must be present if a vehicle is going to move. So a certain driving force is required in order to move a vehicle and this force is reduced by various types of resistance such as air, friction, and inclines. But undesirable or disagreeable forces also occur and they have a negative effect on our movement. These include centrifugal forces, which occur during every cornering maneuver and can cause disagreeable phenomena so that a vehicle skids or overturns. Vehicle dynamics deals with these forces during movement. Vehicle dynamics checks are used in road planning for three reasons (Benger and Kühn, 2002):

1. They deliver the principles for defining the minimum and maximum values for design elements in a planning project (e.g., gradient, radii of curvature, camber, length of acceleration and deceleration lanes, etc.);
2. They are required to reconstruct the driving process and analyze skid marks during local accident investigations; and
3. They finally serve to determine operating expenses (driving time and energy consumption) on existing or planned roads when calculating annual traffic costs or when carrying out cost/benefit analyses within the scope of economic feasibility studies.

4.2 Vehicle Kinematics

4.2.1 Equations of motion

The movement of a vehicle as a function of time can be illustrated in a diagram, where time is an abscissa and the distance covered an ordinate (distance/time diagram) (Figure 4.1). An object – for example a vehicle – moves at a constant speed and covers the distance s_0 by time t_0. At time t_0, the speed is increased for some time until time t_1 – so we are dealing with accelerated motion. Then the journey is continued again at a constant speed until time t_2. From t_2 the speed is reduced – this is decelerated motion – until the vehicle finally comes to a halt at time t_3.

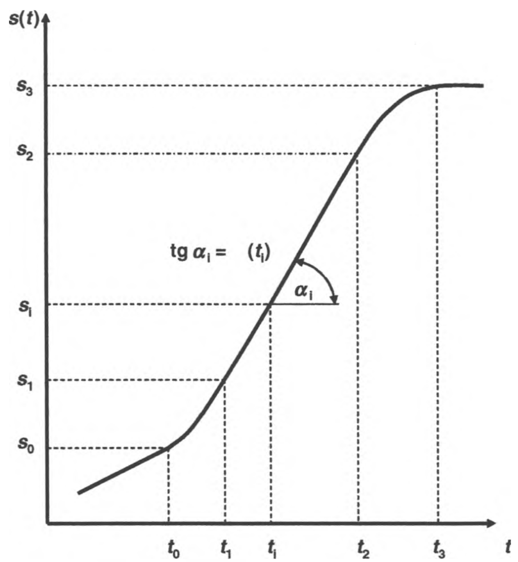


Figure 4.1: Movement as a function of time

The tangent of the angle, which the line of motion forms with the abscissa at any point in time t_i, provides the momentary speed at that time. If we call the distance covered as a function of time $s(t)$, kinematics provides the following equations of motion to describe motion as a function of time (Figure 4.2).

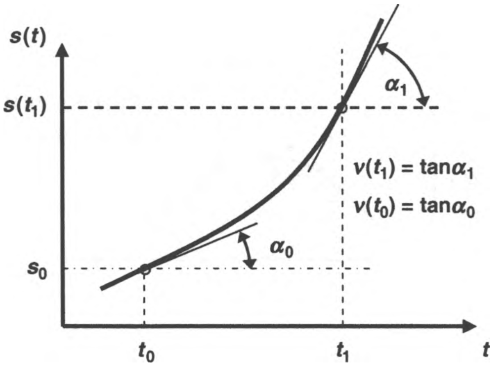


Figure 4.2: Distance/time diagram

Speed as a function of time (Figure 4.3) indicates the change in distance per time unit [m/s].

$$v(t) = \frac{ds}{dt}. \tag{4.1}$$

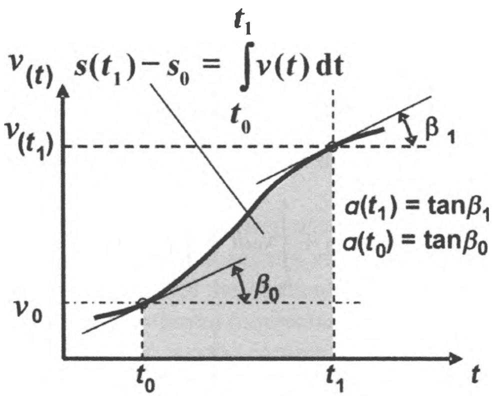


Figure 4.3: Speed/time diagram

Acceleration as a function of time (Figure 4.4) provides the change in speed per time unit [m/s^2].

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}. \tag{4.2}$$

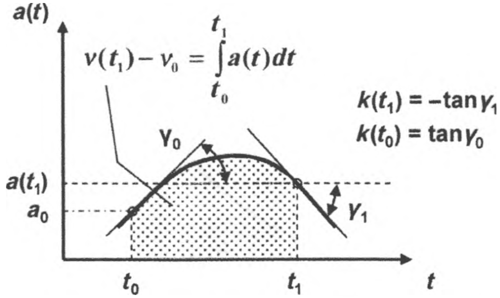


Figure 4.4: Acceleration/time diagram

A jerk as a function of time explains the change in acceleration per time unit [m/s^3].

$$k(t) = \frac{da}{dt} = \frac{d^2v}{dt^2} = \frac{d^3s}{dt^3}. \tag{4.3}$$

If t_0, s_0, v_0, and a_0 are the respective starting conditions, the following equations of motion are produced in different driving states for any time t:

$$s(t) = s_0 + \int_{t_0}^{t} v(t)dt \tag{4.4}$$

$$v(t) = v_0 + \int_{t_0}^{t} a(t)dt \tag{4.5}$$

$$s(t) = s_0 + \int_{t_0}^{t} v_0 dt + \int_{t_0}^{t}\int_{t_0}^{t} a(t)dtdt \tag{4.6}$$

$$a(t) = a_0 + \int_{t_0}^{t} k(t)dt \tag{4.7}$$

$$v(t) = v_0 + \int_{t_0}^{t} a_0 dt + \int_{t_0}^{t}\int_{t_0}^{t} k(t)dtdt \tag{4.8}$$

$$s(t) = s_0 + \int_{t_0}^{t} v_0 dt + \int_{t_0}^{t}\int_{t_0}^{t} a_0 dtdt + \int_{t_0}^{t}\int_{t_0}^{t}\int_{t_0}^{t} k(t)dtdtdt \tag{4.9}$$

For the further study of the movement of vehicles, a distinction must be made between the speed terms shown in Figure 4.5.

The speed of a vehicle at a certain place x at any time t is called the momentary speed. This means the tangent of the angle α, which the vehicle's line of movement forms with the time axis t (Figure 4.5).

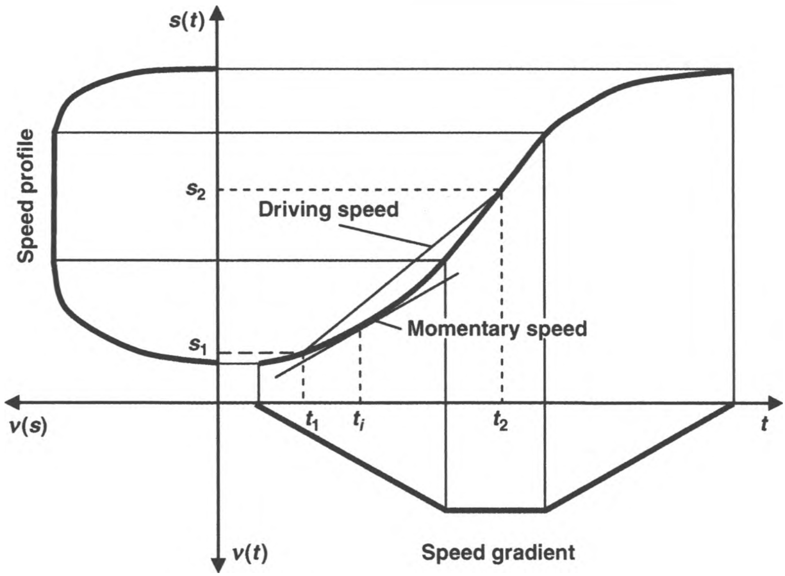


Figure 4.5: Illustration of a momentary speed and driving speed and the speed gradient and speed profile according to Weise and Durth (1997)

The momentary speed can be measured at a certain cross section (locally or space referred) or at a certain time at several cross sections (momentary or time referred). The local speed describes the momentary speed of a vehicle in relation to a cross section and the time referred momentary speed describes the momentary speed of a vehicle in relation to a point in time.

If we want to determine the average speed within a certain period between times t_1 and t_2, the driving speed is the result of the quotient of the distance covered $(s_2 - s_1)$ and the period of time $(t_2 - t_1)$.

The speed gradient can be used to illustrate movement in a distance/time diagram and the relevant speeds, but it can also be used to show the speed gradient $v(t)$ as a function of time in the speed/time diagram (Figure 4.5).

The parabolic shape of the distance/time line corresponds to a straight line in the speed gradient (linear increase in speed). But a straight line in the distance/time diagram corresponds to a parallel line to the time axis (constant speed) on the speed gradient.

The speed $v(s)$ as a function of the distance covered s can be shown as the speed profile. A journey at constant speed is shown in the speed profile by a parallel line to the distance axis, while accelerated or decelerated movement assumes a parabolic course in the speed profile.

Initially this analysis will concentrate on the three different driving conditions with regard to their conditional equations:

- a journey at a constant speed
- a journey with constant acceleration
- a journey with constant changes.

Journey at a constant speed

If we drive at a constant speed, the correlations shown in Figure 4.6 apply.

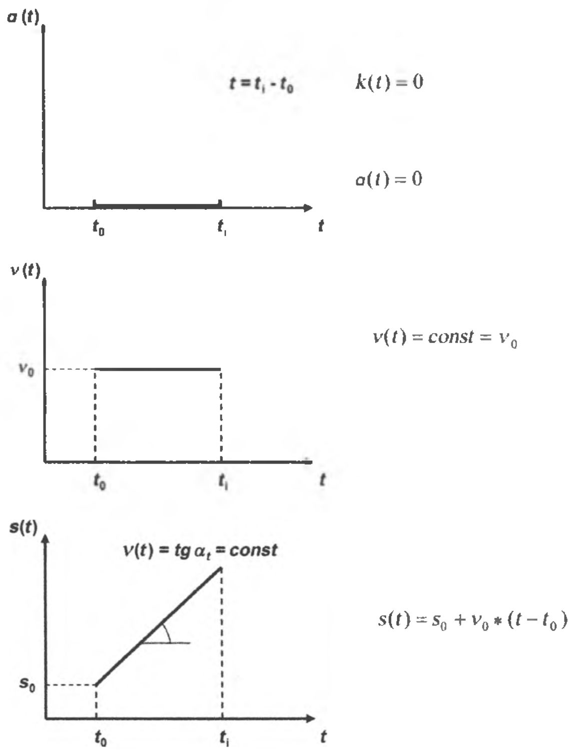


Figure 4.6: Journey at a constant speed

Journey with constant acceleration

If the speed is not constant, but variable during the time being analyzed, accelerated or decelerated movement occurs (Figure 4.7).

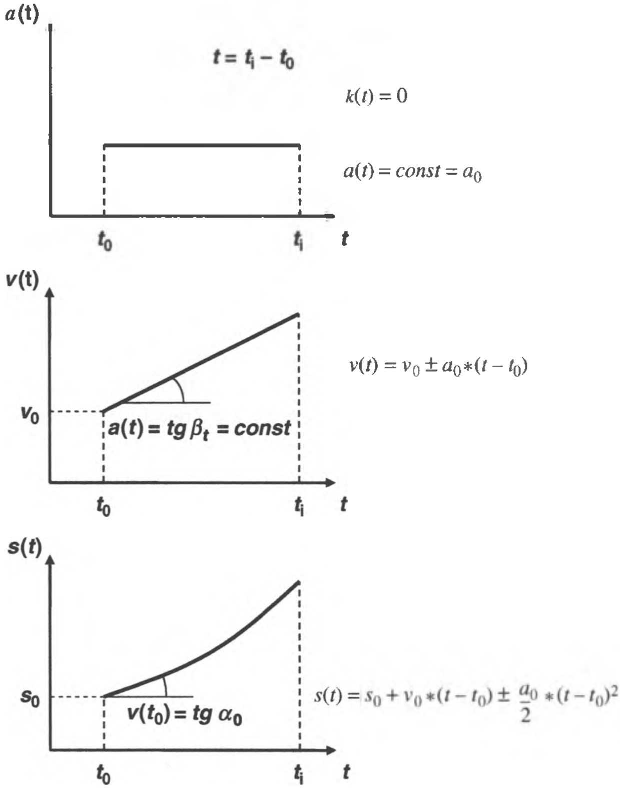


Figure 4.7: Journey with constant acceleration

Journey with constant changes

If, however, the acceleration is not constant during a journey, the following equations apply:

$$k(t) = const \tag{4.10}$$

$$a(t) = \pm a_0 \pm k*(t - t_0) \tag{4.11}$$

$$v(t) = v_0 \pm a_0*(t - t_0) \pm \frac{k}{2}*(t - t_0)^2 \tag{4.12}$$

$$s(t) = s_0 + v_0*(t - t_0) \pm \frac{a_0}{2}*(t - t_0)^2 \pm \frac{k}{6}*(t - t_0)^3. \tag{4.13}$$

The equations of motion for *a* jerk are insignificant for practical purposes, because it is very difficult to attribute certain values to a jerk. The thresholds for a jerk are about 0.3 m/s^3 for persons standing in means of transportation and about 1.2 m/s^3 for persons who are seated.

Function $s(v)$ – i.e. distance as a function of speed – and function $v(s)$ – i.e. speed as a function of the distance covered – can be derived from the equations for a journey with constant acceleration given in Figure 4.7.

Distance as a function of speed s(v)

t (where $t_0 = 0$) can be derived from the equation for speed as a function of time $v(t)$:

$$t = \frac{v - v_0}{\pm a}. \tag{4.14}$$

If this expression is used in the equation for the distance covered $s(t)$, the distance covered can be seen as a function of the speed $s(v)$ as follows:

$$s(v) = \pm\frac{1}{2}a\frac{(v - v_0)^2}{a^2} + v_0\frac{v - v_0}{\pm a} + s_0 \tag{4.15}$$

$$s(v) = \frac{v^2 - 2v_0v + v_0^2}{\pm 2a} + \frac{2v_0v - 2v_0^2}{\pm 2a} + s_0 \tag{4.16}$$

$$s(v) = \frac{v^2 - v_0^2}{\pm 2a} + s_0. \tag{4.17}$$

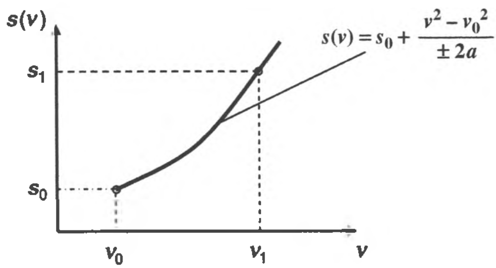


Figure 4.8: Journey with constant acceleration (distance as a function of speed)

Speed as a function of the distance covered v(s)

The following expression ($t_0 = 0$) for the time t is provided from the equation $s(t)$:

$$\left(\frac{v_0}{a}\right)^2 + \frac{2}{\pm a}(s - s_0) = t^2 + 2\frac{v_0}{\pm a}t + \left(\frac{v_0}{a}\right)^2 = \left(t + \frac{v_0}{a}\right)^2 \tag{4.18}$$

$$t = -\frac{v_0}{a} + \frac{1}{a}\sqrt{v_0^2 \pm 2a(s - s_0)}. \tag{4.19}$$

If this expression is used in the equation for speed as a function of time $v(t)$, the following correlation is obtained for $v(s)$:

$$v(s) = -v_0 + \sqrt{v_0^2 \pm 2a(s - s_0)} + v_0 \tag{4.20}$$

$$v(s) = \sqrt{v_0^2 \pm 2a(s - s_0)}. \tag{4.21}$$

4.2.2 Braking path

The braking path is based on two essential processes, the braking process and deceleration (Figure 4.9).

Braking process

The complete braking process comprises the time from the perception of a hazard until the vehicle comes to a halt. The distance covered during a braking maneuver depends on various factors. A distinction must be made between the following time factors when searching for the necessary braking time and stopping distance:

After recognizing the hazard at time 1, the perception and reaction time t_r (the so-called "moment of shock") elapses, before the braking process is started at time 2. The perception and reaction time may last between 0.6 and 1.8 s. During this time the journey continues at an unabated speed.

After initiating the braking process, a certain amount of time elapses until the braking effect starts at time 3. This is called the response time for the brakes t_a. The journey continues at an unabated speed during this period too.

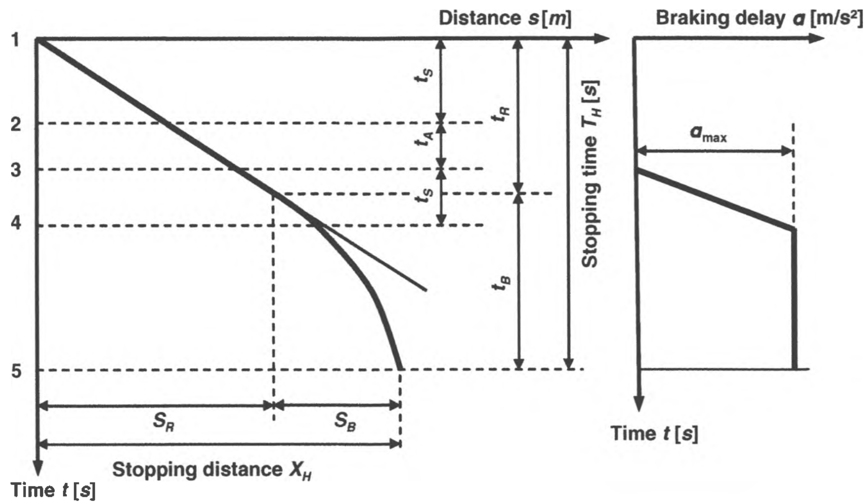
At time 3, deceleration starts and grows in a straight line until maximum deceleration is achieved a_{max} by time 4. This time span corresponds to the pressure build-up time for the brakes. Slight braking occurs, i.e. a reduction in speed can be felt and the time/distance line initially assumes the shape of a parabola.

Now the speed is constantly reduced – depending on the maximum deceleration that can be achieved – until the vehicle comes to a halt at time 5.

The response time and pressure build-up time of the brakes only amount to fractions of seconds, e.g., with:

- Mechanical and hydraulic brakes: 0.1–0.2 s
- Air pressure brakes: 0.2–0.6 s

So the following times for calculating the braking path can be summarized for the reaction and action time t_R:



This means:

1 : perceiving the danger
2 : starting the braking process
3 : starting the braking effect
4 : achieving the full braking effect
5 : vehicle comes to a halt
t_r : perception and reaction time
t_a : response time for brakes
t_s : pressure build-up time for brakes
$0.5 * t_s$: half pressure build-up time for brakes
t_R : reaction and action time [m]
t_R : $t_r + t_a + 0.5 * t_s$ [s]
t_B : pure braking time [s]
S_R : reaction and action distance [m]
S_B : pure braking distance [m]
X_H : stopping distance [m]
T_H : stopping time [s]

Figure 4.9: Time/distance diagram for the braking path

A uniform period of 2 *s* is set for t_R. During this time the distance covered during this journey at a constant speed is calculated as follows:

$$S_R = v{*}t_R, [\text{m}]. \tag{4.22}$$

with t_R : 2 s
v : [m/s].

The distance covered during the pure braking time t_B – on a journey with constant deceleration – is decisively influenced by the maximum deceleration that can be achieved. The following equation emerges for the whole stopping distance X_H:

$$X_H = S_R + S_B = \frac{v}{3.6}{*}2,0 + S_B, \quad v\,[\text{km/h}],\ X_H[\text{m}]. \tag{4.23}$$

Deceleration using brakes

Deceleration using brakes is defined as follows:

$$a = g{*}f_T,\ a\,[\mathrm{m/s^2}]. \tag{4.24}$$

where f_T: tangential component of the coefficient of adhesion between the wheel and road [−]

g : gravitational acceleration [m/s²].

If the journey does not occur on an even road, but in an area with an ascent or descent, the gradient s of the road must also be taken into consideration for the braking deceleration maneuver. The equation is defined as follows:

$$a = g{*}\left(f_T \pm \frac{s}{100}\right),\quad s\,[\%],\ a\,[\mathrm{m/s^2}]. \tag{4.25}$$

Maximum braking deceleration can be determined from brake tests. Because the maximum braking deceleration is only partially reached during deceleration, it is calculated in practice as an average braking deceleration rate, which is roughly

$$a_{mit} = 0.4 - 0.8{*}a_{\max},\quad a\,[\mathrm{m/s^2}]. \tag{4.26}$$

depending on the braking mode. The German Road Traffic Act (StVZO) prescribes the following as the minimum deceleration rate for motorized vehicles in Section 4.1:

$$a_{\min} = 2.5\,\mathrm{m/s^2} \tag{4.27}$$

on dry roads. This figure is far exceeded by automobile manufacturers in Germany, as Table 4.1 makes clear.

The maximum braking deceleration rate can reach values of the order of magnitude of $a_{\max} = 3.75$ to $7.50\,\mathrm{m/s^2}$ in normal braking deceleration maneuvers and values of $a_{\max} = 7.5$ to $15.0\,\mathrm{m/s^2}$ in emergency stop situations.

Example for calculating the braking distance with the help of braking deceleration

A journey is made on an even road ($s = 0\%$) at an initial speed $v_0 = 50$ km/h ($= 13.9$ m/s). Braking maneuvers during the journey on average involve constant deceleration $a_{\mathrm{mit}} = 5.0\,\mathrm{m/s^2}$. How large is

a) he braking time and the total braking period,
b) the braking path and the stopping distance?

Table 4.1: Initial acceleration and average braking deceleration for different means of transportation

Means of transportation	Average initial acceleration [m/s²]	Average deceleration rate	
		Normal deceleration rate [m/s²]	Emergency stop [m/s²]
Car	1.6	up to 3.0	up to 6.0
Bus	0.8–1.1	1.2–1.5	up to 4.5
Streetcar	0.5–1.2	0.6–1.2	up to 4.0
Rapid transit railway	0.6–1.1	0.6–1.2	up to 4.0
Railroad:			
– passenger train	0.20–0.5	0.6–0.8	0.7 – 1.0
– freight train	0.05–0.2	0.2–0.5	0.7 – 1.0

Solution:

a) Braking time t_B:

$$t_B = \frac{v - v_0}{-a} = \frac{0 - 13.9}{-5.0} = 2.8\,s$$

$$t_B = t - t_0.$$

Total braking period:

$$t_R + t_B = 2.0 + 2.8 = 4.8\,s$$

b) Braking path $X_B = X(v)$:

$$X_B = X(v) = \frac{v^2 - v_0^2}{-2a} = \frac{0 - 13.9^2}{-2*5.0} = 19.3\,m$$

or braking path $X_B = X(t)$:

$$X_B = X(t) = v_0*(t - t_0) - \frac{a}{2}(t - t_0)^2$$

$$\text{with } t_B = (t - t_0)$$

$$X_B = 13.9*2.8 - \frac{5,0}{2}*2.8^2$$

$$X_B = 38.6 - 19.3 = 19.3\,m.$$

Stopping distance:

$$X_H = v_0*t_R + X_B$$

$$X_H = 13.9*2.0 + 19.3 = 47.1\,m.$$

Calculating the braking path

As it is generally difficult to set a reliable figure for the average braking deceleration rate to determine the braking path, it is normal to select a different method for calculating the braking path. This option for calculating the braking path involves determining the braking distance in relation to the tangential coefficient of adhesion f_T. Moreover RAS-L (1995) provides the following conditional equation:

$$S_B = \frac{1}{3.6^2 * g} * \int_{v_1}^{v_2} \frac{v}{f_T(v) + \frac{s}{100} + \frac{W_L}{F_G}} dv. \tag{4.28}$$

S_B : pure braking path [m]
v : speed [km/h]
v_0 : speed at the beginning of the braking maneuver [km/h]
g : gravitational acceleration = 9.81 [m/s^2]
f_T : tangential component of the frictional force [−]

$$f_T(v) = 0.241 * \left(\frac{v}{100}\right)^2 - 0.721 * \left(\frac{v}{100} + 0.708\right). \tag{4.29}$$

s : gradient [%]
W_L : air resistance of vehicle [N]
F_G : weight of vehicle [N]

$$\frac{W_L}{F_G} = 0.327 * 10^{-4} * \left(\frac{v}{3.6}\right)^2. \tag{4.30}$$

For the braking maneuver from $v_2 = v_0 = v_{85}$ to $v_1 = 0$, this integral with $f_R = 0$ (f_R is the radial component of the frictional force, as explained in 4.3.2) and $f_T = f(v)$, can be solved as follows:

$$S_B = 147.8 * \ln \left(\frac{0.266 * \left(\frac{v_0}{100}\right)^2 - 0.72 * \frac{v_0}{100} + \frac{s}{100} + 0.708}{\frac{s}{100} + 0.708} \right)$$

$$+ \frac{213}{\sqrt{\frac{1.064 * s}{100} + 0.233}} * \operatorname{arctg} \left(\frac{\frac{v_0}{100} * \sqrt{\frac{1.064 * s}{100} + 0.233}}{2 * \frac{s}{100} - 0.721 * \frac{v_0}{100} + 1.42} \right) \tag{4.31}$$

S_B : pure braking path [m]
v_0 : speed at the beginning of the braking maneuver [km/h]
s : gradient [%]
(+) ascent, (−) descent.

Examples for calculating the braking path:

a) Calculating the pure braking path at an initial speed $v_0 = 50\,\text{km/h}$ with a gradient of $s = 0\%$.

$$S_B = 147.8 * \ln\left(\frac{0.266*\left(\frac{50}{100}\right)^2 - 0.72*\frac{50}{100} + \frac{0}{100} + 0.708}{\frac{0}{100} + 0.708}\right)$$

$$+ \frac{213}{\sqrt{\frac{1.064*0}{100} + 0.233}} * \text{arctg}\left(\frac{\frac{50}{100}*\sqrt{\frac{1.064*0}{100} + 0.233}}{2*\frac{0}{100} - 0.721*\frac{50}{100} + 1.42}\right)$$

$$S_B = 147.8 * \ln\left(\frac{0.067 - 0.360 + 0.708}{0.708}\right) + \frac{213}{0.483} * \text{arctg}\left(\frac{0.241}{1.060}\right)$$

$$S_B = 147.8 * \ln(0.586) + 441.27 * \text{arctg}(0.228)$$

$$S_B = -78.99 + 98.92 = 19.93\,m.$$

b) Calculating the braking distance with an initial speed $v_0 = 80\,\text{km/h}$ with a gradient of $s = -4\%$.

$$S_B = 147.8 * \ln\left(\frac{0.266*\left(\frac{80}{100}\right)^2 - 0.72*\frac{80}{100} - \frac{4}{100} + 0.708}{-\frac{4}{100} + 0.708}\right)$$

$$+ \frac{213}{\sqrt{\frac{1.064* - 4}{100} + 0.233}} * \text{arctg}\left(\frac{\frac{80}{100}*\sqrt{\frac{1.064* - 4}{100} + 0.233}}{2*\frac{-4}{100} - 0.721*\frac{80}{100} + 1.42}\right).$$

$$S_B = 147.8 * \ln\left(\frac{0.170 - 0.576 + 0.708}{-0.04 + 0.708}\right) + \frac{213}{0.436} * \text{arctg}\left(\frac{0.349}{0.763}\right)$$

$$S_B = 147.8 * \ln(0.393) + 488.09 * \text{arctg}(0.457)$$

$$S_B = -138.33 + 209.39 = 71.06\,m.$$

The analysis of the equation for calculating the braking path depending on the speed and the longitudinal slope is summarized in Table 4.2 with selected figures for s and the results are shown graphically in Figure 4.10.

Table 4.2: Braking distances according to RAS-L depending on speed and gradient (+) or (−)

v_0 [km/h]	Braking path $s = -8\%$	Braking path $s = -4\%$	Braking path $s = 0\%$	Braking path $s = +4\%$	Braking path $s = +8\%$
10	0.63	0.59	0.56	0.53	0.51
20	2.84	2.65	2.48	2.34	2.21
30	7.02	6.51	6.06	5.67	5.33
40	13.71	12.60	11.66	10.85	10.15
50	23.54	21.45	19.70	18.23	16.95
60	37.33	33.68	30.69	28.20	26.08
70	56.04	49.99	45.16	41.19	37.88
80	80.82	71.20	63.70	57.66	52.69
90	112.92	98.12	86.88	78.01	70.84
100	153.55	131.48	115.17	102.58	92.55
110	203.64	171.78	148.88	131.56	117.95
120	263.48	219.07	187.98	164.89	146.99
130	332.34	272.84	232.08	202.27	179.44

4.3 Vehicle Dynamics

4.3.1 Driving resistance

Resistance is created as a vehicle travels along a road. This consists of internal and external resistance.

The internal frictional resistance is negligible in the case of a vehicle and is not affected by the road design. This includes the internal frictional resistance which can originate either from the interaction of engine and gearbox or from ball-bearing friction.

The external driving resistance results from the interplay of the environment and vehicle. This includes rolling resistance W_0, gradient resistance W_S, and air resistance W_L (Figure 4.11).

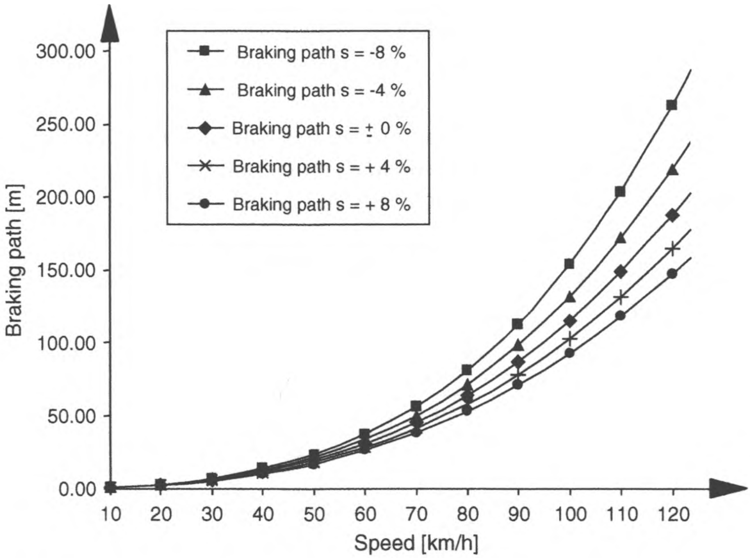


Figure 4.10: Braking paths according to RAS-L depending on speed and gradient

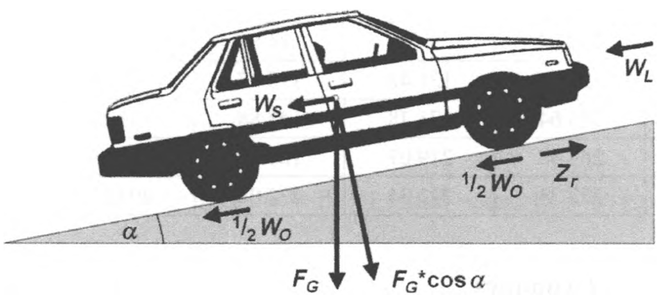


Figure 4.11: Types of driving resistance

Rolling resistance W_0

This resistance occurs as a result of the deformation between the tires and road and depends on the weight, tire pressure, speed, and the type and condition of the road surface.

In the following Table 4.3, typical individual-specific coefficients of rolling resistance are listed for pneumatic tires.

Table 4.3: Specific coefficients of rolling resistance for pneumatic tires

Material	w_0 [‰]	Material	w_0 [‰]
Concrete, asphalt, paving	10–20	Sand track	150–300
Gravel	20–40	Poor rural road	35
Unpaved road	50–150	Wheel on rail	1–2

The rolling resistance W_0 is calculated as follows:

$$W_0 = \frac{w_0 * F_G * \cos\alpha}{1000}, \text{ [N]}. \tag{4.32}$$

w_0 : specific rolling resistance [‰]
F_G : weight of vehicle [N]
α : angle of incline [degree].

For gradients on a road up to $s_{max} = 12\%$, a simplified form of $\cos\alpha$ is acceptable as 1.

Gradient resistance W_S
Gradient resistance originates from the weight of the vehicle when traveling along an inclined slope.

The resistance is calculated as follows:

$$W_S = \pm F_G * \sin\alpha = \pm F_G * \tan\alpha = \pm F_G * \frac{s}{100}, \text{ [N]}. \tag{4.33}$$

F_G : weight of vehicle [N]
s : gradient [%]
(+) ascent, (−) descent.

With a descent, the gradient resistance W_S becomes the driving force down the slope and must therefore be specified as a negative value.

Air resistance W_L
The air mass that a vehicle drives through during a journey generates counter pressure on the front of a vehicle, friction on the side walls, and suction forces and turbulence at the rear. The resulting resistance is called air resistance W_L.

It depends on the size of the front cross-sectional area, the shape of the vehicle, and the relative speed. Air resistance is calculated as follows:

$$W_L = 0.06 * c_w * \frac{A}{2} * v_R^2, \text{ [N]}. \tag{4.34}$$

with density of air at a temperature of 20°C.

c_w : specific aerodynamic drag coefficient [−], (depending on the vehicle's shape)
A : size of cross-sectional area of vehicle [m^2]
v_R : relative speed between vehicle and air [km/h]
– with headwind: $v_R = v + v_{\text{Wind}}$
– with tailwind: $v_R = v - v_{\text{Wind}}$.

The air resistance coefficients c_w and cross-sectional areas are shown for different vehicle types in the Table 4.4:

Air resistance is the most significant type of resistance affecting a vehicle.

Table 4.4: Aerodynamic drag coefficients c_w and cross-sectional areas F for different types of vehicles

Type of vehicle	c_w − value [−]	Cross-sectional area [m^2]
Motorcycle	0.6–0.7	0.5–0.6
Car	0.2–0.4	1.5–4.0
Car (older model)	0.4–0.6	2.0–4.0
Truck	0.8–1.0	4.0–7.0
Truck-trailer	0.8–1.5	4.0–7.0
Bus	0.5–0.7	4.0–7.0

4.3.2 Vehicle-road adhesion

The adhesion between a vehicle and the road is measured by the coefficient of adhesion and it describes the friction which must exist between tires and the roadway to enable any movement. The frictional force has an effect on the surface of the roadway in the direction of movement and can be broken down into two components in terms of vectors (Figure 4.12).

A tangential component f_T (often described as f_1), which is crucial for the movement of the vehicle, and a radial component f_R (also f_2), which only has an effect during cornering and helps partially compensate for the centrifugal forces.

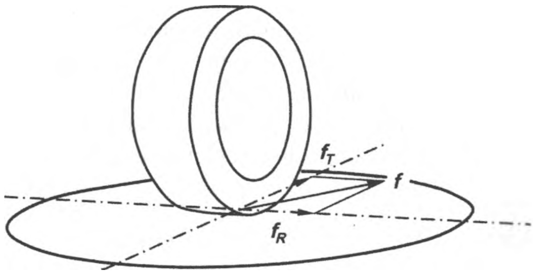


Figure 4.12: Forces between roadway and vehicle

Strictly speaking, there is no clear maximum coefficient of adhesion, because the size of the transferable friction between tire and road depends on a huge number of factors.

These include:

- the road surface (material and degree of pollution);
- the weather (rain, ice, snow);
- the tires (breadth, profile, material, air pressure); and
- driving behavior (speed and manner of driving).

So a wet and clean road was selected as the unfavorable, but realistic condition for taking measurements. Reference values for the coefficient of adhesion f_T for pneumatic tires at a speed of $v = 60$ km/h can be seen in Table 4.5. The following formula is provided to determine the tangential coefficient of adhesion in RAS-L (1995):

$$f_T = 0.241*\left(\frac{v}{100}\right)^2 - 0.721*\left(\frac{v}{100}\right) + 0.708. \tag{4.35}$$

f_T : tangential component of the coefficient of adhesion [−]
v : speed [km/h].

Comment:
Older textbooks often quote a different equation for determining the tangential coefficient of adhesion f_T. This results from the changes made in RAS-L. In the 1984 edition of RAS-L, the relevant measurements for the coefficient of adhesion were carried out with PHOENIX measuring tires with a diagonal structure. Tires have now been improved and measurements in the RAS-L 1995 were based on PIARC measuring tires with a radial structure. This difference has led to equation 4.35. The connection between the old and the new equation can be shown as follows:

$$f_{T(new)} = 0.016 + 1.126 * f_{T(old)}. \tag{4.36}$$

Table 4.5: Coefficient of adhesion f_T for pneumatic tires at $v = 60$ km/h

Road surface	Dry	Wet		Icy
		Clean	Dirty	
Gravel	0.70	0.50	0.40	Dry 0.20
Concrete	0.65	0.50	0.30	
Paving	0.55	0.30	0.20	Wet 0.10
Asphalt	0.55	0.30	0.20	
Dirt track	0.45	–	0.20	

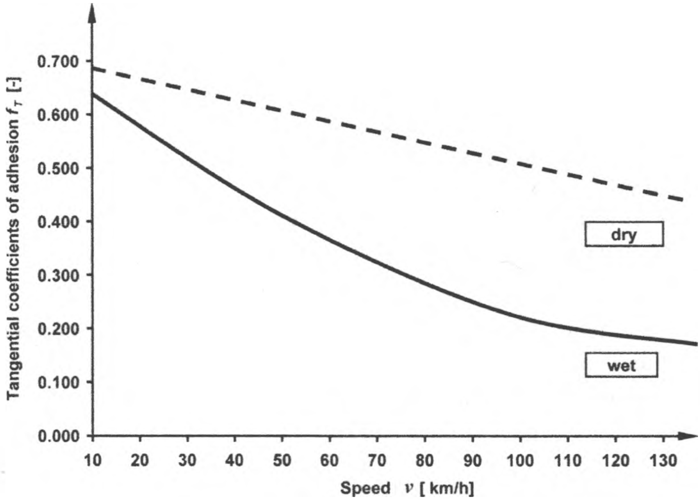


Figure 4.13: Coefficient of adhesion f_T depending on speed

The graphical representation of moisture influence on the tangential coefficient of adhesion f_T can be seen in Figure 4.13 and Table 4.6. The course of the tangential coefficient of adhesion is calculated for a dry road using the following formula:

$$f_T = -0.192 * \left(\frac{v}{100}\right) + 0.7 \tag{4.37}$$

while values under wet conditions are given by (4.35).

Table 4.6: Coefficients of adhesion according to RAS-L

v [km/h]	f_T (wet)	f_T (dry)
10	0.638	0.681
20	0.573	0.662
30	0.513	0.642
40	0.458	0.623
50	0.408	0.604
60	0.362	0.585
70	0.321	0.566
80	0.285	0.546
90	0.254	0.527
100	0.228	0.508
110	0.207	0.489
120	0.190	0.470
130	0.178	0.450

4.3.3 Vehicle motion

The complete external driving resistance is calculated as follows:

$$\sum W_F = W_0 + W_S + W_L, \text{ [N]}. \tag{4.38}$$

To overcome this driving resistance, tractive effort Z is required and this can be calculated from the effective output of the engine N_{mo}.

$$Z = \frac{N_{mo} * 3{,}600 * \eta}{v} \text{ [N]}. \tag{4.39}$$

N_{mo} : engine output [kW]
v : speed [km/h]
η : mechanical efficiency (depending on the gear used) [−].

After subtracting the driving resistance from the tractive effort Z, the remaining force P is either positive or negative. This force is used to overcome the acceleration resistance W_b.

$$P = Z - \sum W_F = Z - W_0 - W_S - W_L, \text{ [N]}. \tag{4.40}$$

If the vehicle is traveling steadily at a constant speed v, $P = 0$. If P is <0, the vehicle is decelerating and if P is > 0, the vehicle is accelerating.

A distinction is made with regard to the vehicle's movement between coasting and wheel spin. The frictional tractive effort Z_r is important for this distinction; this can be determined from the frictional weight G_r of the vehicle and the tangential component of the coefficient of adhesion f_T:

$$Z_r = f_T * G_r, \text{ [N]}. \tag{4.41}$$

G_r = frictional weight of the vehicle [N]; the part of the vehicle weight G lying on the drive wheels (about 0.5–0.6 G for a car)
f_T = tangential component of the coefficient of adhesion

If Z is $\leq Z_r$, coasting is taking place,
If Z is $> Z_r$, wheel spin is taking place (slip).

4.3.4 Cornering

Centrifugal forces are created when a vehicle travels through a bend with a radius R and they can be calculated as follows:

$$F = m * a_R = m * \frac{v^2}{R} = \frac{G}{g} * \frac{v^2}{R}, \text{ [N]}. \tag{4.42}$$

a_R : centrifugal acceleration [m/s^2]
v : speed [m/s]

G : weight of the vehicle [N]
R : radius of the circular arc [m]
g : gravitational acceleration = 9.81 [m/s^2].

Figure 4.14 illustrates the interaction of the forces affecting a vehicle in a bend.

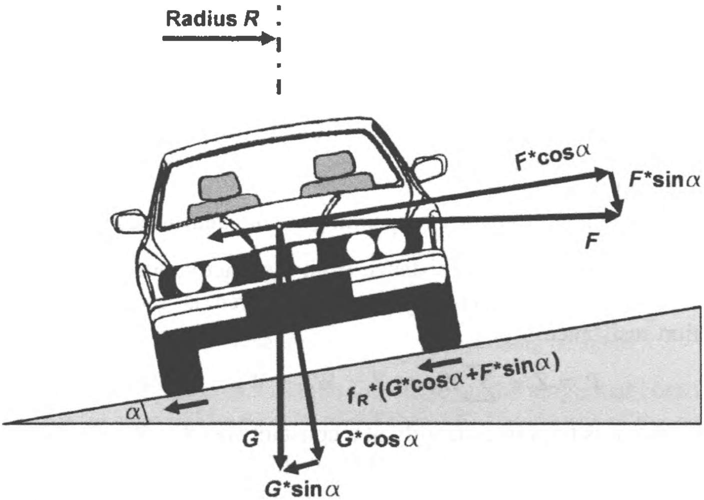


Figure 4.14: Forces acting on a vehicle in a bend

The weight G and the centrifugal forces F act on the vehicle's center of gravity. Both forces can be broken down into a component vertical to and parallel to the road surface. The radial component of the coefficient of adhesion f_R is available to absorb the forces acting in parallel to the road surface. This is supported by the camber on the road, which is directed towards the center point of the bend in order to exploit the weight of the vehicle to absorb the centrifugal forces. This results in forces of the following size affecting the surface of the road:

$$f_R*(G*\cos\alpha + F*\sin\alpha). \quad (4.43)$$

As for determining the necessary camber, the old version of RAS-L assumed that 1/3 of the centrifugal forces were absorbed by the camber on the road and the remaining 2/3 by static friction. The new edition of RAS-L is based on a new link between driving speed, radius of curvature, and camber. Based on this fact, a diagram was developed, from which the necessary camber q can be taken as a function of the radius of curvature R and speed v_{85} (Figure 4.15). Another factor is the road category.

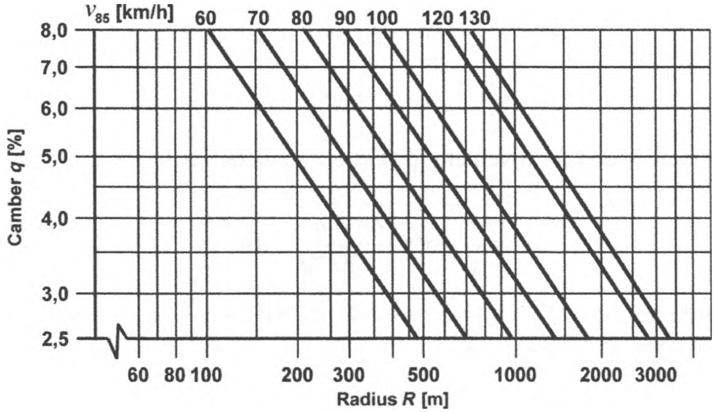


Figure 4.15: Camber q depending on speed v_{85} and the radius R for roads in category A and categories BI/BII according to RAS-L (1995)

Two marginal conditions should be noted here:

1. The minimum camber on the total paved width of the road $q_{\min} = 2.5\%$. For drainage purposes on the road, this is also the minimum camber on straights.
2. The maximum camber in the circular arc area of a bend q_{max} is 8.0% for roads in category A and categories BI/BII.

If the centrifugal forces are too great, drivers face the following unpleasant consequences:

1. The vehicle can skid or
2. The vehicle can turn over.

Vehicle skidding

A vehicle skids in a bend if the components consisting of centrifugal forces and weight that are acting parallel to the road exceed the effective static friction between the tire and the road. The threshold speeds for any incidence of skidding or the threshold radius can be derived from the marginal condition that the lateral cornering force is balanced by the static friction.

Because the angle α is very small, the assumptions can simplified:

$$\cos\alpha \sim 1 \tag{4.44}$$

$$\sin\alpha \sim \tan\alpha = q. \tag{4.45}$$

This provides the following equation if the familiar conditional equation (4.42) is introduced for the centrifugal forces F:

$$\frac{G}{g}*\frac{v^2}{R} - G*q = f_R*\left(G + \frac{G}{g}*\frac{v^2}{R}*q\right) \tag{4.46}$$

$$v^2 - q*g*R = f_R*g*R + f_R*v^2*q \tag{4.47}$$

$$v^2*(1 - f_R*q) = g*R*(f_R + q). \tag{4.48}$$

The threshold speed for a vehicle to skid is calculated as follows:

$$v = \sqrt{g*R*\frac{f_R + q}{1 - f_R*q}}, \quad [\text{m/s}]. \tag{4.49}$$

Because the result of $f_R * q$ is very small, the equation can also be shown in a simplified form as follows:

$$v = \sqrt{g*R*(f_R + q)}, \quad [\text{m/s}]. \tag{4.50}$$

The threshold radius for the skidding situation is calculated as follows:

$$R = \frac{v^2}{g}*\left(\frac{1 - f_R*q}{f_R + q}\right), \quad [\text{m}]. \tag{4.51}$$

or in a simplified form:

$$R = \frac{v^2}{g}*\left(\frac{1}{f_R + q}\right), \quad [\text{m}]. \tag{4.52}$$

Vehicle turns over

A vehicle turns over if the moment of force consisting of centrifugal forces and the weight component of the vehicle at too high a speed is greater than the moment of inertia. The threshold speed, which causes the vehicle to turn over if it is exceeded, can also be derived in a similar way to finding the threshold speed for skidding (Figure 4.16).

$$(F*\cos\alpha - G*\sin\alpha)*h = (G*\cos\alpha + F*\sin\alpha)*\frac{b}{2} \tag{4.53}$$

$$\frac{G}{g}*\frac{v^2}{R}*h - G*q*h = G*\frac{b}{2} + \frac{G}{g}*\frac{v^2}{R}*q*\frac{b}{2} \tag{4.54}$$

$$v^2*\frac{1}{g*R}*\left(h - q*\frac{b}{2}\right) = \frac{b}{2} + q*h \tag{4.55}$$

The threshold speed for a vehicle to turn over is calculated as follows:

$$v = \sqrt{g*R*\left(\frac{\frac{b}{2*h} + q}{1 - \frac{b}{2*h}*q}\right)}, \quad [\text{m/s}]. \tag{4.56}$$

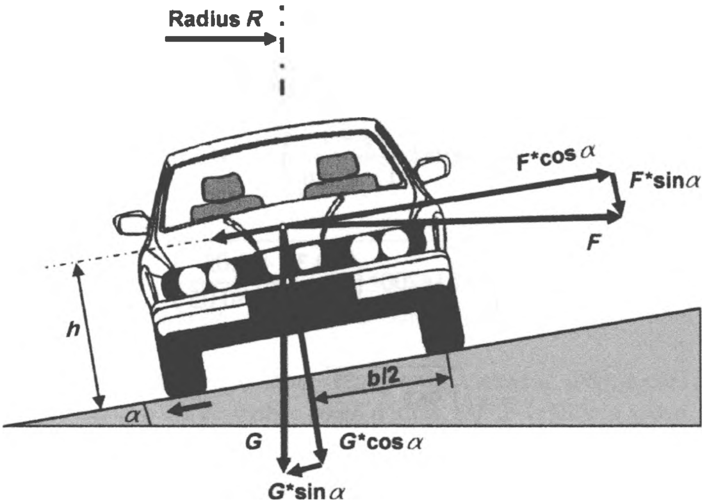


Figure 4.16: Torque forces on a vehicle when cornering

The threshold radius for a vehicle to turn over is:

$$R = \frac{v^2}{g} * \left(\frac{1 - \frac{b}{2*h} * q}{\frac{b}{2*h} + q} \right), \quad [\text{m}]. \tag{4.57}$$

The equation for the threshold speed clearly shows that the danger of turning over does not depend on static friction – in contrast to the risk of skidding. The risk of turning over is reduced by keeping the vehicle track as large as possible and by positioning the center of gravity of the vehicle as low as possible. In cars the risk of skidding is generally important, but the risk of turning over only plays a minor role.

Sample calculation
A car drives into a curve and skids. The radius R is 200 m and the camber q is 5%. The driver of the vehicle claims afterwards that he was not driving faster than 80 km/h.

a) Is this statement correct if measurements show that $f_R = 0.30$ for the radial component of the coefficient of adhesion?
b) How fast can you drive on the same road in winter if the coefficient of adhesion decreases to $f_R = 0.10$ as a result of icy conditions?

Solution:

a) The threshold speed for skidding is calculated as follows:

$$v = \sqrt{g*R*\left(\frac{f_R + q}{1 - f_R*q}\right)}$$

$$v = \sqrt{9.81*200*\left(\frac{0.30 + 0.05}{1 - 0.30*0.05}\right)}$$

$$v = \sqrt{1.962*\frac{0.35}{0.985}}$$

$$v = \sqrt{697} = 26.4\,\text{m/s} = 95\,\text{km/h}$$

or with a simplified calculation:

$$v = \sqrt{687} = 26.2\,\text{m/s} = 94.3\,\text{km/h}$$

The driver's statement is wrong, because the threshold speed is 95 km/h before any skidding occurs. So the driver must have been driving faster than this threshold speed.

b)

$$v = \sqrt{9.81*200*\left(\frac{0.10 + 0.05}{1 - 0.10*0.05}\right)}$$

$$v = \sqrt{1.962*\frac{0.15}{0.995}}$$

$$v = \sqrt{296} = 17.2\,\text{m/s} = 61.9\,\text{km/h}$$

or with a simplified calculation:

$$v = \sqrt{294} = 17.1\,\text{m/s} = 61.6\,\text{km/h}.$$

The maximum speed on this road should be 60 km/h.

4.4 Questions

(1) Sketch and explain the fundamental diagram (distance/time diagram) for the movement of a vehicle.
(2) Explain the term "momentary speed."
(3) What are the constituent elements in stopping distance X_H?
(4) Which figure is normally used for t_R?
(5) How large is the minimum deceleration rate according to the German Road Traffic Act?
(6) Name the crucial sources of driving resistance.
(7) The speed v_R should be taken into account with air resistance. Which speed is this?
(8) How does the centrifugal force change when moving from a curve with a large radius to a curve with a small radius, if the vehicle's speed remains constant?
(9) When does a vehicle start to skid?
(10) When does a vehicle turn over and how can this be prevented by changing the properties of the vehicle?

Chapter 5

Speed Terminology

5.1 Overview

Vehicle speed is the crucial element in road design. A distinction has to be made between the speed at which an individual vehicle can proceed in ideal traffic conditions, independently of other road users, and the speed reached in real traffic conditions when vehicles are following each other. Another distinction exists between the desirable speed of travel depending on the road category and the actual speed reached, which is determined by the road design elements on the horizontal and vertical projections, the cross section, and the given volume of traffic.

The permissible speed v_{zul} has to be distinguished from the four speed terms given in Table 5.1. It sets the maximum speed at which a vehicle may be driven on a section of road. It is governed by the road traffic regulations and is restricted on certain sections of road by traffic signs.

Table 5.1: Crucial speeds (Pietzsch and Wolf, 2000)

	Specification for	
	Individual vehicle	Vehicle in traffic
Speed as a guideline for dimensions	Design speed v_{e}	Average assessment speed v_B
Speed actually reached when traveling	Speed v_{85}	Actual cruising speed v_R

5.2 Design Speed v_{e}

The design speed v_{e} is the main criterion for defining the geometrical elements of the alignment of a road. It is selected depending on the network function of the desired level of traffic quality and the purpose of the road. The v_e is the maximum speed that can be reached safely along the track in wet road conditions, when the speed is limited only by geometric characteristics (i.e., perfect road conditions); referring to this, the road planning allows minimum radii and minimum length of crest and sag quadratic parabolas with maximum camber to ensure that the vehicle comes to a standstill in good time in front of an obstacle (stopping sight distance) during an emergency braking maneuver.

The choice of design speed depends on the road category (Table 5.2) and should take into consideration the degree of difficulty of the terrain and the accumulation of constraints. It also determines the threshold values and benchmarks for the design elements on the horizontal and vertical projections including the minimum radii of curvature, the minimum clothoid parameters, the maximum gradient, and the minimum length of crest and sag quadratic parabolas.

The v_e crucially affects the characteristics of the route and therefore the safety and quality of the traffic flow and economic aspects. So the speed should remain constant over fairly long associated sections of roads.

5.3 Speed v_{85}

The speed v_{85} is used to check the design input and to verify existing road tracks, assessing road traffic design elements related to safety on the horizontal and vertical projections and the cross section. It should describe actual driving behavior and aims to be the speed that 85% of unimpeded driving vehicles will not exceed on a wet road.

The speed v_{85} determines the cross slope (q in %), the necessary stopping sight distance (min S_h in meter), the passing sight distance (min $S_Ü$ in meter), and the minimum radii with negative camber (min R in meter).

When determining speed v_{85} according to RAS-L, a distinction must be made between undivided and divided highways. On undivided single-lane highways occurring in category A, there is an interrelation between the geometry of the section and any alteration in speed v_{85}. To determine parameters, RAS-L sets out a procedure, whereby v_{85} can be determined for an associated section of a road or an individual bend. A distinction is made between new building work (Case 1) and modification and upgrading work (Case 2).

Case 1: Normal case for new building work

The speed has to be determined depending on the bendiness and the road width for both directions of traffic along various sections (Figure 5.1).

Table 5.2: Design and operating features of roads depending on their road category (RAS-L)

Road function		Design and operating features				
Category group	Road category	Type of traffic	Permissible speed (km/h)	Cross section	Intersections	Design speed (km/h)
A Roads with no extra lanes outside urban areas mainly with a connecting function	**A I** Long-distance road	Vehicles Vehicles	- ≤ 100(120)	Divided highways Undivided highways	Different levels (different levels) same level	120 100 100 90 (80)
	A II National/regional road	Vehicles General	- ≤ 100	Divided highways Undivided highways	Different levels (same level) same level	100 90 (80) 90 80 (70)
	A III Road linking communities	General	≤ 100 ≤ 100	Divided highways Undivided highways	(Different levels) same level same level	(90) 80 70 80 70 60
	A IV Road opening up sites	General	≤ 100	Undivided highways	Same level	70 60 (50)
	A V Less important road	General	≤ 100	Undivided highways	Same level	(50) None
	A VI Business route	General	≤ 100	Undivided highways	Same level	None
B Roads with no extra lanes near a city and within urban areas mainly with a connecting function	**B I** City highway		≤ 100	Divided highways	Different levels	100 90 80 (70)
	B II Fast trunk road		≤ 80	Divided highways	Different levels (same level)	80 70 (60)
	B III Main road	General General	≤ 70 ≤ 70	Divided highways Undivided highways	Same level Same level	70 60 (50) 70 60 (50)
	B VI Main local Distribution road	General	≤ 60	Undivided highways	Same level	60 50
C Roads with extra lanes inside urban areas mainly with a connecting function	**C III** Main road	General General	50 50	Divided highways Undivided highways	Same level Same level	(70) (60) 50 (40) None (60) 50 (40)
	C IV Main local Distribution road	General	50	Undivided highways	Same level	50 (40)
D Roads with extra lanes within urban areas mainly with site development function	**D IV** Distribution road	General	≤ 50	Undivided highways	Same level	None
	D V Residential road	General	≤ 50	Undivided highways	Same level	None
E Roads with extra lanes within urban areas mainly with a residential function	**E V** Residential road	General	Step speed	Undivided highways	Same level	None
	E VI Residential track	General	Step speed			None

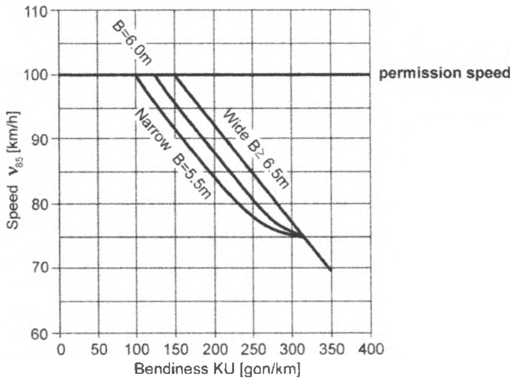


Figure 5.1: Relation between bendiness KU, road width B, and speed v_{85} on undivided two-lane roads in category group A (RAS-L)

The bendiness is the total of the absolute changes in direction related to the length of the section of road and it can be calculated using the formula:

$$\mathrm{KU} = \frac{\sum_{i=1}^{n} |\gamma_i|}{L} \tag{5.1}$$

where

KU [gon/km]: bendiness
γ_i [gon]: sum of the angle of total changes of direction in a bend $\gamma_i = \tau_{1i} + \alpha_{1i} + \tau_{2i}$
τ_i [gon]: angle of change in direction in the transitional curve
α_i [gon]: angle of change in direction in the circular arc
L [km]: length of the section of road

The section under examination must be divided into sections of similar curvature. The limit on the single sections should be estimated from the total line of the absolute changes in direction.

Case 2: Normal case for short sections being modified or upgraded

The procedure for Case 1 is not suitable for modification and upgrading work, so the speed v_{85} has to be estimated on short sections of road from the road width and radius of curvature according to Figure 5.2.

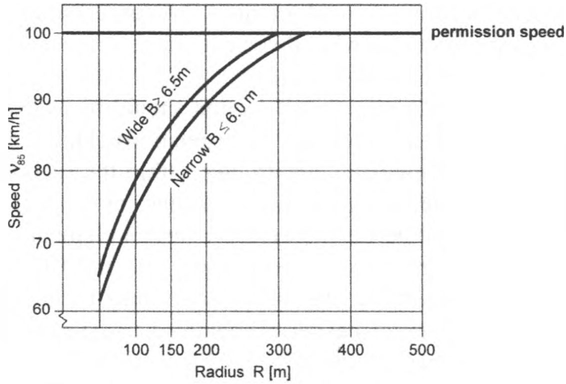


Figure 5.2: Relationship between the radius R, road width B, and speed v_{85} on undivided two-lane roads in category group A, (RAS-L)

Roads with an undivided 2 + 1 cross section occupy a special position when it comes to determining the speed v_{85} because the above-mentioned procedure cannot be used because of the high speeds in the passing lane. The v_{85} can be determined using the following equation, but v_{85} may not exceed 100 km/h.

$$\begin{aligned} v_{85} &= v_e + 20 \text{ km/h} \\ v_e &< 100 \text{ km/h} \end{aligned} \tag{5.2}$$

On divided highways in category group A, there is still no accurate knowledge about the interrelation between road characteristics and driving speed. So the following equations are used to determine v_{85}:

$$v_{85} = v_e + 20 \text{ km/h} \qquad (v_e < 100 \text{ km/h}) \tag{5.3}$$

$$v_{85} = v_e + 10 \text{ km/h} \qquad (v_e \geq 100 \text{ km/h}) \tag{5.4}$$

On roads in category BI and BII, the speed v_{85} is equated with the permissible maximum speed: $v_{85} = zul\,v$.

The speed v_{85} and the design speed v_e defined in the preceding section should relate to each other in a balanced manner, which enables the features of the route to be harmonized with driver behavior. This particularly applies to roads with very small radii of curvature or a large degree of bendiness.

In the case of average speeds, a road section planned with minimum values also permits higher speeds used by 85% of road users than that achieved by selecting v_e. So it is difficult to coordinate v_{85} and v_e. If it is impossible to select a higher v_e for the section of road in question for economic reasons, the transition between the sections with diverse road features must be designed carefully.

The actual speed driven along a road section should be as even as possible. This can be guaranteed by the necessary radii relations (Figure 5.3), because a well-balanced sequence of elements for driving within the sections at the same design speed promotes a steady and economic manner of driving.

The principle of balance between successive design elements should also be taken into consideration for roads in road category BI and BII, even if driving behavior is largely affected by the permissible maximum speed in these categories, not by the choice of design elements. If the speeds determined for adjacent sections deviate by more than 10 km/h, checks should be made to see whether the speed figures can be adjusted or whether an additional section can create a transition between the different speeds.

When upgrading sections of existing roads, the design elements in the adjacent road sections must be taken into consideration. The design of the transition sections requires special care if there are huge differences in the road features.

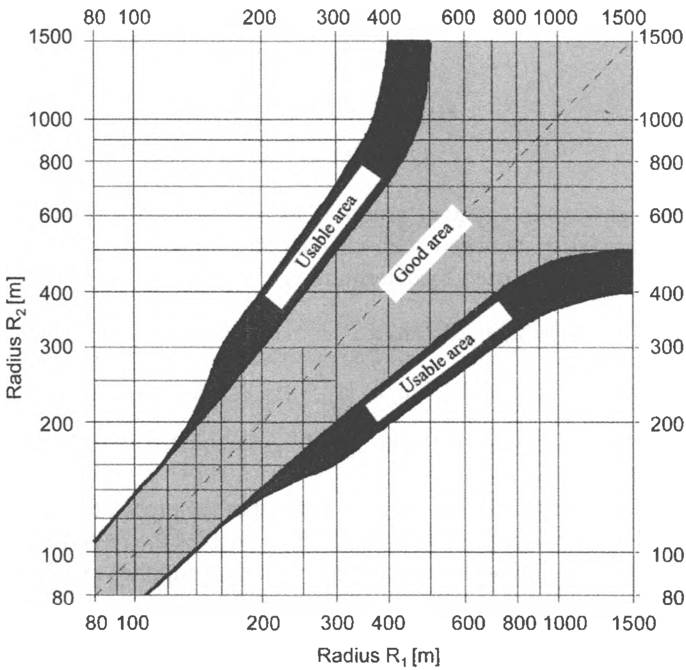


Figure 5.3: Coordinating sequence of radii on roads in category group A (Lippold, 1997)

5.4 Average Assessment Speed v_B

The average assessment speed v_B describes the desired average speed of travel for all vehicles in existing road conditions. It is used in RAS-N and provides evidence of traffic quality when setting the dimensions of cross sections of roads. Speed ranges for the average assessment speed v_B are given for the standard cross sections of roads without any extra lanes (Table 5.3). The lower values are valid in difficult terrain conditions and the values in brackets for extreme topography. It is essential that v_B is $< v_{zul}$ in every case.

Table 5.3: Average assessment speed v_B depending on the standard cross section (Pietzsch and Wolf, 2000)

Standard cross section	Assessment speed (km/h)
RQ 35.5/RQ 29.5	110–80 (70)
RQ 33/RQ 26	100–70 (60)
RQ 26/RQ 15.5	90–60 (50)
RQ 10.5	80–50 (40)
RQ 9.5	60–50 (40)

5.5 Actual Cruising Speed v_R

The actual cruising speed v_R is the expected figure for the average speed of vehicles that should be reached on a fairly long section of road in the road network using the traffic assessment number method in RAS-Q.

It is also a measure of traffic quality within the quality stages A, B, C, D, and E, and therefore provides a target value for the desired average cruising speed for cars in existing road and traffic conditions.

The cruising speed v_R is an important indicator for selecting the design speed and is taken from the basic table for road network design in RAS-N (Table 5.4).

5.6 Intersection Speed v_K

The intersection speed v_K must be based on the dimensions at intersections. Generally, it matches the permissible speed v_{zul} within built-up areas. The intersection speed v_K is taken from the RAS-K-1 for roads in category groups A to C.

All the design elements for the intersection must be assessed using the intersection speed. The only exception is the visibility conditions – where the v_{85} speed is the standard figure.

Table 5.4: Recommended cruising speeds according to RAS-N

	Road function		Standard distance range km	Targets for speeds in km/h in				
	Category group	Road category		Workday traffic			Vacation traffic	Sunday traffic
				Connection below	Connection in	Connection above		
				Standard distance range				
	1	2	3	4	5	6	7	8
A	Roads with no extra lanes outside urban areas mainly with a connecting function	A I Long-distance road	100–200	60–90	70–100	90–110	60–90	60–80
		A II National/regional road	50–100	50–80	60–90	70–90	50–80	50–70
		A III Road linking communities	25–50	40–60	50–80	60–80	40–70	40–60
		A IV Road opening up sites	0–25	–	40–60	50–70	40–60	40–50
		A V Less important road	–	None			None	None
		A VI Business route	–	None			None	None
B	Roads with no extra lanes near a city and within urban areas mainly with a connecting function	B I City highway	–	50–70			40–60	40–50
		B II Main road	–	40–60			30–50	30–40
		B III Main local distribution road	–	30–50			30–40	30
C	Roads with extra lanes inside urban areas mainly with a connecting function	C III Main road	–	30–50			30–50	30–40
		C IV Main local distribution road	–	30–40			30–40	30
D	Roads with extra lanes within urban areas mainly with site development function	D IV Distribution road	–	20–30			20–30	20–30
		D V Residential road	–	None			None	None
E	Roads with extra lanes within urban areas mainly with residential function	E V Residential road	–	None			None	None
		E VI Residential track	–	None			None	None

5.7 Questions

(1) What do you understand by the design speed v_e?
(2) Is the v_e normally higher or lower than the maximum speed allowed by the German Road Traffic Act on rural roads?
(3) How is v_e selected?
(4) What evidence does v_{85} provide?
(5) What do you understand by the term 'sequence of radii'?
(6) Explain the connection between cruising speed and traffic quality.

Chapter 6

Sight Distances

6.1 Principles

Sight distances are very important for drivers and design engineers. On the one hand, the German Road Traffic Regulations oblige drivers to adapt their speed to the existing sight distances. On the other hand, sight distance provides the design engineer with a single-dimensional parameter, which typically describes the three-dimensional road ahead. If these two issues are joined together, this creates the false assumption that the engineer can influence the speed with sight distance. However, there is no direct connection between sight distance and drivers' choice of speed. This is based on how difficult it is to assess the speed of your own vehicle and the speed of oncoming vehicles and the necessary braking paths caused as a result.

But there are connections between sight distance and the average speed of traffic. Different types of sight distances are used depending on their relevance for drivers or design engineers and the reasons for their use.

6.2 Meteorological Sight Distance

Meteorological sight distance indicates the visibility found in certain weather conditions. It is not possible to subdivide them into rain, fog, or snow sight distance, as it is impracticable to standardize these particular weather conditions. Therefore meteorological sight distance cannot be taken into account in the design process. The arrangement of traffic control equipment like road markers and marker posts or other guidance aids, e.g. bend signs, can counteract reductions in visibility caused by precipitation.

6.3 Physiological and Psychological Perception of Sight Distance

The parameters for the physiological perception of sight distance depend on the physical features of the human eye and the optical features of the visible objects and their backgrounds. The existing light conditions – e.g. lighting and oncoming light – and the prevalent weather conditions – e.g. rain or fog – are other factors.

Various types of physiological perception of sight distance are mentioned in the literature on the subject:

Aulhorn (1971) defines sight distances as the "distance from which an object can be perceived with particular visual acuity."

Lorenz (1971) describes the perception of sight distance as the distance needed to guarantee safe, not sudden braking. He specifies this as the distance that the driver will cover in the next 10 s.

Dilling (1973) includes traffic in the process by which drivers determine sight distance. He characterizes the section of road where the road is clearly visible and his starting point is the largest distance between two vehicles where the driver still has eye contact with the moving vehicle ahead.

Physiological sight distances cannot be incorporated in road design because of the huge number of factors that play a part.

Hiersche (1968) defines the depth of the road traffic area as a psychological perception of sight distances where drivers have the impression that they can perceive the area adequately and perfectly for the driving maneuver. Hiersche demands that traffic space should be designed in such a way that the information to be absorbed by drivers justifies the selected speed. It is not possible to include the perception of psychological sight distances in road design because of their subjective nature.

6.4 Geometrical Sight Distances

6.4.1 Overview

Geometrical sight distances represent minimum set values that must be heeded and, by comparing the sight distances available, they primarily serve as a means of checking the design. They are calculated according to defined driving and geometrical model images. The calculation models used contain various simplifications compared with the actual conditions. Meteorological, physiological, and psychological perception factors are not taken into account in these calculations. Geometrical sight distances therefore represent technical checking factors for the road design process.

Geometrical sight distances are divided into the required stopping sight distance and the necessary passing sight distance.

6.4.2 Stopping sight distance

According to RAS-L, the stopping sight distance describes the distance at which a driver needs to stop his vehicle driving at speed v_{85} just in time in front of an obstacle that unexpectedly appears on the roadway. Because the ability of the human eye to perceive things is limited, the obstacle that unexpectedly appears must be of at least a certain size and form a contrast with the background. But these parameters cannot be quantified in a universally valid manner at the moment. So it is not possible to take them into account in the calculation model for the necessary stopping sight distance. The necessary stopping sight distance S_H consists of the distance during the reaction and action time S_R and the braking path S_B.

The distance S_R represents the area that the vehicle covers during the reaction and action time at a constant speed.

The distance S_B represents the pure braking path. Its size depends on the braking deceleration rate that can be achieved and is calculated according to equation (6.1). The calculation model assumes a constant braking deceleration rate.

$$S_B = \frac{1}{3.6^2 * g} * \int_{v_2}^{v_1} \frac{v}{f_T(v) + \frac{s}{100} + \frac{W_L}{F_G}} * dv, \quad [\mathrm{m}]. \tag{6.1}$$

This includes:

v	: speed	[m/s]
v_1	: speed at the beginning of the braking maneuver	[km/h]
v_2	: speed at the end of the braking maneuver	[km/h]
g	: gravitational acceleration	[m/s^2]
f_T	: tangential coefficient of adhesion	[−]
s	: gradient	[%]
W_L	: air resistance of the vehicle	[N]
F_G	: weight of the vehicle	[N].

The braking deceleration rate that can be achieved is determined by the sources of resistance to driving and the tangential adhesion using the following defined marginal conditions:

–	Vehicle:	passenger car
–	Reaction and action time:	$t_R = 2\,s$
–	Speed at the beginning of the braking maneuver:	$v_1 = v_{85}$
–	Speed at the end of the braking maneuver:	$v_2 = 0\,\mathrm{km/h}$
–	Tangential adhesion:	$f_T = \frac{f_{max}}{t_R} = 0.$

The required stopping distance can be calculated with equation (6.2):

$$S_H = 0.56 * v_{85} + \frac{v_{85}^2}{254 * \left(f_T \pm \frac{s}{100}\right)}, \quad [\mathrm{m}]. \tag{6.2}$$

6.4.3 Passing sight distance

Another application of the equations of motion can be seen in connection with a situation where one vehicle passes another. These checks are important for road design, where – depending on the road category – sufficient passing sight distance must be guaranteed on certain road sections. A passing maneuver can be clearly shown in a distance/time diagram (Figure 6.1).

Vehicle 1 driving at a certain speed v_1 wishes to pass vehicle 2 (speed v_2) in a section measuring s. The passing vehicle accelerates to speed $v_1 + \Delta v$ when there is a possibility to pass and moves back into the lane after the passing maneuver at a distance of s_b in front of the vehicle that has just been passed and continues its journey at speed v_1. While the vehicle being passed drives at a constant speed during the passing maneuver, the passing vehicle first accelerates and then decelerates.

Vehicle 3 is driving in the opposite direction at speed v_3.

The following sight distance with regard to vehicle 3 must be available for vehicle 1 to pass a vehicle that is moving at a slower speed:

$$S_{\ddot{U}} = s_1 + s_3 + s_4, \quad [\text{m}]. \tag{6.3}$$

This involves:

$S_{\ddot{U}}$: passing sight distance [m]
s_1 : distance covered by the passing vehicle [m]
s_3 : distance covered by the oncoming vehicle [m]
s_4 : safety margin [m].

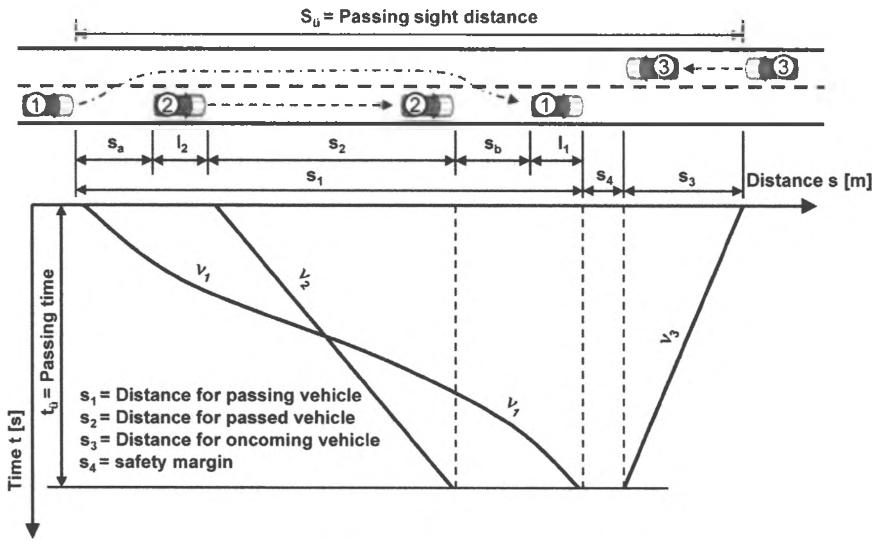


Figure 6.1: Passing with constant acceleration and deceleration

The length of the passing maneuver s_1 can be calculated as follows:

$$s_1 = \frac{v_2}{3.6} * \sqrt{2*d*\left(\frac{a_2 - a_1}{a_1*a_2}\right)} + d, \quad [\mathrm{m}]. \tag{6.4}$$

This involves:

s_1 : passing distance [m]
v_2 : speed of the vehicle being passed [km/h]
a_1 : acceleration of the passing vehicle [m/s^2]
a_2 : deceleration of the passing vehicle [m/s^2]
d : $s_a + s_b + l_1 + l_2$ [m]
l_1 : length of the passing vehicle [m]
l_2 : length of the vehicle being passed [m].

To calculate the distances s_a and s_b, it has been assumed for the purposes of simplification that they are covered at speed v_1 during a period which roughly corresponds to the action time required for braking and acceleration. Then the distances can be calculated as follows:

$$s_a = s_b = \frac{v_1}{3.6} * t_A, \quad [\mathrm{m}] \tag{6.5}$$

where t_A = action time = 1 s.

Example for calculating the passing distance s_1:

v_1 : 90 km/h	v_2 : 70 km/h
a_1 : 1.0 m/s^2	a_2 : −0.5 m/s
l_1 : l_2 = 5.0 m.	

On the basis of this information, the passing distance s_1 is calculated as follows:

$$s_a = s_b = \frac{90}{3.6} * 1 = 25.0\,\mathrm{m}$$

$$d = 2*25.0 + 2*5.0 = 60.0\,\mathrm{m}$$

$$s_1 = \frac{70}{3.6} * \sqrt{2*60.0*\left(\frac{-0.5 - 1.0}{1.0*(-0.5)}\right)} + 60.0$$

$$s_1 = 19.44*\sqrt{120*3.0} + 60.0 = 19.44*18.97 + 60.0$$

$$s_1 = 368.85 + 60.0 = 428.85 = 430\,\mathrm{m}.$$

Model for Calculating the Necessary Passing Sight Distance According to RAS-L:

In RAS-L (1995), the model for calculating the passing sight distance $S_{Ü}$ is simplified as follows (Figure 6.2).

Instead of the passing vehicle accelerating and then decelerating, it is assumed that the vehicle drives at a constant speed. During the passing maneuver, this speed v_1 is set at $1.10 * v_{85}$. It is assumed that the vehicle being passed continues to travel during the passing maneuver consistently at the speed $v_2 = 0.85 * v_{85}$. The speed of any oncoming vehicle is assumed to be $v_3 = v_{85}$.

With the help of this model, the passing sight distance required $S_{Ü}$ can be calculated depending on v_{85} by using the following formula:

$$S_{Ü} = \frac{(v_1 + v_3)*d}{v_1 - v_2} + s_4, \quad [\text{m}]. \tag{6.6}$$

This involves:

v_1 : speed of the passing vehicle,	$v_1 = 1.10 * v_{85}$	[km/h]
v_2 : speed of the vehicle being passed,	$v_2 = 0.85 * v_{85}$	[km/h]
v_3 : speed of the oncoming vehicle,	$v_3 = 1.00 * v_{85}$	[km/h]

s_4 : safety margin (assumed to be 25 m), [m]

d : $s_a + s_b + l_1 + l_2$ [m].

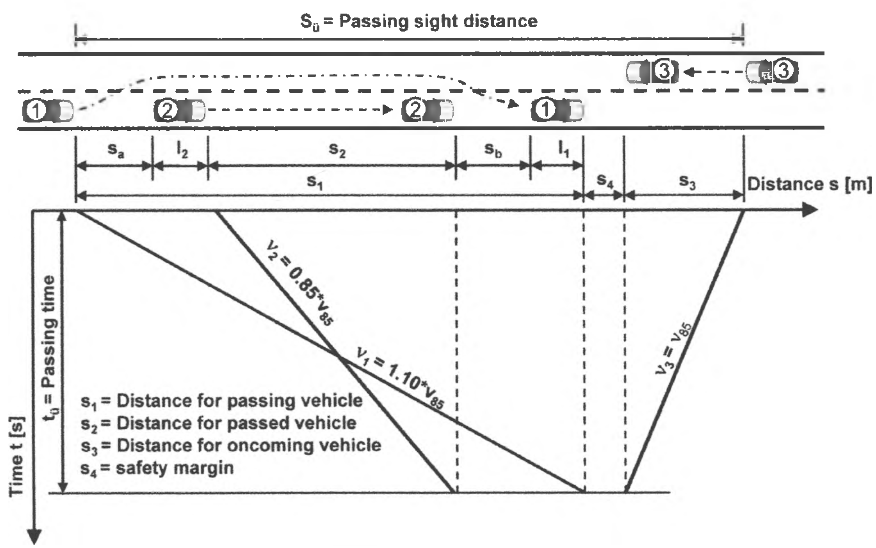


Figure 6.2: Model for assessing passing sight distance according to RAS-L

If the set speeds are entered in the above equation, this provides the following equation for the passing distance $S_{\ddot{U}}$:

$$S_{\ddot{U}} = \frac{(1.10*v_{85} + 1.00*v_{85})*d}{1.10*v_{85} - 0.85*v_{85}} + s_4, \quad [\text{m}].$$

$$S_{\ddot{U}} = \frac{2.10*v_{85}*d}{0.25*v_{85}} + s_4 = 8.40*d + s_4, \quad [\text{m}].$$

It is clear that speed is involved to determine the distances s_a and s_b and they in turn are needed to determine d.

The following example should make clear the difference between the model for passing sight distance used in RAS-L and the actual driving behavior during a passing maneuver.

Example: Passing Maneuvers Involving Two Cars

The vehicle lengths of both passenger cars are given as 5 m and their speed is $v_{85} = 80$ km/h, $s_4 = 25$ m.

1. Calculation with constant acceleration and deceleration

   Acceleration $a_1 = 1.5\ \text{m/s}^2$

   Deceleration $a_2 = -1.5\ \text{m/s}^2$

$$d = s_a + s_b + l_1 + l_2$$

$$l_1 = l_2 = 5.00\ \text{m}$$

$$s_a = s_b = \frac{v_1}{3.6}*t_A$$

$$v_1 = 1.10*v_{85} = 1.10*80 = 88\ \text{km/h}$$

$$s_a = s_b = \frac{88}{3.6}*1.0 = 24.50\ \text{m}$$

$$d = 2*24.50 + 2*5.0 = 59.0\ \text{m}$$

Passing distance s_1:

$$s_1 = \frac{v_2}{3.6}*\sqrt{2*d*\left(\frac{a_2 - a_1}{a_1*a_2}\right)} + d$$

$$s_1 = \frac{0.85*80}{3.6}*\sqrt{2*59*\left(\frac{-1.5 - 1.5}{1.5*(-1.5)}\right)} + 59$$

$$s_1 = 18.90*\sqrt{157} + 59$$

$$s_1 = 18.90*12.54 + 59 = 237 + 59 = 296\,\text{m}.$$

Passing time $t_{Ü}$:

$$t_{Ü} = \frac{s_1}{v_1}*3.6 = \frac{296}{88}*3.6 = 12.1\,\text{s}.$$

Distance covered by the oncoming vehicle s_3:

$$s_3 = t_{Ü}*v_3 = 12.1*\frac{80}{3.6}$$

$$s_3 = 269\,\text{m}.$$

Passing sight distance $S_{Ü}$:

$$S_{Ü} = s_1 + s_3 + s_4$$

$$S_{Ü} = 296 + 269 + 25 = 590\,\text{m}.$$

2. Calculation at constant speeds

$$S_{Ü} = \frac{(1.10*v_{85} + 1.00*v_{85})*d}{1.10*v_{85} - 0.85*v_{85}} + s_4$$

$$S_{Ü} = \frac{2.10*v_{85}*d}{0.25*v_{85}} + s_4 = 8.40*d + s_4$$

$$d = 59\,\text{m}, \text{ with } s_4 = 25\,\text{m}$$

$$S_{Ü} = 8.40*59 + 25 = 495 + 25 = 520\,\text{m}.$$

3. Passing sight distance $S_{Ü}$ according to RAS-L

$$S_{Ü} = 525\,\text{m}$$

Analysis:
The passing sight distance $S_{Ü}$ calculated as movement with constant acceleration and constant deceleration is much longer than the passing sight distance $S_{Ü}$ using the model in RAS-L (1995). The main reason for this is the higher value for distance s_1, because it is calculated at a constant speed ($1.10 * v_{85}$) in the RAS-L (1995) model.

Another crucial factor is the assumption about the figures for acceleration and deceleration. In practice, these figures are usually higher than those assumed in the sample calculation.

The model for the passing sight distance $S_{Ü}$ (driving at a constant speed) provides lower v_{85} speed figures for $S_{Ü}$ figures than the values listed in Table 6.1 in RAS-L (1995). The reason for this is that actual driving behavior at lower speeds differs markedly from the driving behavior assumed in the model (speed of the passing vehicle is constant $= 1.10 * v_{85}$ and speed of the vehicle being passed is also constant = 0.85 * v_{85}). So it makes sense to retain the specified values in RAS-L (1995) when designing roads, particularly for lower v_{85} speeds.

Table 6.1: Comparison of the figures for passing sight distance $S_Ü$ according to Table 15 in RAS-L (1995)

v_{85} [km/h]	$S_Ü$[m] RAS-L (1995)	$S_Ü$ [m] (calculated values)
60	475	416
70	500	470
80	525	520
90	575	571
100	625	620

6.5 Existing Sight Distances

The existing sight distances are calculated by taking into account the following geometrical marginal conditions:

- eye level in the road cross section,
- height of eye level above the surface of the road,
- position of the target point in the road cross section,
- height of the target point above the surface of the road.

In general, the middle of the right lane is assumed to be the eye level. This roughly corresponds to the actual driver's position in the cross section of the road. The eye level height depends on the vehicle in question. For calculation purposes, a height of 1.0 m has been used. Experiments (Durth, 1984 and Durth and Levin, 1991) proved that the average eye level is greater than 1.0 m for all types of cars. So there is extra safety potential in the calculation models. The specifications are different for the position and height of the target point.

Checks on sight distances during the design process are mainly used to guarantee safety levels. The stopping sight distance should be present along the whole road on all road types for safety reasons in order to allow a vehicle to stop in good time in front of an obstacle that suddenly appears.

The passing sight distance should be guaranteed for an adequate part of the road (more than 20%) on rural roads with oncoming traffic in order to allow drivers to pass slow moving vehicles and guarantee a high level of efficiency.

In order to make passing dependent on traffic volumes and make it safe, a passing lane is increasingly being used alternating between traffic flows in both directions (2 + 1 cross section), i.e. passing is only possible where there is a two-lane cross section in one direction without the need to use the oncoming traffic's lane.

Checks on existing sight distances can take place in simplified form and separately on the horizontal and vertical projections (preliminary assessment) or, what is better, by using a three-dimensional model using the visual ray procedure. Suitable CAD modules in road design programs are available for this purpose.

6.6 Questions

(1) Name and explain the essential model assumptions for calculating stopping sight distance.
(2) Does the stopping sight distance have to be provided on all rural roads?
(3) Which simplified assumption is made between cars 1, 2 and 3 to determine the necessary stopping sight distance?
(4) What length of a road is roughly required for passing sight distance?

Chapter 7

Alignment

7.1 Principles

The three-dimensional measurements of a roadway are calculated and displayed by superimposing the three design stages – the horizontal projection, the vertical projection, and the cross section. The horizontal projection represents the projection of the three-dimensional route in the form of a map (Figure 7.1). The developed view of the vertical section through the central axis of the route is shown on the vertical projection. The heights are generally shown enlarged 10-fold with respect to the horizontal scale, to allow a clearer representation of height differences. The third design level, the cross section, consists of the roadway configuration at right angles to the road axis.

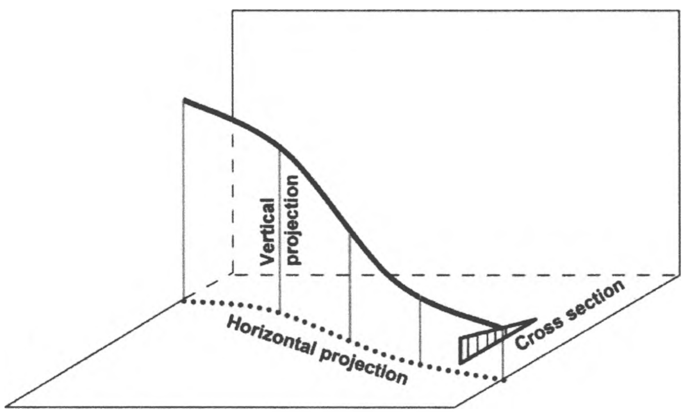
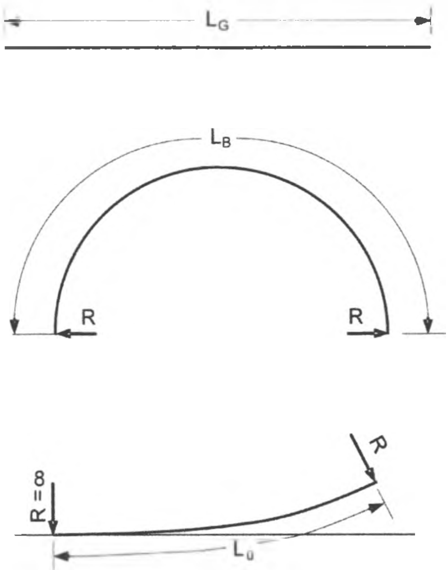


Figure 7.1: Illustration of the design levels

7.2 Design Elements on the Horizontal Projection

7.2.1 Review

Axis calculations are made on the horizontal projection using the following design elements: straight, circular arc, and transition curve (Figure 7.2).



Straight
L_G: Length of the straight

Circular arc
R: Radius of the circular arc
L_B: Length of the circular arc

Transition curve
$L_{\ddot{U}}$: Length of the transition curve
R: Radius at end of the transition curve

Figure 7.2: Design elements on the horizontal projection

All three design elements can also be defined using curvature K:

$$K = \frac{d\tau}{dL} = \frac{1}{R}, \quad \left[\frac{1}{\mathrm{m}}\right]. \tag{7.1}$$

where τ = angle of change of direction on the horizontal projection, [gon] and R = radius of curvature, [m].

In terms of the route, the curvature of the sequence of elements is depicted as a curvature graph. For this purpose K must be multiplied by the scale factor C:

$$K = C * \frac{1}{R}, \quad \left[\frac{1}{\mathrm{m}}\right]. \tag{7.2}$$

This means that there is a characteristic curvature graph for each design element.

7.2.2 Straights

The location of a straight is determined by two points or one point and a direction. The radius figure tends to move towards infinity and so the curvature is virtually zero.

The significance of the straight as an autonomous route element has fundamentally changed. In the early days of the road design, long straights with small transition radii at the intersection points were used (extreme limit road design), but the route for modern roads is chosen with a well-harmonized sequence of elements (road design with balanced radii).

Advantages of straights:

- Shortest connection between two points
- Traffic aspects (overtaking space, arrangement of intersections)
- Grouping of different forms of transport (road, rail)
- Landscape areas (flat valleys).

Disadvantages of a straight:

- Incorrect assessment of speed of oncoming and following traffic
- Risk of drivers being blinded at night by oncoming vehicles
- Fitting the straight into the landscape in hilly areas
- Risk of drivers becoming tired because of the monotonous driving style.

Designing routes with straights today is an exceptional practice because the conditions for using straights in road design are limited. According to the German design guidelines, RAS-L, the following limits must be noted for roads in category A:

The maximum length of straights with a constant gradient should not exceed 20 times the design speed v_e to prevent drivers from being blinded by lights.

$$\max L_G = 20 * v_e, \quad [\mathrm{m}]. \tag{7.3}$$

The minimum length of straights between circular arcs curving in the same direction should be six times the design speed v_e if they cannot be avoided.

$$\min L_G = 6 * v_e, \quad [\mathrm{m}]. \tag{7.4}$$

The maximum length of an intermediate straight between two clothoid branches of curved circular arcs going in the opposite direction (S-shaped clothoid) must not exceed the following length:

$$\max L_Z \leq 0.08 * (A_1 + A_2), \quad [\mathrm{m}]. \tag{7.5}$$

where L_Z, is the length of the intermediate straight [m] and A_1 and A_2, clothoid parameters with the dimension in meters [m].

7.2.3 Circular arcs principles

The circular arc is the most frequently used road design element in the modern road design. It has the same radius *R* at each of its points and therefore has the same curvature *K* (Figure 7.3).

The location of the circular arc may be set in different ways:

- Three points located on the periphery of the circle
- Two points located on the periphery of the circle and the radius when specifying the direction of curvature
- The centre point of the circle and radius

Taking into account the restrictions in the surrounding area, the radii selected should be as large as possible to guarantee adequate passing sight distances. On the other hand, radii should only have dimensions that ensure that their size and sequence fit into the structure of the surrounding area harmoniously.

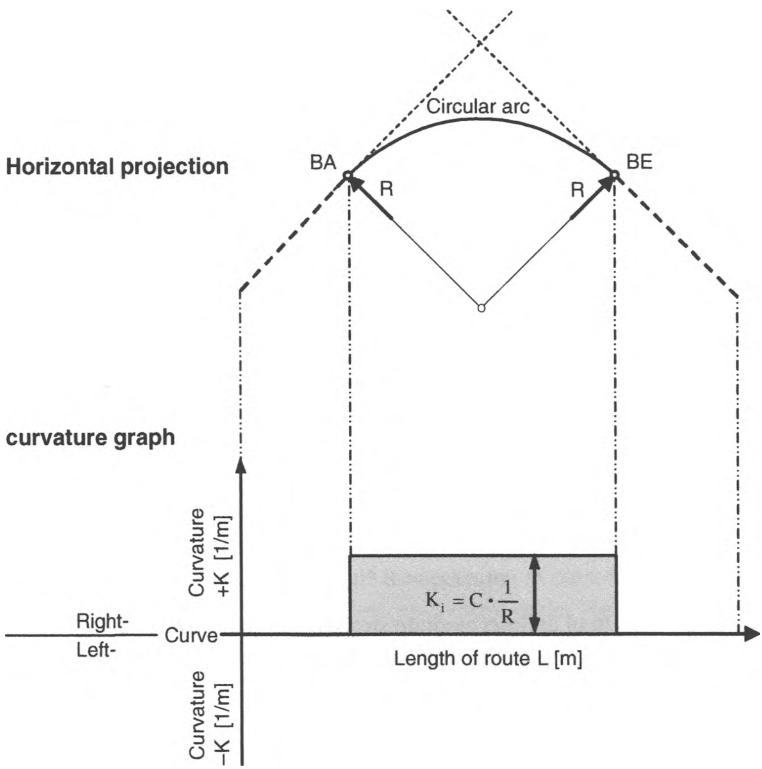


Figure 7.3: Horizontal projection and curvature of the circular arc

Design elements

The following correlations apply to the circular arc (Figure 7.4):

Tangent length: $$T_B = R * \tan \frac{\alpha\,[gon]}{2}, \quad [\mathrm{m}]. \tag{7.6}$$

Abscissas at the central point in the arc:

$$X = R * \sin \frac{\alpha\,[gon]}{2}, \quad [\mathrm{m}]. \tag{7.7}$$

Ordinate of the central point of the arc:

$$Y = R * \left(1 - \cos \frac{\alpha[gon]}{2}\right), \quad [\mathrm{m}]. \tag{7.8}$$

Apex distance: $$a_s = R * \sec \frac{\alpha[gon]}{200} - 1, \quad [\mathrm{m}]. \tag{7.9}$$

Length of the circular arc:

$$L_B = \frac{R * \pi * \alpha[\mathrm{gon}]}{200} = R * \alpha, \quad [\mathrm{rad}]. \tag{7.10}$$

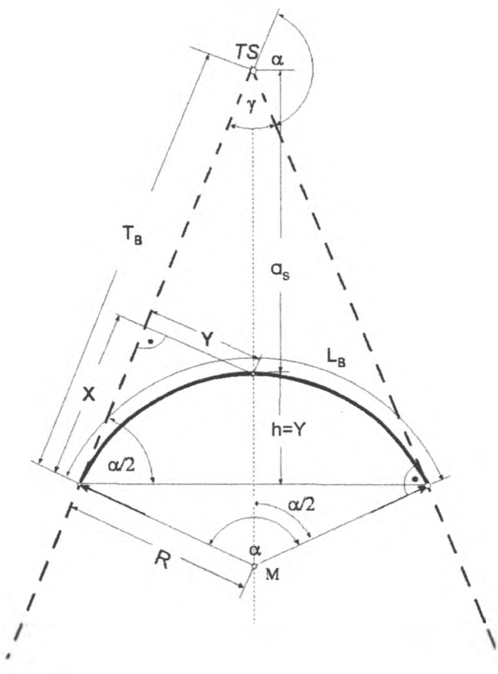


Figure 7.4: Design elements of a circular arc

Route limits

The minimum radii (min R) for a circular arc and the minimum lengths (min L_B) of a circular arc are produced according to RAS-L (1995) by using the following equations:

$$\min R = \frac{v_e^2}{3.6^2 * g * \left(\max f_R * n + q\right)} = \frac{v_e^2}{127 * \left(\max f_R * n + q\right)}, \quad [\mathrm{m}]. \tag{7.11}$$

$$\min L_B = \frac{2s * v_e}{3.6}, \quad [\mathrm{m}]. \tag{7.12}$$

g: acceleration of gravity, [m/s²],
max f_R: max. radial coefficient of adhesion, [–],
max $f_R = 0.925 * \max f_T$, [–],
max f_T: max. tangential coefficient of adhesion, [–],
max $f_T = 0.241 * (v/100)^2 - 0.721 * (v/100) + 0.708$[–],
n: exploitation of the max. radial coefficient of adhesion, [–],
q: camber, [%]
(a negative figure if directed towards the outside of the curve).

The minimum radii and the minimum length of the circular arcs depend on the design speed v_e, the exploitation n of the radial adhesion and the limits for the camber q (Table 7.1).

Table 7.1: Minimum radii for curves and minimum length of circular arcs for category group A and BI/II (RAS-L)

v_e [km/h]	min R [m]	min L_B [m]
50	80	30
60	120	35
70	180	40
80	250	45
90	340	50
100	450	55
120	720	65

For traffic safety reasons, the radii of consecutive curves must be in a balanced ratio to each other according to the principles of balanced road design. The correct size of the radii for adjacent circular arcs must be selected depending on which road category group is required. The relationships between consecutive circular arc radii can be seen in Figure 7.5.

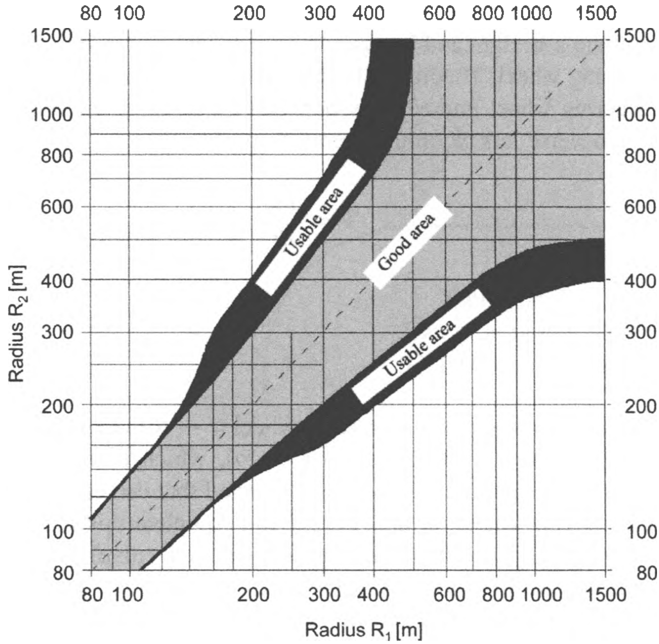


Figure 7.5: Coordinating sequence of radii for roads in category group A (Lippold, 1997)

In the case of a "straight–transition curve–circular arc" sequence of elements, the minimum radii for the circular arcs listed in Table 7.2 should be followed depending on the length of the straight L, if the selected design speed does not require any larger radii.

Table 7.2: Minimum curve radii in a sequence of elements: straight–transition curve–circular arc (RAS-L)

Straight length L	Minimum radius for circular arc
$L \geq 300\,\text{m}$	min $R > 400\,\text{m}$
$L < 300\,\text{m}$	min $R >$ L

7.2.4 Transition curve principles

The transition curve is the third design element on the horizontal projection in addition to the straight and circular arc. As part of modern road design, its layout is important for the following reasons:

(1) *Safety*

Driving along a straight and in a circular arc occurs with a smooth movement of the steering wheel. Theoretically the driver would have to change the turn on the steering wheel immediately between the straight and the circular arc and the transverse jerk Ψ would increase considerably:

$$\Psi = \frac{da_R}{dt} \tag{7.13}$$

Ψ: transverse jerk, [m/s^3]
a_R: radial acceleration, [m/s^2]
t: time for turning wheel, [s].

A sudden movement of the steering wheel ($t \longrightarrow \infty$) would produce a very high figure for the transverse jerk ($\Psi \rightarrow \infty$), which could endanger road safety. Practical experiments have demonstrated that drivers actually start to move the steering wheel on the straight and complete this maneuver in the circular arc – i.e. the actual driving line (steered curve) differs from the theoretical driving line (middle of the lane) (Figure 7.6). This would mean that drivers could use the opposite lane if the radii were smaller and the lanes narrow; this should be avoided at all costs for road safety reasons.

By inserting a transition curve between the straight and circular arc, the curvature of which changes constantly, the theoretical driving line can be adapted to the steered line. The gradual change in curvature in the transition produces an even change in radial acceleration and the lateral jerk is restricted to a value of $\Psi \leq 0.5\,\text{m/s}^3$.

(2) *Esthetics*

Uneven points, e.g. sharp bends in the alignment, are perceived by the human eye and are viewed as esthetic shortcomings. Transition curves counteract this because of the gradual change in curvature and contribute towards good alignment from an optical point of view. This is why transition curves should be used even with large radii of curvature and slight changes of direction.

(3) *Civil engineering issues*

Greater camber going towards the inner side of the bend is required in the circular arc to proportionally accommodate the centrifugal forces when going round a bend. The necessary distortion in the road should also follow the change in curvature in the transition curve for driving performance reasons.

Route planning limits

According to RAS-L, transition curves may be waived on two conditions. Firstly, care must be taken that the figures do not fall below the subsequent minimum values for circular arc radii:

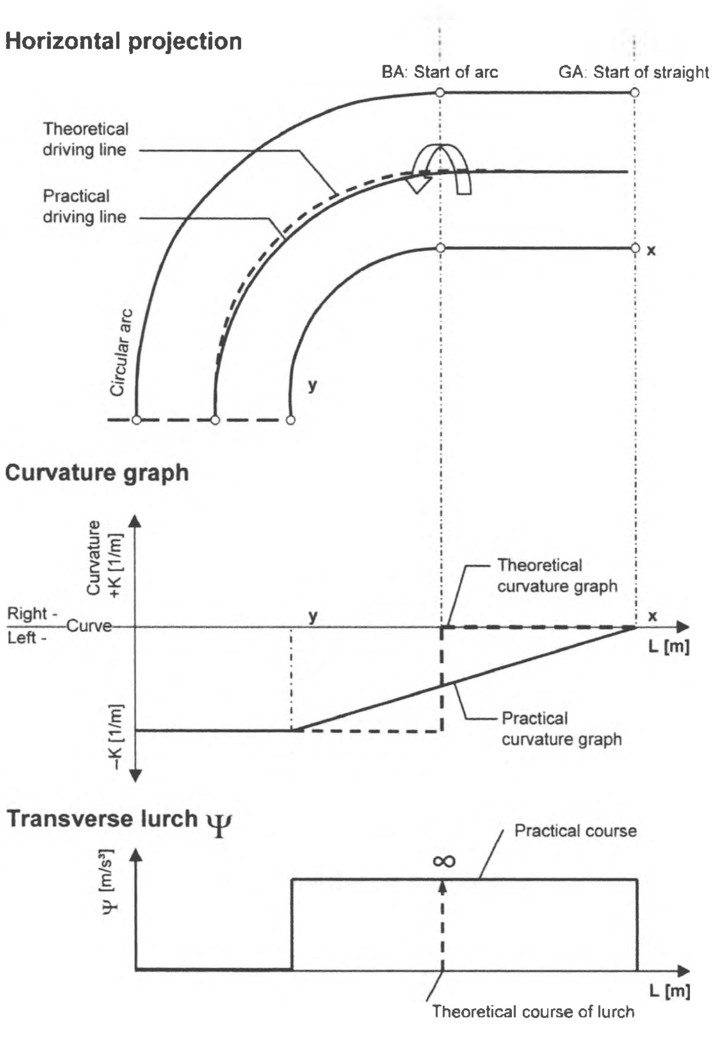


Figure 7.6: Relationship between driving line and curvature graph (Weise and Durth, 1997)

$$\begin{aligned} v_e &\leq 80 \text{ km/h} \\ \min R &= 1500 \text{ m} \end{aligned} \tag{7.14}$$

and

$$\begin{aligned} v_e &> 80 \text{ km/h} \\ \min R &= 3000 \text{ m} \end{aligned} \tag{7.15}$$

or that the change in angle in the curve is $\gamma < 10$ gon and the minimum arc length (min L_B) corresponds to twice the design speed v_e:

$$\gamma < 10, \quad [\text{gon}].$$
$$\min L_B = 2 * v_e, \quad [\text{m}]. \tag{7.16}$$

The following mathematical curves were tested in the past to see whether they were suitable for use as transition curves:

"Basket" curve: Two circular arcs going in the same direction with different radii, but a common tangent line at the meeting point form a "basket" curve.

Parabola: The parabola is a good transition curve for large radii and arc lengths. But smaller radii are used in road building, where a sharp bend occurs between a transition curve and a circular arc.

Sine line: The sine line has constant curvature and could be used in the range between 0 and 100 gon.

Steering wheel curve: The steering wheel curve represents the ideal curve. Its practical application failed, however, because a general mathematical formula could not be found for it.

Clothoid: In the end the clothoid gained the upper hand as the suitable mathematical function for theoretically describing the course of the steering wheel curve. It found its way into the standards of road design with the development of tables (Kasper, et al., 1964).

7.2.5 The clothoid as a standard transition curve

A clothoid has a constant and monotonically increasing curvature in the same direction from $K = 0$ as far as $K = \infty$, since the increase in curvature K is proportional to the length of the arc $L_Ü$ (Figure 7.7).

The product of any arc length $L_Ü$ and its associated radius R is constant.

$$L_Ü * R = \text{konst}, \quad [\text{m}]. \tag{7.17}$$

This constant can be replaced by the parameter A. Then the formation rule is:

$$A = \sqrt{L_Ü * R}, \quad [\text{m}]. \tag{7.18}$$

A: clothoid parameter, [m]
R: radius at the end of the clothoid section, [m]
$L_Ü$: length of the clothoid as far as the radius of the circle R, [m].

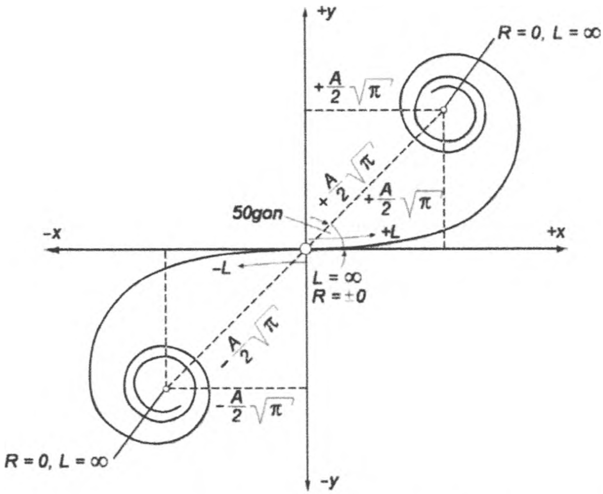


Figure 7.7: The graph of a clothoid (Weise and Durth, 1997)

The curvature alters in a linear fashion with the arc length – the driver can therefore turn his steering wheel with a constant angular velocity (Figure 7.8).

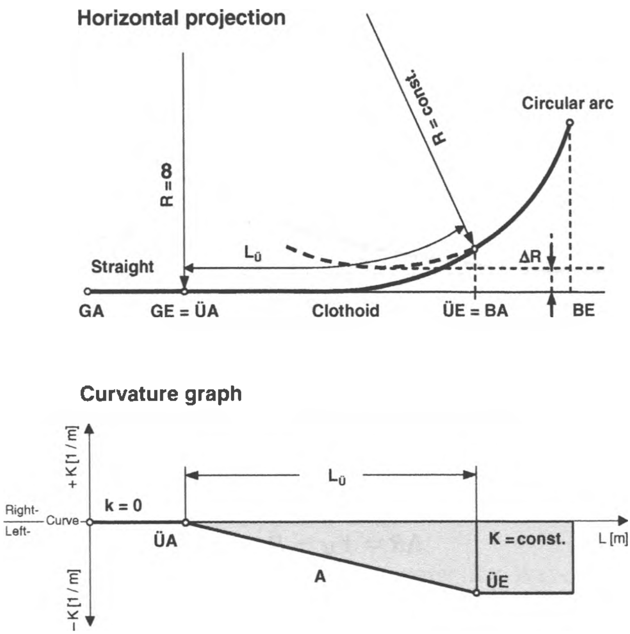


Figure 7.8: Horizontal projection and clothoid curvature graph

$$K = \frac{d\tau}{dL} = \frac{1}{R} = \frac{L_{\ddot{U}}}{A^2}, \quad \left[\frac{1}{\text{m}}\right]. \tag{7.19}$$

All clothoids are geometrically similar and can be reduced to a unit clothoid with the parameter $A = 1$.

As with the radius R, the parameter A serves as a modification factor (enlargement or reduction) so that an infinite number of clothoids can be created. A transition curve and straight or circular arc should form a common point, the same tangent direction, and the same curvature at the main points (transition points for design elements).

Design elements

For practical applications, a clothoid is viewed in the 1st quadrant of the coordinate system. The following correlations apply (Figure 7.9):

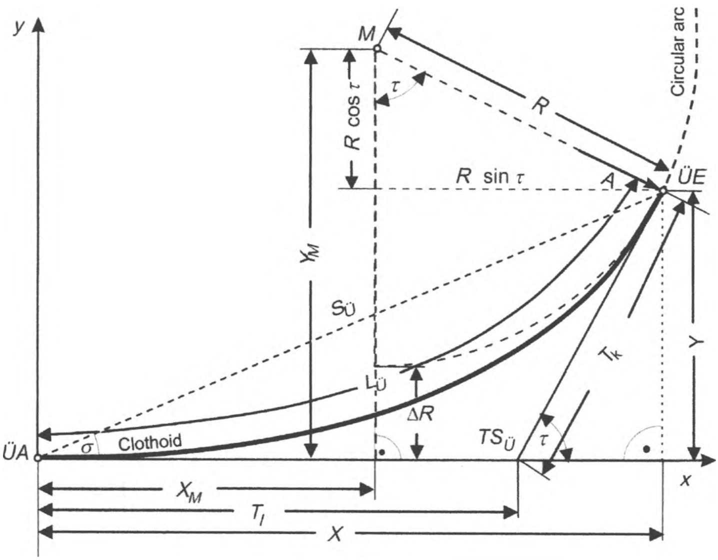


Figure 7.9: Design elements of a clothoid (Weise and Durth, 1997)

– Tangent retraction:

$$\Delta R = Y_M - R, \quad [\text{m}]. \tag{7.20}$$

– Coordinates of the middle point of the circle:

$$X_M = X - R \sin \tau, \quad [\text{m}]. \tag{7.21}$$

$$Y_M = Y + R\cos\tau = R + \Delta R, \quad [\text{m}]. \tag{7.22}$$

– Tangent length:

$$T_k = Y/\sin\tau, \quad [\text{m}]. \tag{7.23}$$

$$T_l = X - Yctg\tau, \quad [\text{m}]. \tag{7.24}$$

– Tangent angle:

$$\tau = L_{\ddot{U}}/2R = L_{\ddot{U}}^2/2A^2 = A^2/2R^2, \quad [\text{gon}] \tag{7.25}$$

ΔR represents the amount of retraction between the straight and circular arc where the clothoid is arranged.

Fresnel integrals can be used to calculate the clothoid:

$$X = \int_0^L \cos\frac{L^2}{2 * R^2}dL \tag{7.26}$$

$$Y = \int_0^L \sin\frac{L^2}{2 * R^2}dL \tag{7.27}$$

Tables have been drawn up for clothoids because of their significance in road design and their resulting widespread use. The unified clothoid where the parameter A = 1 served as the basic figure. An infinite number of clothoids can be derived from it by multiplying the length factors from the tables with the selected clothoid parameter A.

Clothoid rulers represent a simpler option. They have a convex and concave curve and already contain some inflection tangents and radii. Simple curves can be depicted very quickly if they are used.

Route planning limits

RAS-L names two crucial criteria that dictate the minimum parameters:

Scale factor: A change of direction of at least $\tau = 3.5$ gon must be used so that the clothoid appears as a curve. This means that A_{min} must have a value of *R*/3. An upper limit of $\tau = 31.8$ gon is recommended. This means that $A_{max} = R$ and therefore:

$$R/3 \leq A \leq R. \tag{7.28}$$

Raising and lowering road edges: The parameter for the clothoid must be large enough so that all the edges of the road can be lowered or raised within the clothoid without exceeding the permissible raised or lowered edge gradients. Evidence of this must be provided when setting the angle of torsion.

The following minimum value for the clothoid parameter emerges when these two criteria are taken into consideration:

$$\min A = \sqrt{R * \frac{(q_e - q_a)}{\max \Delta s} * a} \tag{7.29}$$

$\min A$:	minimum clothoid parameter, [m]
a:	distance at the edge of the road from the axis of rotation, [m]
q_e:	camber at the end of the clothoid, [%]
q_a:	camber at the start of the clothoid, [%]
R:	radius at the end of the clothoid, [m]
$\max \Delta s$:	max. raised or lowered edge gradient, [%].

7.2.6 Horizontal alignment curves

The linking of the design elements on the horizontal projection provides horizontal alignment curves for various applications in road design (Table 7.3).

Table 7.3: Various combinations for design elements on the horizontal projection (Weise and Durth, 1997)

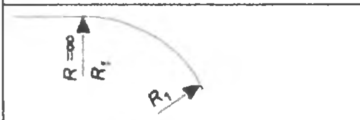
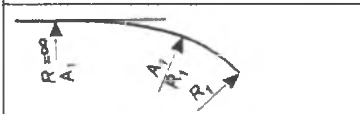
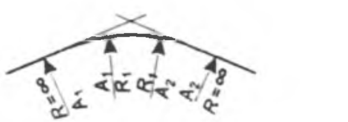
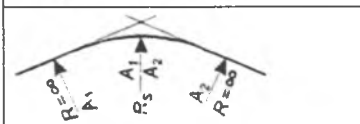
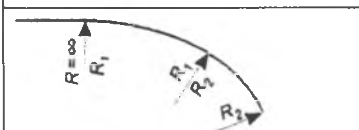
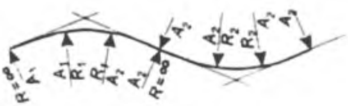
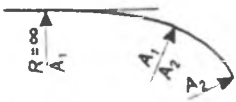
Image	Sequence of elements	Description	Use
	Straight-circular arc	Circular arc without transition curve	Limited use
	Straight-clothoid-circular arc	Simple clothoid	Useful
	Clothoid-circular arc-clothoid	Combined curve	Useful
	Clothoid-clothoid	Apex clothoid	Limited use
	Circular arc – circular arc (in same direction)	"Basket" curve	To be avoided

Table 7.3: (Continued)

Image	Sequence of elements	Description	Use
	Circular arc-clothoid-clothoid-circular arc (in different directions)	Inflection line	Useful
	Circular arc-clothoid section-circular arc (in same direction)	Egg-shaped line	Useful
	Clothoid-clothoid (in same direction)	"Basket" clothoid	To be avoided

7.2.7 Combined curves

Principles

The clothoid–circular arc–clothoid sequence of elements is known as a combined curve. A distinction is made between symmetric combined curves – both of the clothoid parameters are of the same size here – and asymmetric combined curves with different parameters. If the aim is to provide a gradual transition between the design elements, the clothoid parameter must be larger if the connecting curve radius is smaller – but the limits must still be followed.

But if drivers are to recognize the subsequent circular arc at an early stage, a small parameter must be selected. This makes it easier to recognize the transition (Figure 7.10).

Design elements

The principles from circular arc geometry can be used to calculate combined curves:

– Tangent length of the circular arc

$$T_B = R * \tan \frac{\alpha\,[\text{gon}]}{2}, \quad [\text{m}]. \tag{7.30}$$

– Tangent length of the substitute circle

$$T' = (\text{R} + \text{AR})^* \tan \frac{\beta}{2}, \quad [\text{m}]. \tag{7.31}$$

– Total tangent length on the combined curve

$$T_G = T' + X_M, \quad [\text{m}]. \tag{7.32}$$

– Centre point angle of the circular arc

$$\alpha = \beta - 2 * \tau, \quad [\text{gon}]. \tag{7.33}$$

– Length of the circular arc

$$L_B = \frac{R * \pi * \alpha[\text{gon}]}{200}, \quad [\text{m}].$$
$$L_B = R * \alpha[\text{rad}], \quad [\text{m}]. \tag{7.34}$$

– Total length of the combined curve

$$L_G = L_B + 2 * L_{\ddot{U}}, \quad [\text{m}]. \tag{7.35}$$

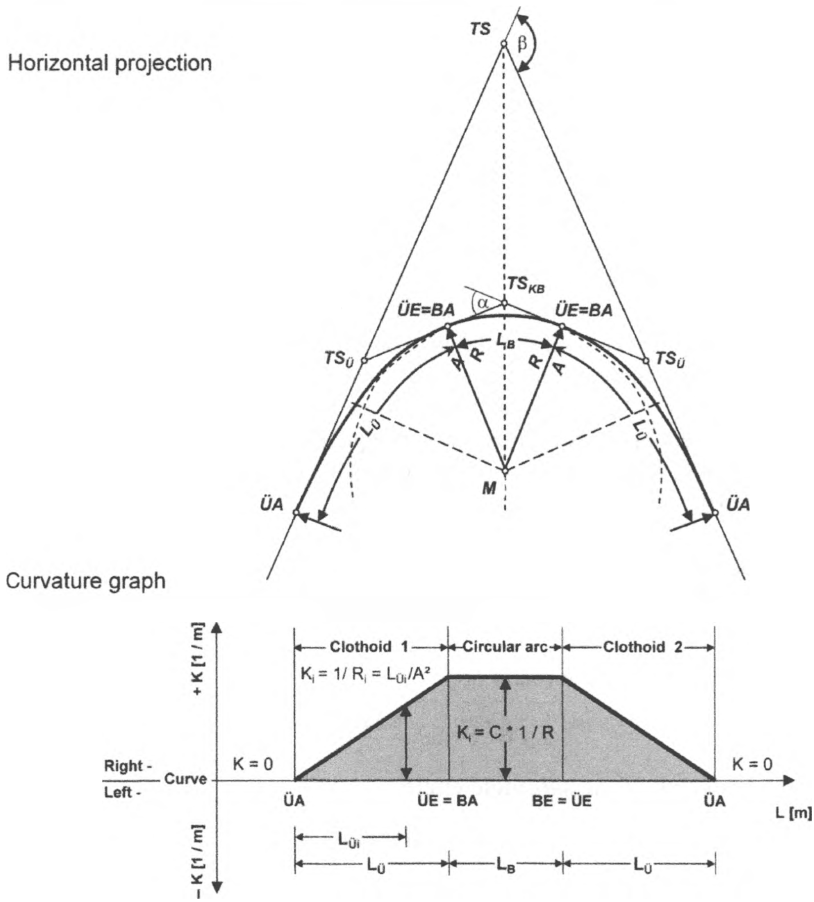


Figure 7.10: Horizontal projection and curvature graph for a symmetric combined curve, (Weise and Durth, 1997)

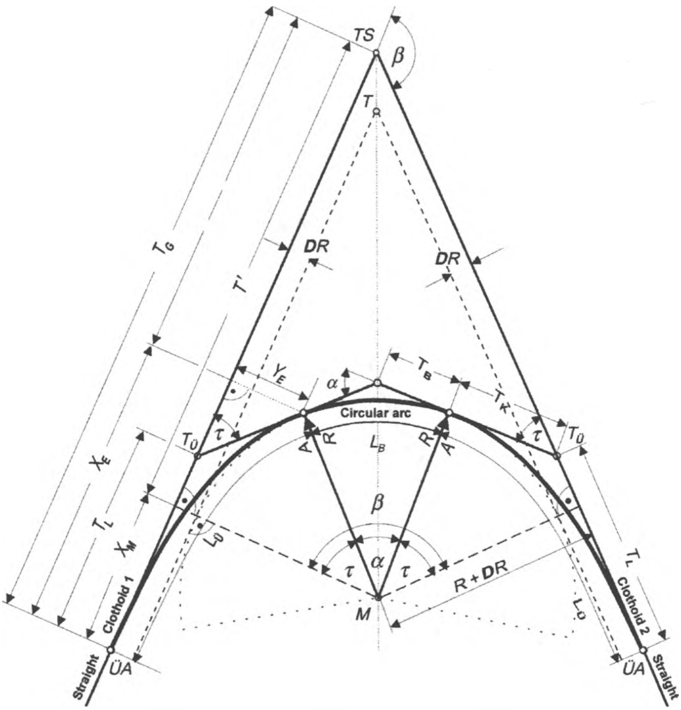


Figure 7.11: Symmetric combined curve (Weise and Durth, 1997)

There is a mathematical connection between symmetric and asymmetric combined curves using the following approaches (Figure 7.11):

$$\sin\beta = \frac{\Delta R_2 - \Delta R_1}{d}, \quad d = \frac{\Delta R_2 - \Delta R_1}{\sin\beta}. \tag{7.36}$$

The adjustment factor d now makes it possible to calculate all the dimensions in an asymmetric combined curve (Figure 7.12):

– Tangent length of the substitute circles

$$T'_1 = (R + \Delta R_1) * \tan\frac{\beta}{2}, \quad [\text{m}]. \tag{7.37}$$

$$T'_2 = (R + \Delta R_2) * \tan\frac{\beta}{2}, \quad [\text{m}]. \tag{7.38}$$

– Total tangent lengths

$$T_{G1} = X_{M1} + T'_1 + d, \quad [\text{m}]. \tag{7.39}$$

– Centre point angle of the circular arc

$$T_{G2} = X_{M2} + T'_2 - d\,, \quad [\text{m}]. \tag{7.40}$$

– Total length of the combined curve

$$L_G = L_B + L_{\ddot{U}1} + L_{\ddot{U}2}\,, \quad [\text{m}] \tag{7.41}$$

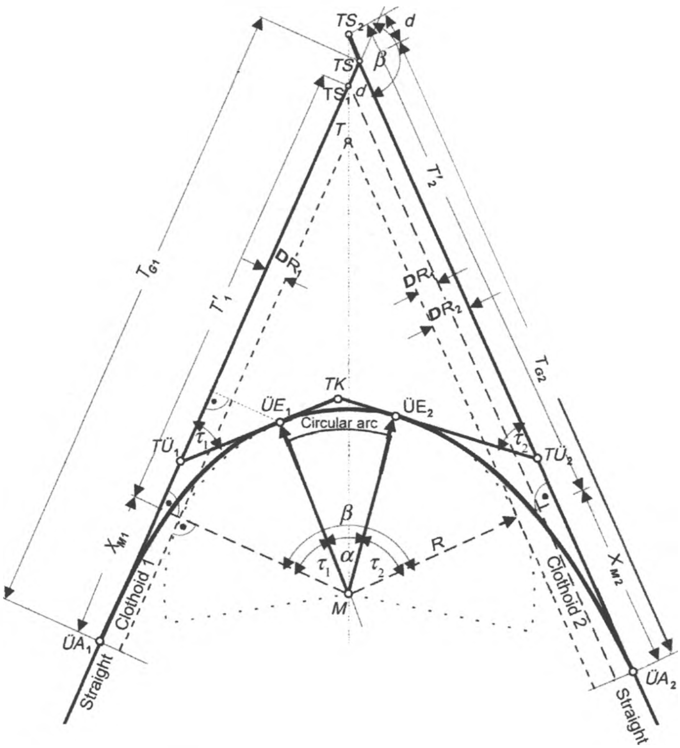


Figure 7.12: Asymmetric combined curve (Weise and Durth, 1997)

Route planning limits

The minimum curve radii can be taken from Section 7.2.3 (Table 7.1). The following is also valid:

– Length of the circular arc:

$$\min L_B \geq \frac{v_e * 2}{3.6}, \quad [\text{m}]. \tag{7.42}$$

– Angle of the circular arc:

$$\min \alpha \geq \frac{\min L_B * 200}{\pi * R}, \quad [\text{gon}]. \tag{7.43}$$

7.2.8 Apex clothoids

Principles
An apex clothoid (Figure 7.13) consists of two clothoid branches, which are connected to each other via a joint abutting radius. If there are two equal clothoid parameters, we have a symmetric apex clothoid – otherwise it is asymmetric. Apex clothoids can be used if the change in direction is slight (β < 5 gon) and the change in radius is large ($R > 600$ m).

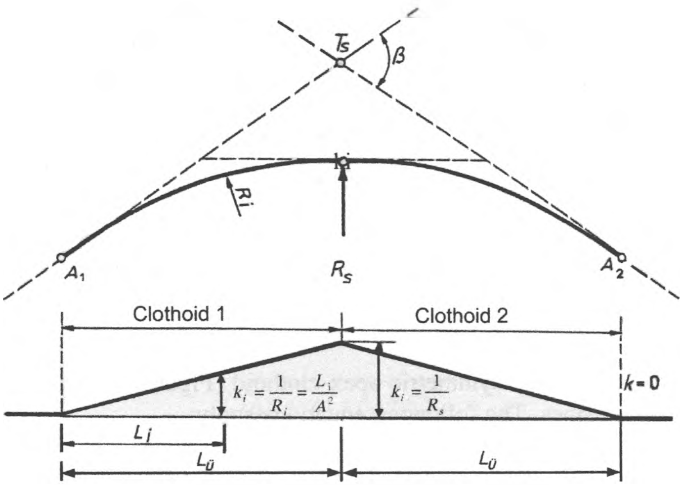


Figure 7.13: Horizontal projection and curvature graph of an apex clothoid

Design elements
The following correlations concerning a symmetric apex clothoid (Figure 7.14) are provided by the geometry of the design elements:

$$\frac{\tan\beta}{2} = \frac{s}{Y_E}, \quad [-] \tag{7.44}$$

$$s = Y_E * \tan\beta, \quad [\mathrm{m}] \tag{7.45}$$

$$T_G = X_E + s, \quad [\mathrm{m}] \tag{7.46}$$

$$L_G = 2 * L_{\ddot{U}}, \quad [\mathrm{m}] \tag{7.47}$$

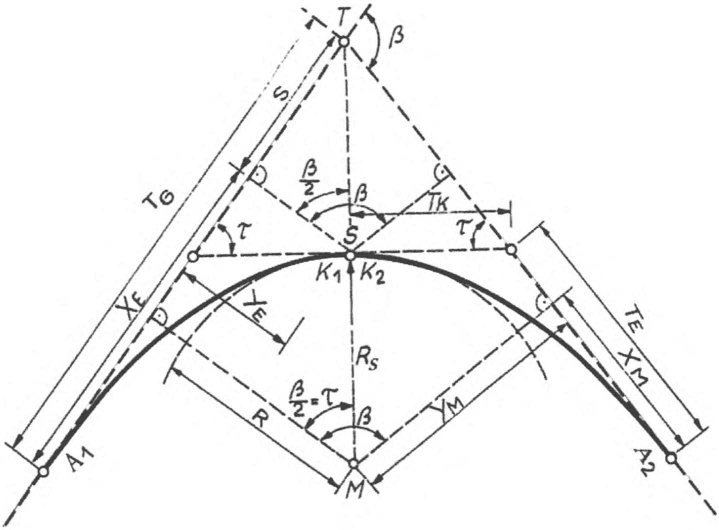


Figure 7.14: Symmetric apex clothoid (Weise, 1987)

The main points of the asymmetric apex clothoid (Figures 7.15) are calculated using the law of sines. The following equations apply:

$$\frac{C_1}{\sin \tau_2} = \frac{T_{K1} + T_{K2}}{\sin(200 - \beta)} = \frac{T_{K1} + T_{K2}}{\sin \beta}. \tag{7.48}$$

$$\frac{C_2}{\sin \tau_1} = \frac{T_{K1} + T_{K2}}{\sin(200 - \beta)} = \frac{T_{K1} + T_{K2}}{\sin \beta}. \tag{7.49}$$

$$C_1 = \frac{\sin \tau_2 (T_{K1} + T_{K2})}{\sin \beta}, \quad [\mathrm{m}]. \tag{7.50}$$

$$C_2 = \frac{\sin \tau_1 (T_{K1} + T_{K2})}{\sin \beta}, \quad [\mathrm{m}]. \tag{7.51}$$

$$T_{G1} = T_{L1} + C_1, \quad T_{G2} = T_{L2} + C_2, \quad [\mathrm{m}]. \tag{7.52}$$

Route planning limits

Round clothoid parameters are often used. The abutting radius is calculated as follows (Figure 7.15):

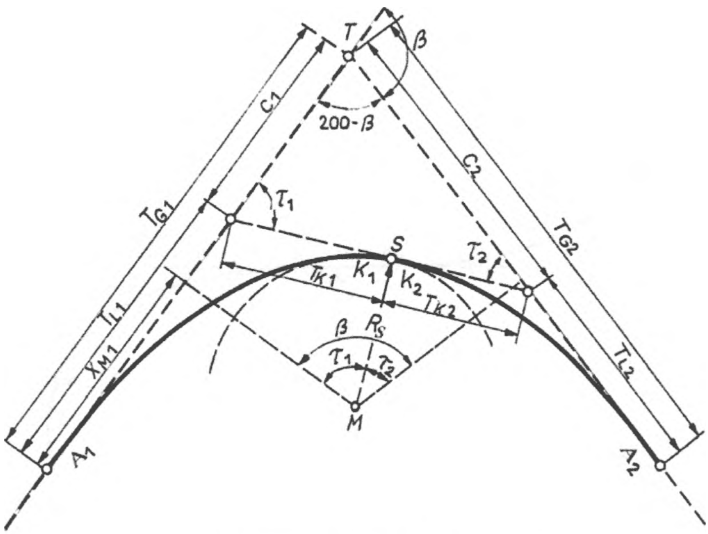


Figure 7.15: Asymmetric apex clothoid (Weise, 1987)

$$R_s = \sqrt{\frac{A_1^2 + A_2^2 * 63.662}{\beta[\text{gon}] * 2}}, \quad [\text{m}]. \tag{7.53}$$

In the apex of the curve area there is a section with a constant transverse gradient. Its length is calculated as follows:

$$L_{konstq} = 0.3 * v_e\,, \quad [\text{m}]. \tag{7.54}$$

7.2.9 Inflection lines principles

The S-shaped clothoid consists of two clothoid branches with curvature in the opposite direction and abutting at their zero point. If the connected line segments include the two adjacent circles, an inflection line (S-shaped) is created (Figure 7.16). This horizontal projection curve allows a constant torsion in the road from the camber in the first circle to the opposing camber in the second circle. The two circles should not be too far away from each other, not touch each other, or intersect. Both clothoid branches should have almost equal parameters in the interests of harmonious alignment and raising or lowering road edges. If the parameters are not equal, the following condition must be followed if A_2 is < 200 m.

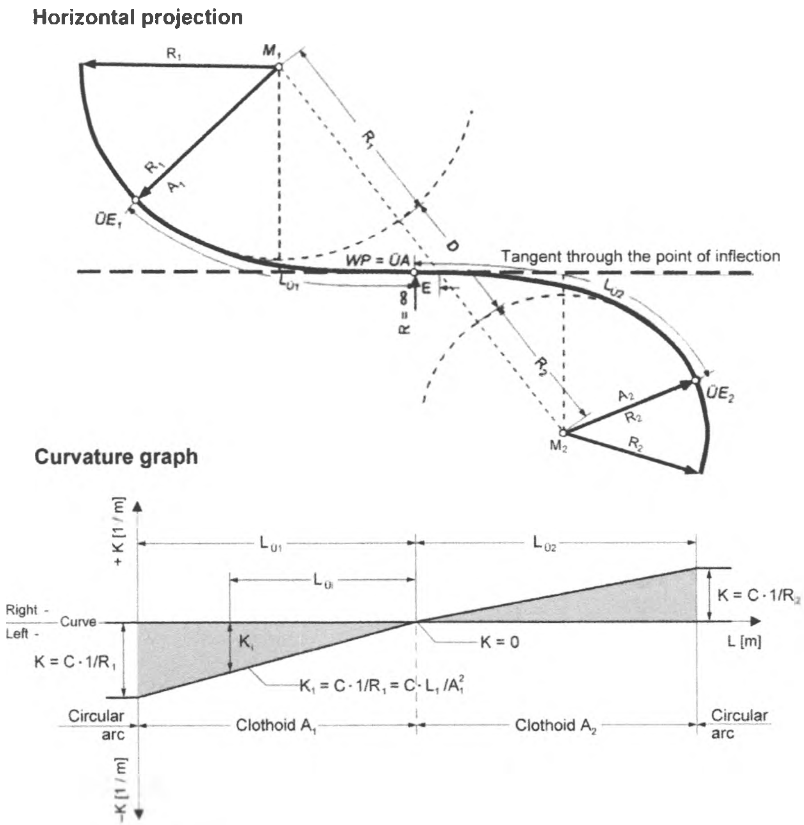


Figure 7.16: Horizontal projection and curvature graph of an inflection line (Weise and Durth, 1997)

$$\frac{A_1}{A_2} < 1.5 \tag{7.55}$$

A_1: larger parameter
A_2: smaller parameter.

Design elements

When designing an inflection line, the location of the two circular arcs in relation to each other is critical. Either the S-shaped clothoid is a given factor and the location of the circular arcs has to be found or the converse is true.

The distance between the two circles from each other can be determined using the distance between the central points $\overline{M_1M_2}$ in the circles:

$$D = \overline{M_1M_2} - (R_1 + R_2) \tag{7.56}$$

Symmetric inflection lines ($A_1 = A_2$) can be calculated using the following procedure. If the clothoid parameter has been set, the design elements can be determined using tables.

Various procedures are available to manually calculate the design elements of an inflection line, depending on the initial conditions:

(1) Approximate calculations using the "substitute radius" procedure: Substitute radius of the inflection line:

$$R' = \frac{R_1 * R_2}{R_1 + R_2} \tag{7.57}$$

(R' is an auxiliary variable, which does not appear in the design).

A graphical solution using the nomograms for symmetric and asymmetric S-shaped clothoids according to Osterloch: After calculating the parameters, the clothoid elements are calculated using the approaches for a combined curve or inflection line.

(2) Calculations using the S-tables in line with Kasper, et al., (1964) (Figure 7.17).

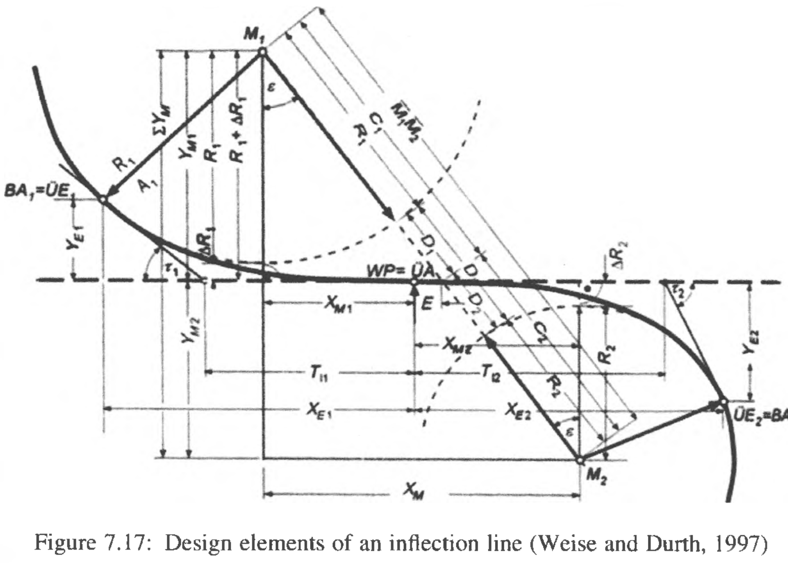


Figure 7.17: Design elements of an inflection line (Weise and Durth, 1997)

(3) Exact calculations for special cases with substitution tables (Kasper, et al., 1964)

$$A = \sqrt{R' * L} \quad A = \sqrt[4]{24 * D * R'} \quad A = \sqrt{\frac{L^3}{24 * D}} \tag{7.58}$$

$$L = \frac{A^2}{R'} \quad L = \sqrt[3]{24 * A^2 * D} \quad L = \sqrt{24 * D * R'} \tag{7.59}$$

$$R' = \frac{A^2}{L} \quad R' = \sqrt[3]{\frac{A^4}{24 * D}} \quad R' = \frac{L^2}{24 * D} \tag{7.60}$$

$$D = \frac{A^4}{24 * R'3} \quad D = \frac{L^3}{24 * A^2} \quad D = \frac{L^2}{24 * R'} \tag{7.61}$$

The following equations are used to calculate the individual parameters:

$$\sum Y_M = Y_{M1} + Y_{M2} = (R_1 + \Delta R_1) + R_2 + \Delta R_2 \tag{7.62}$$

$$\sum X_M = X_{M1} + X_{M2} \tag{7.63}$$

$$\tan \varepsilon [gon] = \frac{\sum X_M}{\sum Y_M}, \quad [-] \tag{7.64}$$

$$E = (R_1 + \Delta R_1) * \tan \varepsilon - X_{M1}, \quad [m] \tag{7.65}$$

$$E = X_{M2} - (R_2 + \Delta R_2) * \tan \varepsilon, \quad [m] \tag{7.66}$$

$$\overline{M_1 M_2} = \frac{\sum X_M}{\sin \varepsilon}, \quad [m] \tag{7.67}$$

$$D = \overline{M_1 M_2} - (R_1 + R_2), \quad [m] \tag{7.68}$$

$$D = D_1 + D_2, \quad [m] \tag{7.69}$$

$$D_1 = \frac{X_{M1} + E}{\sin \varepsilon} - R_1, \quad [m] \tag{7.70}$$

$$D_2 = \frac{X_{M2} + E}{\sin \varepsilon} - R_2, \quad [m]. \tag{7.71}$$

Route planning limits

Attempts should be made to achieve equally large final radii when designing an inflection line. If this is impossible, the following limits must be followed:

$$A_1 = A_2 \rightarrow R_1 \leq 2R_2 \tag{7.72}$$

$$A_1 \neq A_2 \rightarrow R_1 \leq 3R_2 \tag{7.73}$$

$$A_1 = A_2 \rightarrow R_1 = R_2. \tag{7.74}$$

Overlapping is only possible if the clothoids do not collide at their original points.

$$l_s \leq \frac{A_1 + A_2}{40}, \quad [\mathrm{m}]. \tag{7.75}$$

If it is necessary to insert a straight in the point of inflection to move apart the clothoid origins for civil engineering reasons, an inflection line can be created if the following relations concerning maximum length of the straight applies:

$$l_z \leq 3\, l_s \tag{7.76}$$

or

$$l_z \leq 0.08(A_1 + A_2), \quad [m]. \tag{7.77}$$

7.2.10 Egg-shaped line principles

An egg-shaped line is used as a transition curve between two circles going in the same direction (Figure 7.18).

But the circles must satisfy three conditions:

(1) They must lie within each other
(2) They may not intersect
(3) They may not have a joint centre point.

If the connected line segments enclose the two adjacent circles, the egg-shaped line is produced as a horizontal projection curve. The radius of the larger circle is marked by R_1 and the smaller circle by R_2. The clothoid part of the egg-shaped clothoid does not begin at the clothoid origin – as is the case with most clothoid applications – but at the point with radius R_1.

Design elements

As is the case with an inflection line, there are also four procedures for manual calculations:

(1) Approximate calculations using the "substitute radius" procedure

$$R = \frac{R_1 * R_2}{R_1 - R_2}, \quad [\mathrm{m}]. \tag{7.78}$$

(2) Calculations using the egg clothoid tables
(3) Graphical procedure using a nomogram according to Osterloch
(4) Exact calculations using substitute and unit tables.

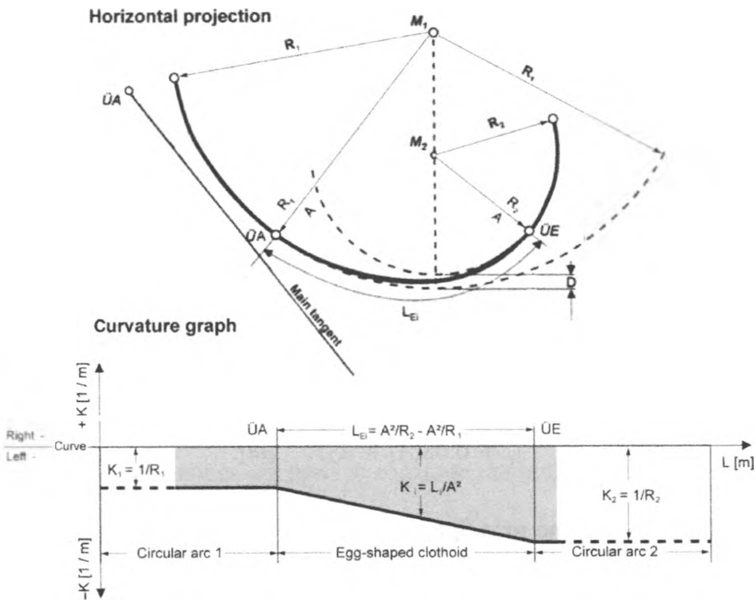


Figure 7.18: Horizontal projection and curvature graph of an egg-shaped line (Weise and Durth, 1997)

The length of the egg-shaped clothoid L_{Ei} can be calculated using the partial clothoids with the same parameter:

$$L_{Ei} = L_{K1(R1)} - L_{K1(R2)}, \quad [\text{m}]. \tag{7.79}$$

$$L_{Ei} = \frac{A^2}{R_1} - \frac{A^2}{R_2} = A^2 * \left(\frac{1}{R_1} - \frac{1}{R_2}\right), \quad (R_1 < R_2), \quad [\text{m}]. \tag{7.80}$$

Route planning limits

The parameter for the egg-shaped clothoid should lie within the following limits (Figure 7.19):

$$\frac{R_2}{2} \leq A \leq R_2, \quad R_1 < R_2, \quad [\text{m}] \tag{7.81}$$

with an angle of: $\tau \geq 3.5$ gon.

The following approach should be used to calculate the remaining angle:

$$\tau_{Ei} = \tau_{K1(R1)} - \tau_{K1(R2)}. \tag{7.82}$$

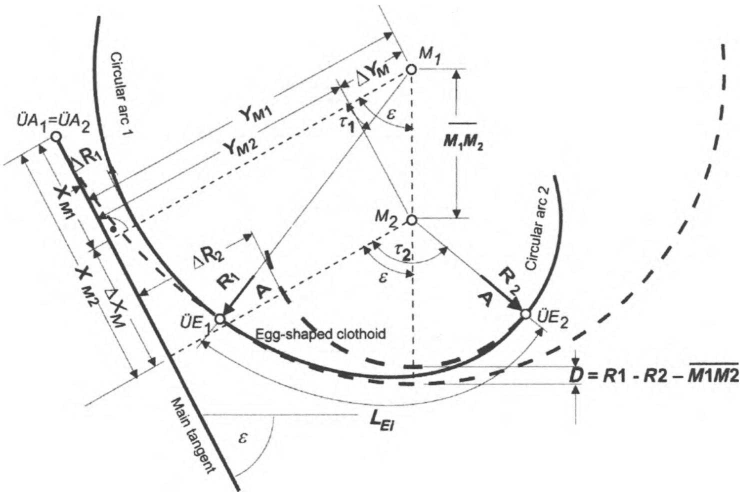


Figure 7.19: Design elements of an egg-shaped line (Weise and Durth, 1997)

7.2.11 Calculation examples (Weise, Kühn, et al., 1980)

1st Example:
Calculate the clothoid parameter and the appropriate determining factors from the graphical values of ΔR and R

Given that: ΔR: 0.60 m
R: 250.00 m
Find: A and appropriate determining factor
Solution:
Table entry: $\frac{\Delta R}{R} = \frac{\Delta r}{r} = \frac{0.60}{250.00} = 0.0024.$
Found 0.002401 (exact enough without interpolating)

$$\frac{l}{r} = 0.240100$$

$$L = 0.240100 * 250 = 60.025 \text{ m}$$

$$A = \sqrt{R * L} = \sqrt{250 * 60.025} = 122.50 \text{ m}.$$

2nd Example:
Calculate a sequence from a symmetric and asymmetric combined curve (Table 7.4).

Table 7.4: Determining factors for a clothoid

	Unit	Description	Value 1 (unit clothoid)	Value 2 (clothoid considered)
1	m	A		122.5
2	m	R	250	250
3	m	L		60.03
4	m	ΔR	0.6	0.6
5	m	$Y_M = R + \Delta R$		250.6
6	m	X_M	0.244882	30
7	gon	τ	7.64262	7.643
8	gon	σ	2.5472	2.547
9	m	X	0.4892943	59.94
10	m	Y	0.019588	2.4
11	m	T_K	0.163558	20.04
12	m	T_L	0.326914	40.05

Input
A road consisting of two combined curves has to be calculated. Curve 1 (at tangent intersection point T_2) is to be designed as a symmetric combined curve. Three straight sections (L_{G1}, L_{G2}, and L_{G3}) with straight lengths are also specified (Figure 7.20).

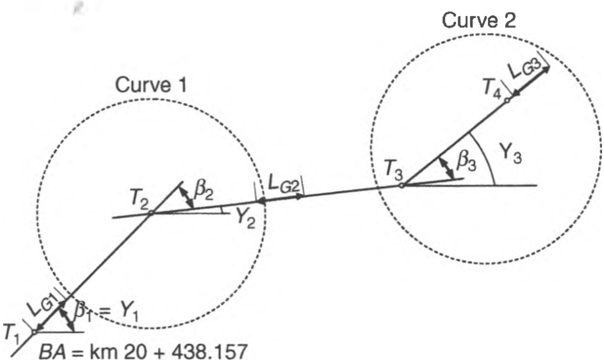


Figure 7.20: Geometrical exercise

Given: coordinates of the tangent intersection points

T_i	T_1	T_2	T_3	T_4
X_i	125.145	337.547	784.829	956.523
Y_i	47.307	259.439	308.819	956.523

- Road category: A II $\Rightarrow$: $v_e = 80$ km/h
- Straight lengths L_{G1}: approx. 80 m, L_{G2}: approx. 100 m, L_{G3}: approx. 70 m.

Find:

(1) Limits for $R_{\min}$, $A_{\min}$, $L_{G\min}$, $L_{G\max}$
(2) Determine tangent lengths and tangent angle
(3) Select circular arc radius R and clothoid parameters A ($A_1 = A_2$)
(4) Determine design elements for the clothoid
(5) Calculate the main points of the combined curves

Solution for curve 1

Circular arc with transition curve (Figure 7.21, symmetric combined curve $A_1 = A_2 = A$)

(a) Selected: R and A (or τ or ΔR)

$$L_{\ddot{U}} = \frac{A^2}{R}.$$

(b) Selected: R and $L_{\ddot{U}}$,

$$T_B = R^* \tan\frac{\beta}{2} \quad \text{and} \quad \frac{L_{\ddot{U}}}{2} \approx T_1T_2 - T_B \quad \text{and} \quad A = \sqrt{R * L_{\ddot{U}}}.$$

Approach for curve 1

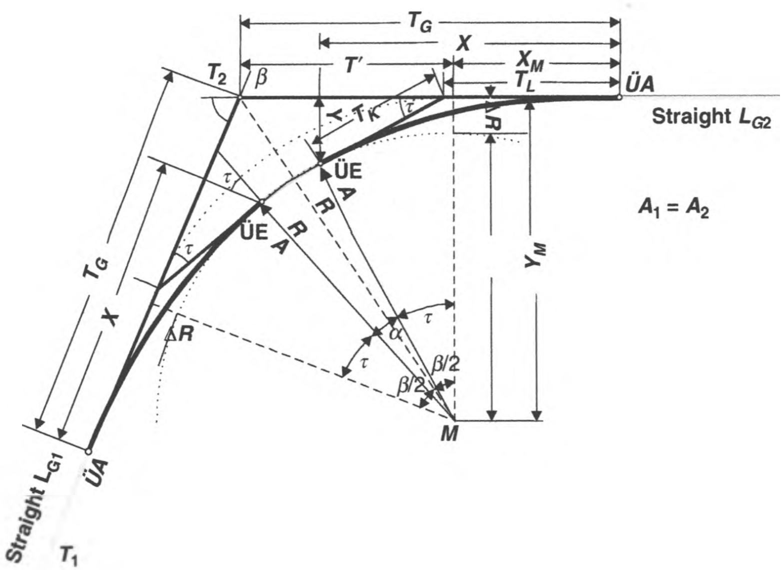


Figure 7.21: Symmetric combined curve

Tangent lengths and angle (Figure 7.22)

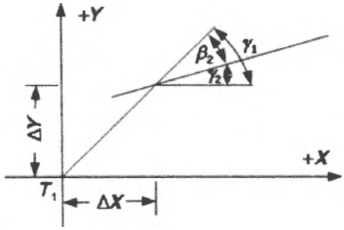


Figure 7.22: Tangent lengths and angle

$$\overline{T_1T_2} = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$
$$= \sqrt{(337.547 - 125.415)^2 + (259.439 - 47.307)^2} = 300.00m$$
$$\overline{T_2T_3} = \sqrt{(X_3 - X_2)^2 + (Y_3 - Y_2)^2}$$
$$= \sqrt{(784.829 - 337.547)^2 + (308.819 - 259.439)^2} = 450.00m$$
$$\overline{T_3T_4} = \sqrt{(X_4 - X_3)^2 + (Y_4 - Y_3)^2}$$
$$= \sqrt{(956.523 - 784.829)^2 + (446.373 - 308.819)^2} = 220.00m$$
$$\tan\gamma_1 = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{259.439 - 47.307}{337.547 - 125.415} = 1.00000 \quad \Rightarrow \quad \gamma_1 = 50.00\text{gon}$$
$$\tan\gamma_2 = \frac{Y_3 - Y_2}{X_3 - X_2} = \frac{308.819 - 259.439}{784.829 - 337.547} = 0.11040 \quad \Rightarrow \quad \gamma_2 = 7.000\text{gon}$$
$$\tan\gamma_3 = \frac{Y_4 - Y_3}{X_4 - X_3} = \frac{446.373 - 308.819}{956.523 - 784.829} = 0.80116 \quad \Rightarrow \quad \gamma_3 = 43.00\text{gon}$$
$$\beta_2 = \gamma_3 - \gamma_2 = 50.000 - 7.000 = 43.000\text{gon}$$
$$\beta_3 = \gamma_3 - \gamma_2 = 43.000 - 7.000 = 36.000\text{gon}.$$

Limits:

RAS-L stipulates for $v_e = 80\,\text{km/h}$

$R_{min} = 250\,\text{m}$
$R_{min} = 375\,\text{m}$
($q_{max} = 8\%$ and $v_{85} = 100\,\text{km/h}$)
$A_{min} = 80\,\text{m}$
$L_{Gmax} = 1600\,\text{m}$

Input parameters:

Circular arc radius R_2 selected: $R_2 = 400\,\text{m}$

Conditions: $R_{min} \leq R_2$ and $A_{min} \leq A_{1,2}$ or $R_2/3 \leq A_{1,2} \leq R_2$

Clothoid parameter A_1 is determined by the length of tangent T_1T_2, which sets the possible length of the transition curve:

Note: If a circular arc with a radius $R_2 = 400\,\text{m}$ is set for the tangents that intersect, a tangent intersection, which can be assumed to be approximately $\frac{1}{2}\,L_{\ddot{u}}$ of the incoming clothoid, is formed from the point of intersection as far as the previous possible start of the combined curve. So it is possible to select a transition curve, the length of which is no more than double the size of part of the tangent in order to adhere to the set restrictions on length. Select the same clothoid parameter for the outgoing clothoid, as the ensuing straight L_{G2} is not exactly set as regards its location. It is also important to note that the length of the remaining circular arc may not fall below a minimum value (2 s at v_e). If this condition is not met, it helps to select a shorter clothoid length or rather choose a larger radius for the circular arc.

(Generally attempts should be made to avoid this fixed method so as to be more flexible in the alignment, i.e. solution a).

The conditions $R_{min} \leq R_2$ and $A_{min} \leq A_{1,2}$ or $R_2/3 \leq A_{1,2} \leq R_2$ must be followed.

$$\overline{T_1T_2} = \frac{L_{\ddot{U}1}}{2} + L_{G1} + T_B^*$$

$$T_B^* = R_2 \cdot \tan\frac{\beta_2}{2} = 400 \cdot \tan\frac{43}{2} = 140.47 \text{ m}$$

$$\frac{L_{\ddot{U}1}}{2} = \overline{T_1T_2} - L_{G1} - T_B^* = 300 - 80 - 140.47 = 79.53 \text{ m}$$

$$L_{\ddot{U}1} = 159.06 \text{ m} \approx 160 \text{ m}$$

$$A = \sqrt{R \cdot L_{\ddot{U}}}$$

$$A_1 = \sqrt{R_2 \cdot L_{\ddot{U}1}} = \sqrt{400 \cdot 160} = 252.98 \approx 250 \text{ m.}$$

In the example it is advantageous to work with round circular arc radii and round clothoid parameters as the starting points for the calculations because the peripheral conditions L_{G1}, L_{G2}, and L_{G3} only have to be met approximately.

Design elements for the clothoid:

- Selected $A_1 = A_2 = 250 \text{ m} > A_{min} = 80 \text{ m}$

- Input factors:
  R_2 (selected) 400.00 m
  $A_1 = A_2$ (selected) 250.00 m.
- Table values or calculated figures (from clothoid tables; see Figure 7.23, or calculations)

$\Delta R = \Delta R_1 = \Delta R_2$	2.540 m
$L_{\ddot{U}} = L_{\ddot{U}1} = L_{\ddot{U}2}$	156.250 m
$\tau = \tau_1 = \tau_2$	12.434 gon
$X_M = X_{M1} = X_{M2}$	78.026 m
$X = X_1 = X_2$	155.655 m
$Y = Y_1 = Y_2$	10.145 m
$T_K = T_{K1} = T_{K2}$	52.273 m
$T_L = T_{L1} = T_{L2}$	104.376 m
$Y_M = R + \Delta R = Y_{M1} = Y_{M2}$	402.54 m.

Design elements of a combined curve:

- $\beta_2 = 43.00$ gon (from the preliminary graphical draft)

$$T' = T'_1 = T'_2 = (R + \Delta R) * \tan\frac{\beta}{2} = (R_2 + \Delta R_1) + \tan\frac{\beta_2}{2}$$

$$= (400.00 + 2.540) * \tan\frac{43,00}{2} = 141.362\text{m}$$

$$T_G = T_{G1} = T_{G2} = T' + X_M = T'_1 + Y_{M1} = 141.362 + 78.062 = 219.388\text{m}$$

$$\alpha_2 = \beta_2 - (\tau_1 + \tau_2) = 43.00 - (12.434 + 12.434) = 18.132\text{gon}$$

$$L_B = R * \alpha * \frac{\pi}{200} = R_2 * \alpha_2 * \frac{\pi}{200} = 400.00 * 18.132 * \frac{\pi}{200} = 113.927\text{m}$$

$$L_\Sigma = L_{\ddot{U}1} + L_B + L_{\ddot{U}2} = 156.25 + 113.927 + 156.25 = 426.427\text{m}.$$

- Check:

$$\overline{T_1T_2} - T_{G1} = 300.00 - 219.388 = 80.612 \text{ m} \approx 80.00 \text{ m}.$$

- Minimum length of circular arc (2 s at v_e):

$$L_{B\,\min} = \frac{v_e}{3.6} * 2 = \frac{80.00}{3.6} * 2 = 44.444 \approx 50.00\text{m} < L_B.$$

Continuation $A = 240$

L	τ^g	τ°	R	ΔR	X_M	X	Y	T_K	T_L	L
338,824	63,4417	57 05 51	170	27,160	163,956	306,687	104,815	124,830	238,873	338,824
360,000	71,6197	64 27 28	160	32,265	172,668	317,030	123,276	136,630	258,119	360,000
384,000	81,4873	73 20 19	150	38,645	181,979	325,681	145,638	152,020	282,095	384,000
411,429	93,5442	84 11 23	140	46,667	191,766	331,047	172,494	173,385	313,494	411,429
443,077	108,4891	97 38 25	130	56,783	201,742	330,588	204,067	205,894	357,962	443,077

$$K = \frac{\rho^{cc}}{6A^3} = 1{,}842\,072$$

$A^2 = 62\,500$ $\frac{1}{A} = 0{,}004\,000\,000$ **$A = 250$**

L	τ^g	τ°	R	ΔR	X_M	X	Y	T_K	T_L	L
39,063	0,7771	0 41 58	1600	0,040	19,531	39,062	0,159	13,021	26,042	39,063
44,643	1,0150	0 54 49	1400	0,059	22,321	44,642	0,237	14,882	29,762	44,643
52,083	1,3816	1 14 36	1200	0,094	26,041	52,081	0,377	17,362	34,725	52,083
62,500	1,9894	1 47 26	1000	0,163	31,249	62,494	0,651	20,835	41,669	62,500
69,444	2,4561	2 12 38	900	0,223	34,721	69,434	0,893	23,152	46,300	69,444
78,125	3,1083	2 47 52	800	0,318	39,059	78,106	1,271	26,048	52,090	78,125
83,333	3,5368	3 10 59	750	0,386	41,662	83,308	1,543	27,786	55,565	83,333
89,286	4,0601	3 39 15	700	0,475	44,637	89,249	1,898	29,773	59,537	89,286
96,154	4,7087	4 14 16	650	0,593	48,068	96,101	2,370	32,068	64,121	96,154
104,167	5,5262	4 58 25	600	0,753	52,070	104,088	3,012	34,747	69,472	104,167
113,636	6,5767	5 55 09	550	0,978	56,798	113,515	3,910	37,917	75,800	113,636
125,000	7,9578	7 09 43	500	1,302	62,468	124,805	5,203	41,729	83,402	125,000
131,579	8,8175	7 56 09	475	1,518	65,747	131,327	6,066	43,940	87,808	131,579
138,889	9,8244	8 50 31	450	1,785	69,389	138,559	7,132	46,401	92,709	138,889
147,059	11,0142	9 54 46	425	2,118	73,456	146,619	8,463	49,160	98,193	147,059
156,250	12,4340	11 11 26	400	2,540	78,026	155,655	10,145	52,273	104,376	156,250
[illegible]	[illegible]	[illegible]	375	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
178,572	16,2403	14 36 59	350	3,787	89,092	177,413	15,114	59,895	119,456	178,572
192,308	18,8349	16 57 05	325	4,727	95,874	190,631	18,847	64,642	128,798	192,308
208,333	22,1048	19 53 40	300	6,002	103,750	205,835	23,906	70,252	139,776	208,333
227,273	26,3066	23 40 33	275	7,779	112,993	223,423	30,925	77,012	152,893	227,273
250,000	31,8310	28 38 52	250	10,324	123,966	243,822	40,929	85,370	168,903	250,000
277,778	39,2975	35 22 04	225	14,096	137,143	267,378	55,619	96,090	189,021	277,778
312,500	49,7359	44 45 44	200	19,907	153,125	293,958	77,000	110,628	215,409	312,500
328,947	55,1090	49 35 53	190	23,104	160,450	305,138	89,957	118,128	228,574	328,947
347,222	61,4024	55 15 44	180	26,999	168,365	316,283	104,431	127,081	243,869	347,222
357,143	64,9612	58 27 54	175	29,265	172,550	321,706	112,736	132,270	252,526	357,143
367,647	68,8386	61 57 17	170	31,779	176,887	326,925	121,850	138,062	262,013	367,647
390,625	77,7124	69 56 28	160	37,687	186,003	336,298	142,810	152,032	284,153	390,625
416,667	88,4195	79 34 39	150	45,035	195,633	343,158	167,900	170,716	312,274	416,667
446,429	101,5019	91 21 06	140	54,211	205,581	345,542	197,515	197,568	350,203	446,429

$$K = \frac{\rho^{cc}}{6A^3} = 1{,}697\,65$$

Figure 7.23: Extract from clothoid table (Kasper, et al., 1964)

Solution for curve 2

Curve 2 (at tangent intersection point T_3) is to be calculated as an asymmetric combined curve (Figure 7.24, $A_3 \neq A_4$).

Design elements for an asymmetric combined curve:

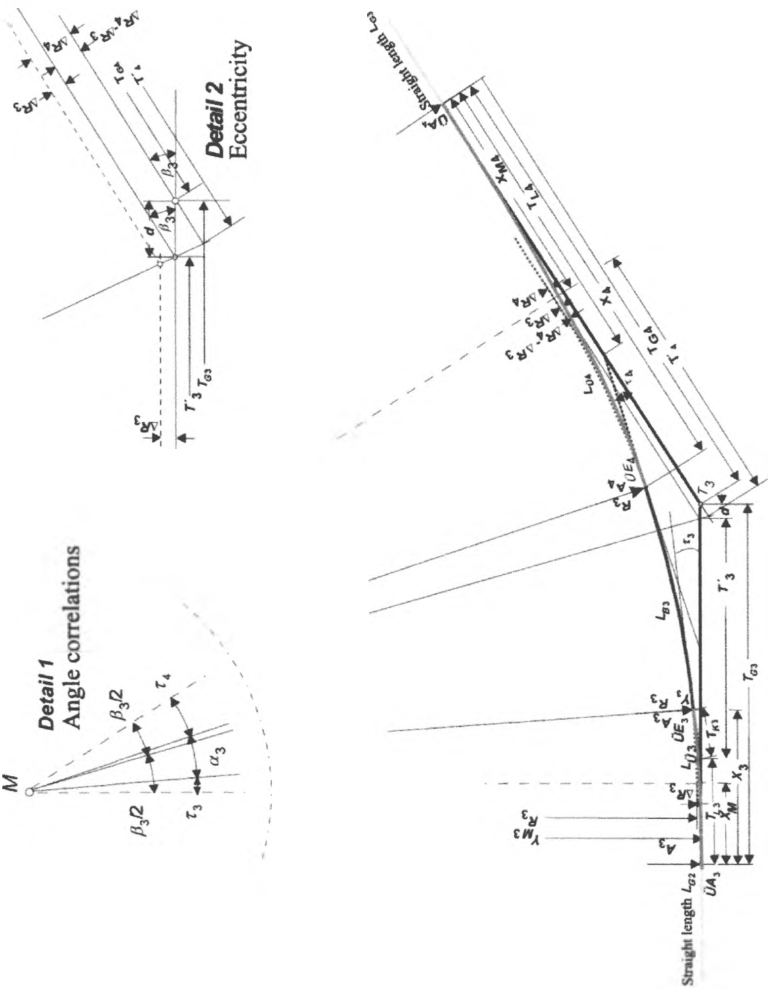


Figure 7.24: Asymmetric combined curve

Input parameters:

Circular arc radius R_3 selected:	$R_3 = 300\,\text{m}$
Clothoid parameter A_3 and A_4:	provided by the restriction on length for tangent T_2T_3, which determines the possible length of the transition curve (solution b)

Note: A circular arc with a radius $R_3 = 300\,\text{m}$ (selected) is initially set for the tangents that intersect (tangent angle $\beta_3 = 36\,\text{gon}$); then tangent sections, which can be assumed to be approximately LÜ3/2 or LÜ4/2 of the incoming clothoid or the outgoing clothoid, are formed from the point of intersection as far as the previous possible start of the combined curve or the subsequent possible end of the combined curve. So it is possible to select transition curves, the length of which cannot be more than double the size of these parts of the tangent in order to adhere to the specified restrictions on length. Furthermore, it should be noted that the length of the remaining circular arc should not fall below a minimum figure (see RAS-L 95). If this condition is not met, a shorter clothoid length should be selected – or rather a larger circular arc.

These conditions should also be met: $R_{min} \leq R_2$ and $A_{min} \leq A_{1,2}$ or $R_{2/3} \leq A_{1,2} \leq R_2$.

$$\overline{T_2T_3} = T_{G2} + L_{G2} + \frac{L_{\ddot{U}3}}{2} + T^*_{B3}$$

$$T^*_{B3} = R_3 * \tan\frac{\beta_3}{2} = 300 * \tan\frac{36}{2} = 87.16 \text{ m}$$

$$\frac{L_{\ddot{U}3}}{2} = \overline{T_2T_3} - L_{G2} - T^*_{B3} - T_{G2} = 450 - 100 - 87.16 - 219.39 = 43.45 \text{ m}$$

$$L_{\ddot{U}3} = 86.90 \approx 80 \text{ m}$$

$$A_3 = \sqrt{R_3 * L_{\ddot{U}3}} = \sqrt{300 * 80} = 154.92 \approx 150 \text{ m}$$

$$\overline{T_3T_4} = T^*_{B3} + \frac{L_{\ddot{U}4}}{2} + L_{G3}$$

$$\frac{L_{\ddot{U}4}}{2} = \overline{T_3T_4} - L_{G3} - T^*_{B3} = 220 - 70 - 87.16 = 62.84 \text{ m}$$

$$L_{\ddot{U}4} = 125.68 \text{ m} \approx 130 \text{ m}$$

$$A_4 = \sqrt{R_3 * L_{\ddot{U}4}} = \sqrt{300 * 130} = 197.48 \text{ m} \approx 200 \text{ m}$$

In the example it is advantageous to work with round circular arc radii and round clothoid parameters as the input parameters for the calculations, because the peripheral conditions L_{G1}, L_{G2}, and L_{G3} only have to be satisfied approximately.

Design elements for the clothoid:

Selected

$$A_3 = 150 \text{ m} \geq A_{\min} = 80 \text{ m}$$

$$A_4 = 200 \text{ m} \geq A_{\min} = 80 \text{ m}.$$

This means that the following input data have been set:

$R_3 = 300.00$ m (selected)
$A_3 = 150.00$ m (selected) and $A_4 = 200.00$ m (selected).

Table figures or calculated figures (from clothoid tables, see Figure 7.25 and 7.26, or calculations):

for $A_3 = 150$ m

$$\Delta R_3 = 0.781 \text{ m}$$

$$L_{\ddot{U}3} = 75.00 \text{ m}$$

$$\tau_3 = 7.9578 \text{ gon}$$

$$X_{M3} = 37.481 \text{ m}$$

$$Y_3 = 3.122 \text{m}$$

$$T_{K3} = 25.037 \text{m}$$

$$T_{L3} = 50.041 \text{m}$$

$$Y_{M3} = R_3 + \Delta R_3 = 300.781 \text{ m}$$

for $A_3 = 200$ m

$$\Delta R_4 = 2.465 \text{ m}$$

$$L_{\ddot{U}4} = 133.333 \text{ m}$$

$$\tau_4 = 14.1471 gon$$

$$X_{M4} = 66.557 \text{ m}$$

$$X_4 = 132.676 \text{ m}$$

$$Y_4 = 9.842 \text{ m}$$

$$T_{K4} = 44.654 \text{ m}$$

$$T_{L4} = 89.120 \text{ m}$$

$$Y_{M4} = R_4 + \Delta R_4 = 302.465 \text{ m}$$

Continuation $A = 140$

L	τ^g	τ^o	R	ΔR	X_M	X	Y	T_K	T_L	L
156,800	39,9288	35 56 09	125	8,081	77,383	150,743	31,872	54,308	106,772	156,800
163,333	43,3255	38 59 35	120	9,111	80,412	155,929	35,845	56,966	111,654	163,333
178,182	51,5609	46 24 17	110	11,748	87,178	166,843	45,897	63,373	123,144	178,182
196,000	62,3887	56 08 59	100	15,469	94,945	177,995	59,766	71,965	137,909	196,000
206,316	69,1288	62 12 57	95	17,902	99,234	183,281	68,619	77,561	147,127	206,316
217,778	77,0232	69 19 15	90	20,844	103,787	187,988	79,062	84,507	158,146	217,778
230,588	86,351	77 42 58	85	24,417	108,374	191,628	91,333	93,473	171,741	230,588
245,000	97,4824	87 44 03	80	28,771	113,326	193,463	105,608	105,692	189,285	245,000
261,333	110,9133	99 49 29	75	34,083	118,307	192,407	121,877	123,690	213,507	261,333

$$K = \frac{\rho^{cc}}{6A^3} = 5{,}413\,43$$

$A^2 = 22\,500$ $\frac{1}{A} = 0{,}006\,666\,667$ **$A = 150$**

L	τ^g	τ^o	R	ΔR	X_M	X	Y	T_K	T_L	L
28,125	1,1191	1 00 26	800	0,041	14,062	28,124	0,165	9,375	18,751	28,125
32,143	1,4616	1 18 56	700	0,061	16,071	32,141	0,246	10,715	21,429	32,143
37,500	1,9894	1 47 26	600	0,098	18,749	37,496	0,391	12,501	25,001	37,500
45,000	2,8648	2 34 42	500	0,169	22,499	44,991	0,675	15,003	30,003	45,000
47,368	3,1743	2 51 25	475	0,197	23,682	47,357	0,787	15,793	31,583	47,368
50,000	3,5368	3 10 59	450	0,232	24,997	49,985	0,926	16,671	33,339	50,000
52,941	3,9651	3 34 07	425	0,275	26,467	52,922	1,099	17,654	35,301	52,941
56,250	4,4762	4 01 43	400	0,330	28,120	56,222	1,318	18,759	37,510	56,250
60,000	5,0930	4 35 02	375	0,400	29,994	59,962	1,599	20,012	40,013	60,000
64,286	5,8465	5 15 43	350	0,492	32,134	64,232	1,967	21,446	42,876	64,286
69,231	6,7806	6 06 09	325	0,614	34,602	69,152	2,456	23,102	46,181	69,231
75,000	7,9578	7 09 43	300	0,781	37,481	74,883	3,122	25,037	50,041	75,000
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
90,000	11,4592	10 18 48	250	1,349	44,951	89,709	5,388	30,093	60,101	90,000
100,000	14,1471	12 43 57	225	1,849	49,918	99,507	7,381	33,491	66,840	100,000
112,500	17,9049	16 06 52	200	2,629	56,102	111,613	10,487	37,785	75,313	112,500
118,421	19,8393	17 51 19	190	3,065	59,019	117,276	12,216	39,842	79,353	118,421
125,000	22,1048	19 53 40	180	3,601	62,250	123,501	14,343	42,151	83,866	125,000
128,571	23,3861	21 02 51	175	3,917	63,998	126,847	15,592	43,416	86,328	128,571
132,353	24,7819	22 18 13	170	4,270	65,844	130,361	16,989	44,764	88,946	132,353
140,625	27,9765	25 10 44	160	5,115	69,862	137,933	20,317	47,755	94,716	140,625
150,000	31,8310	28 38 52	150	6,195	74,379	146,293	24,557	51,222	101,342	150,000
160,714	36,5407	32 53 12	140	7,597	79,483	155,500	30,033	55,311	109,052	160,714
173,077	42,3785	38 08 26	130	9,451	85,276	165,563	37,206	60,244	118,182	173,077
180,000	45,8366	41 15 11	125	10,602	88,467	170,890	41,627	63,129	123,429	180,000
187,500	49,7359	44 45 44	120	11,944	91,875	176,375	46,740	66,377	129,245	187,500
204,545	59,1898	53 16 25	110	15,368	99,396	187,557	59,584	74,343	143,098	204,545
225,000	71,6197	64 27 28	100	20,165	107,917	198,144	77,048	85,394	161,324	225,000
236,842	79,3567	71 25 17	95	23,782	112,545	202,505	88,014	92,853	173,011	236,842
250,000	88,4195	79 34 39	90	27,021	117,380	205,895	100,740	102,430	187,365	250,000
264,706	99,1277	89 12 54	85	31,522	122,349	207,541	115,357	115,368	205,760	264,706

$$K = \frac{\rho^{cc}}{6A^3} = 4{,}715\,70$$

Figure 7.25: Extract from clothoid table (Kasper, et al., 1964)

Continuation A = 140

L	τ^g	τ^c	R	ΔR	X_M	X	Y	T_K	T_L	L
277,692	67.9940	61 21 41	130	23.733	133.731	247.645	91,094	103,058	197.554	277,692
288,800	73.5423	66 11 17	125	26.514	138,210	252,569	101,047	110,449	207,977	288,800
300,833	79.7985	71 49 07	120	29,718	142,874	256,883	112,275	118,175	220,009	300,833
328,182	94,9669	85 28 13	110	37.703	152,645	262,301	139,016	139.451	251,288	328,182
361,000	114.9099	103 25 08	100	48,401	162,580	259,849	171,608	176,424	300,792	361,000

$$K = \frac{\rho^{\prime\prime}}{6A^3} = 2{,}939\ 15$$

$A^2 = 40\,000$ $\frac{1}{A} = 0{,}005\,000\,000$ **$A = 200$**

L	τ^g	τ^c	R	ΔR	X_M	X	Y	T_K	T_L	L
30,769	0.7534	0 40 41	1300	0.030	15.385	30.769	0,121	10,257	20.513	30,769
36,364	1,0523	0 56 49	1100	0.050	18,182	36,363	0,200	12,122	24.245	36,364
44.444	1,5719	1 24 53	900	0.091	22,222	44.442	0,366	14.816	29,631	44.444
53.333	2,2635	2 01 14	750	0,158	26,666	53.327	0,632	17,780	35.558	53.333
57.143	2.5985	2 20 19	700	0,194	28,570	57.133	0,777	19.051	38,098	57.143
61,538	3,0136	2 42 44	650	0.243	30,767	61,525	0,971	20,517	41,030	61,538
66,667	3.5368	3 10 59	600	0.309	33.330	66,646	1,234	22,229	44.452	66,667
72,727	4.2091	3 47 17	550	0,401	36.358	72,695	1,601	24.252	48,496	72,727
80,000	5,0930	4 35 01	500	0.533	39.991	79.949	2,132	26,683	53.351	80,000
84,211	5,6432	5 04 44	475	0.622	42.094	84.144	2,487	28,091	56,163	84,211
88,889	6,2876	5 39 32	450	0.731	44.430	88,802	2,924	29,657	59.290	88,889
94,118	7.0491	6 20 39	425	0,868	47.039	94.002	3.471	31,409	62,785	94,118
100,000	7.9578	7 09 43	400	1.041	49.974	99,844	4.162	33.383	66,721	100,000
106,667	9,0541	8 08 55	375	1,263	53.297	106,451	5,049	35,624	71,187	106,667
114,286	10,3938	9 21 16	350	1.554	57.092	113,982	6,208	38,192	76,297	114,286
133,333	14.1471	12 43 57	300	2,465	66,557	132,676	9,842	44.654	89,120	133,333
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
160,000	20,3718	18 20 05	250	4.251	79.728	158,360	16,942	53,859	107,244	160,000
177,778	25,1504	22 38 07	225	5.820	88,426	175,023	23,151	60,154	119,502	177,778
200,000	31,8310	28 38 52	200	8,259	99.172	195,058	32,743	68,296	135,122	200,000
210,526	35,2698	31 44 34	190	9,614	104,195	204,156	38,034	72,294	142,676	210,526
222,222	39,2975	35 22 04	180	11,277	109,715	213,903	44.495	76,872	151,217	222,222
228,571	41,5752	37 25 04	175	12,252	112,680	219,014	48,262	79,427	155,930	228,571
235,294	44,0568	39 39 04	170	13.340	115,794	224,273	52,449	82,194	160,987	235,294
250,000	49.7359	44 45 44	160	15.926	122,500	235.166	62,320	88,502	172,327	250,000
266,667	56,5884	50 55 46	150	19,205	129,898	246,353	74,664	96,170	185,740	266,667
285,714	64,0612	58 27 54	140	23,412	138,040	257,365	90,189	105,816	201,021	285,714
307,692	75,3397	67 48 21	130	28,871	146,937	267,305	109,764	118,547	222,524	307,692
320,000	81,4873	73 20 19	125	32,204	151,649	271,401	121,365	126,683	235,079	320,000
333.333	88,4195	79 34 39	120	36,028	156,506	274,526	134,320	136,573	249,820	333,333
363,636	105,2264	94 42 14	110	45.474	166,462	276,191	164,404	165,050	289,626	363,636

$$K = \frac{\rho^{\prime\prime}}{6A^2} = 2{,}652\ 58$$

Figure 7.26: Extract from clothoid table (Kasper, et al., 1964)

Design elements for combined curve 2:

- β_3 (from the preliminary graphical draft) = 36.00 gon

$$T'_3 = (R_3 + \Delta R_3) * \tan\frac{\beta_3}{2} = (300.00 + 0.781) * \tan\frac{36.00}{2} = 87.385 \text{ m}$$

$$T'_4 = (R_3 + \Delta R_4) * \tan\frac{\beta_3}{2} = (300.00 + 2.465) * \tan\frac{36.00}{2} = 87.874 \text{ m}$$

$$\Delta R_4 > \Delta R_3 \quad \Rightarrow \quad d = \frac{\Delta R_4 - \Delta R_3}{\sin\beta_3} = \frac{2.465 - 0.781}{0.5358} = 3.143 \text{ m}$$

$$T_{G3} = T_3 + X_{M3} + d = 87.385 + 37.481 + 3.143 = 128.009 \text{ m}$$

$$T_{G4} = T'_4 + X_{M4} - d = 87.374 + 66.557 - 3.143 = 151.288 \text{ m}$$

$$\alpha_3 = \beta_3 - (\tau_3 + \tau_4) = 36.00 - (7.9578 + 14.1471) = 13.895 \text{gon}$$

$$L_{B3} = R_3 * \alpha_3 * \frac{\pi}{200} = 300.00 * 13.895 * \frac{\pi}{200} = 65.479 \text{ m}$$

$$L_\Sigma = L_{Ü3} + L_{B3} + L_{Ü4} = 75.00 + 65.479 + 133.333 = 273.812 \text{ m}$$

- *Check:*

$$\overline{T_2T_3} - T_{G2} - T_{G3} = 450.00 - 129.388 - 128.009 = 102.603 \text{ m} \approx 100.00\text{m}$$

$$\overline{T_3T_4} - T_{G4} = 220.00 - 151.288 = 68.710 \text{ m} \approx 70.00 \text{ m}$$

- Minimum length for arc (2 s at v_e):

$$L_{B\,\min} = \frac{v_e}{3.6} * 2 = 44.444 \approx 50.00\text{m} < L_{B3}$$

Note: In the case of an asymmetric combined curve, the tangent length T_G on the side of the larger tangent retractions is smaller by the length d, but larger by d on the other side (Figure 7.24).

3rd Example: calculating a symmetric inflection line

Input (Figure 7.27)

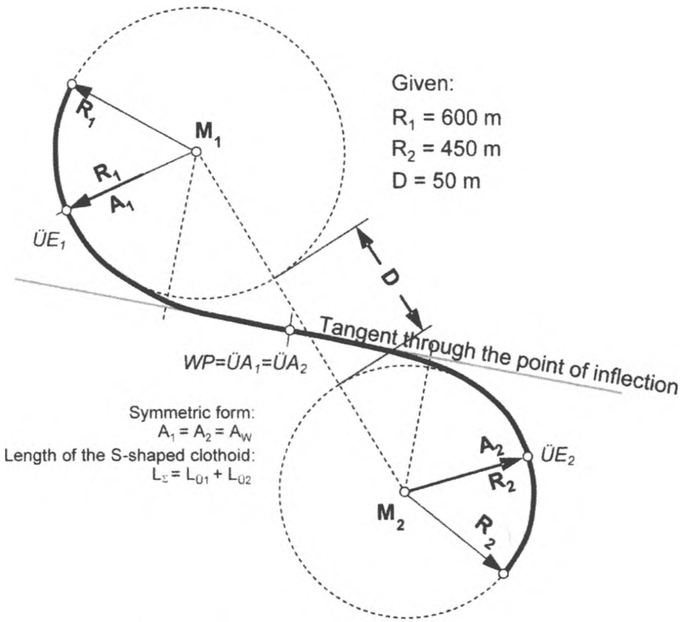


Figure 7.27: Geometrical interrelations

- Given that:
  - Road category: A II $\Rightarrow v_e = 80\,\text{km/h}$
  - Circular arc radius $R_1 = R_g = 600\,\text{m}$
  - Circular arc radius $R_2 = R_k = 450\,\text{m}$
  - Gap between circles $D = 50.00\,\text{m}$
- Looking for:
  1. Limits $R_{\min}$, $A_{\min}$, $L_{Z\max}$
  2. Calculate the clothoid parameters for the inflection clothoid ($A_1 = A_2 = A_w$)
  3. Design elements for the inflection line.

Solution

Limits:

$$L_Z \leq 0.08(A_1 + A_2) \leq 84 \text{ m}$$
$$R_1 \geq R_{\min} = 250 \text{ m}$$
$$R_2 \geq R_{\min} = 250 \text{ m}$$
$$R_2 : R_1 = 1 : 1.33 < 1 : 2$$
$$A_1 = A_2 = A_W \geq A_{\min} = 80 \text{ m.}$$

Clothoid parameter A_W:

Approximate solution for A_W with substitute radius

- Calculating the substitute radius R':

$$R' = \frac{R_g * R_k}{R_g + R_k} = \frac{600 * 450}{600 + 450} = 257.14 \text{ m}$$

$$A_W \approx \sqrt[4]{24 * D * (R')^3} = \sqrt[4]{24 * 50 * 257.14^3} \approx 377.93 \text{ m}$$

$$\Rightarrow \text{selected} : A_W = 375.00 \text{ m.}$$

- Calculations using nomograms in line with Osterloch (1964) (Figure 7.28)

$$\frac{D}{R_g} = \frac{50}{600} = 0.08333$$

$$\frac{R_k}{R_g} = \frac{450}{600} = 0.75 \Rightarrow \text{Extracting} : \frac{A_W}{R_g} = 0.63 * R_g = 378 \text{ m}$$

$$\Rightarrow \text{selected} : A_W = 375.00 \text{ m.}$$

The numerical value for the clothoid parameter is usually not a round number. But as it is normally hard to determine the inflection tangent or the two circles absolutely, the next closest round number can be selected for A_W. This means that the gap between the circles D has to be recalculated (Figure 7.29).

7.3 Design Elements on the Vertical Projection

7.3.1 General issues

The vertical projection represents the drawing of the vertical cross section through the three-dimensional roadway, where the lengths of the road are depicted as

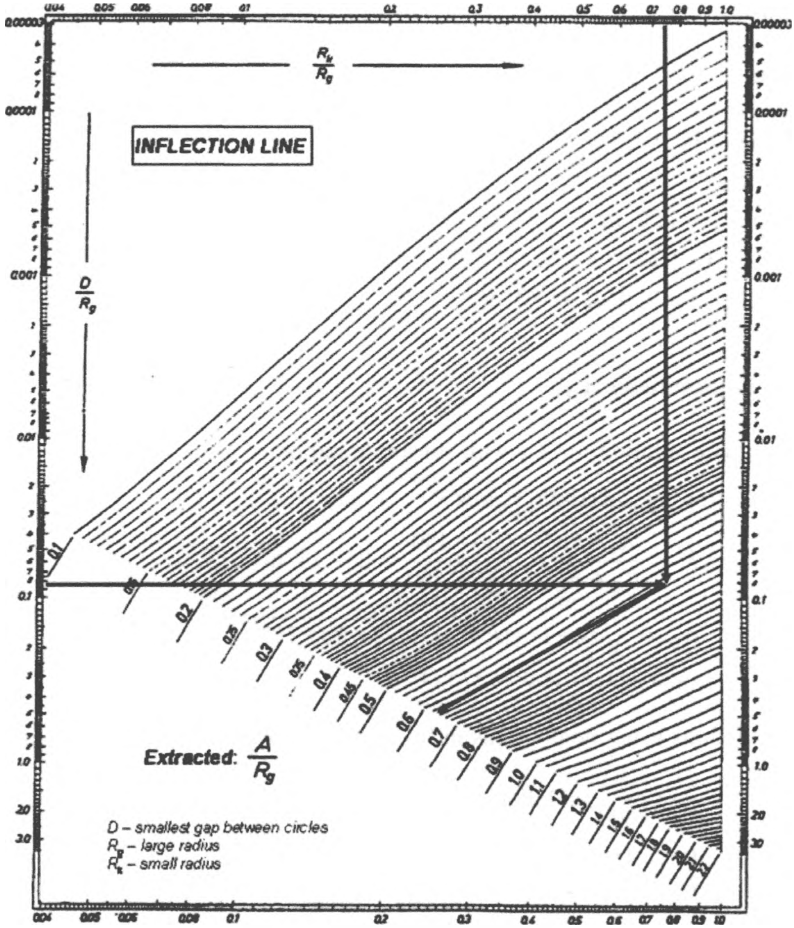


Figure 7.28: Nomogram for symmetrical inflection clothoid (Osterloch, 1964)

simplified projections on the horizontal projection level. The heights on the other hand are generally enlarged 10-fold.

The reference line is normally the road axis taken from the horizontal projection. The edge of the road is only selected as the reference line in special cases. The main constituent parts of the vertical projection are (Figure 7.30):

- Longitudinal profile of the terrain:
  Height of the ground above the road axis.
- Gradient (inclination along the road axis):
  Height changes along the road axis.

No.	Unit	Description	Numerical value
1	m	$R_g = R_1$ (from prelim. draft)	600.00
2	m	$R_k = R_2$ (from prelim. draft)	450.00
3	m	D (from prelim. draft)	50.00
4	m	(Clothoid parameter selected) A_w	375.00
5	m	R_1	3.810
6	m	L_1	234.375
7	gon	1	12.4340
8	m	X_{M1}	117.039
9	m	X_1	233.483
10	m	Y_1	15.217
11	m	T_{K1}	78.410
12	m	T_{L1}	156.563
13	m	R_2	9.003
14	m	L_2	312.500
15	gon	2	22.1048
16	m	X_{M2}	155.624
17	m	X_2	308.753
18	m	Y_2	35.859
19	m	T_{K2}	105.378
20	m	T_{L2}	209.664
21	m	$Y_{M1} = R_1 + \Delta R_1 = 1 + 5$	603.810
22	m	$Y_{M2} = R_2 + \Delta R_2 = 2 + 13$	459.003
23	m	$\Sigma X_M = X_{M1} + X_{M2} = 8 + 16$	272.663
24	m	$\Sigma Y_M = Y_{M1} + Y_{M2} = 21 + 22$	1062.813
25	-	$\tan \varepsilon = \Sigma X_M / \Sigma Y_M = 23 / 24$	0.25655
26	m	$E = (R_1 + \Delta R_1) \tan \varepsilon - X_{M1} = 21*25-8$	37.868
27	-	$\sin \varepsilon$	0.24850
28	m	$X_{M1} + E = 8 + 26$	154.907
29	m	$X_{M2} - E = 16 - 26$	117.756
30	m	$C_1 = (X_{M1} + E) / \sin \varepsilon = 28 / 27$	623.368
31	m	$C_2 = (X_{M2} - E) / \sin \varepsilon = 29 / 27$	473.867
32	m	$M = C_1 + C_2 = 30 + 31$	1097.235
33	m	$D_1 = C_1 - R_1 = 30 - 1$	23.368
34	m	$D_2 = C_2 - R_2 = 31 - 2$	23.867
35	m	$D = D_1 + D_2 = M - (R_1 + R_2) = 32-1-2$	47.235

Figure 7.29: Calculating the determining factors

- Incline graph:
  Lengths of the road with gradients and quadratic parabolas with determining factors.
- Altitude data:
  Altitude of terrain and gradients at characteristic points from the horizontal projection.
- Distance points:
  Taking over distance points along the axis from the horizontal projection.

The following special graphs are normally included in a vertical projection:

(1) General curvature graph
(2) Raising or lowering road edge graph
(3) Sight distance graphs

While the general curvature graph illustrates the sequence of elements on the horizontal projection, the raising or lowering road edge graph shows the height changes at the edge of the road.

The visibility graphs help depict the areas with adequate visibility for overtaking as well as adherence to the stopping sight distance.

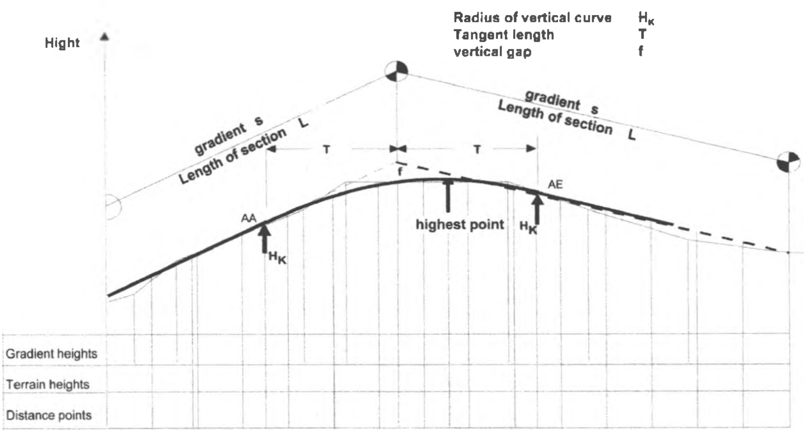


Figure 7.30: Vertical projection with important components

7.3.2 Gradients on roads

The gradient of a road (ascent, descent) is the first design element in the vertical projection.

The gradient s [%] of the incline is the ratio between the difference in height ΔH and the difference in length ΔL between two points on the tangent. The gradient s is calculated from the ratio between the difference in height between the

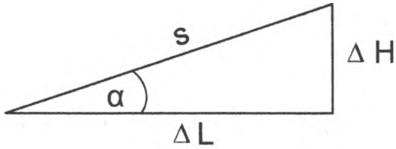


Figure 7.31: Calculating the gradient

change in gradient points ΔH (tangent intersection points) and their distance from each other ΔL (Figure 7.31).

$$\tan \alpha = \frac{\Delta H}{\Delta L}, \quad [-] \tag{7.83}$$

$$s = \frac{\Delta H}{\Delta L} * 100 = 100 * \tan \alpha, \quad [\%] \tag{7.84}$$

- Maximum gradient
  The following values emerge depending on the design speed and the road category (Table 7.5).
- Minimum gradient
  - for raising or lowering the edge of the road on roads without marked edges
    $s > 0.7\%$ (better $s \geq 1.0\%$)
  - for raising or lowering the edge of the road on roads with marked edges
    $s = 0.5\%$ (for all curbstones)
    $s - \Delta s \geq 0.5\%$.
- Minimum gradient on bridges
  $s = 0.5\%$.
  The gradient between the maximum and minimum gradient is dictated by the existing terrain situation, the necessary intersections, and any overpasses or underpasses over or under traffic routes.

Table 7.5: Max. gradient according to RAS-L

v_E [km/h]	max s [%] on roads in category group	
	A	B I, B II
50	9.0	12.0
60	8.0	10.0
70	7.0	8.0
80	6.0	7.0
90	5.0	6.0
100	4.5	5.0
120	4.5	–

- Calculating intersection points
  The following two cases must be taken into account for calculating two tangents (Figures 7.32 and 7.33).

The distance points and heights of the tangent intersection points can be seen graphically on the vertical projection.

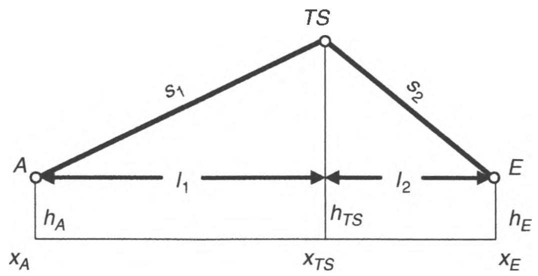


Figure 7.32: Case 1: Tangent intersection point known (h_{TS}, x_{TS})

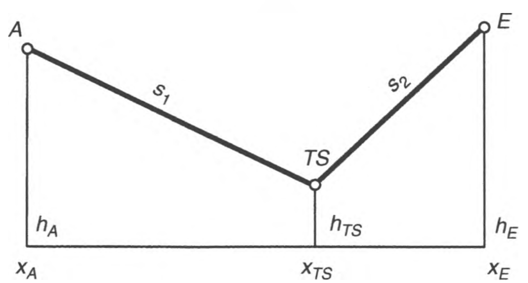


Figure 7.33: Case 2: Tangent intersection point unknown

7.3.3 Quadratic parabolas

The intersection points of the gradients must be rounded off, but a difference must be made between a change of gradient and an alteration in gradient and between rounding off a crest or a sag.
A crest (Figure 7.34) can be formed at the transition point between

- an ascent in a descent
- an ascent that goes from steep to shallow
- a descent that goes from gradual to steep

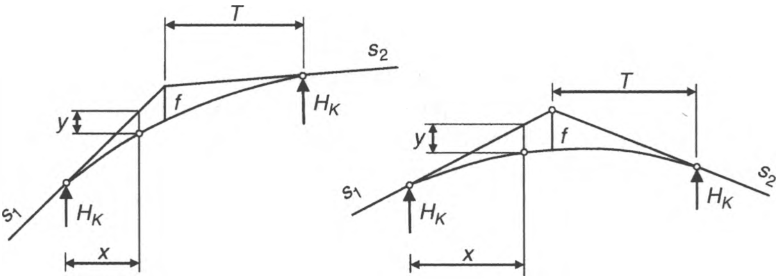


Figure 7.34: Gradient situation on a crest

A sag (Figure 7.35) is formed at the transition point between

- a descent to an ascent
- an ascent that goes from gradual to steep
- a descent that goes from steep to gradual

A gradient change or gradient alteration is understood to be the change or alteration in the increase of the tangent to the gradient.

In road design, the quadratic parabola radius is described as H on the vertical projection ($H_K =$ crest radius, $H_W =$ sag radius). It is not necessary to interpolate a transition curve on the vertical projection as a very small gradient and a very large quadratic parabola radius can normally be chosen.

The choice of quadratic parabola radius depends on:

- Safety issues (e.g. sight distance)
- Driving factors (e.g. centrifugal acceleration)
- Esthetic aspects (e.g. sharp bends or uneven stretches).

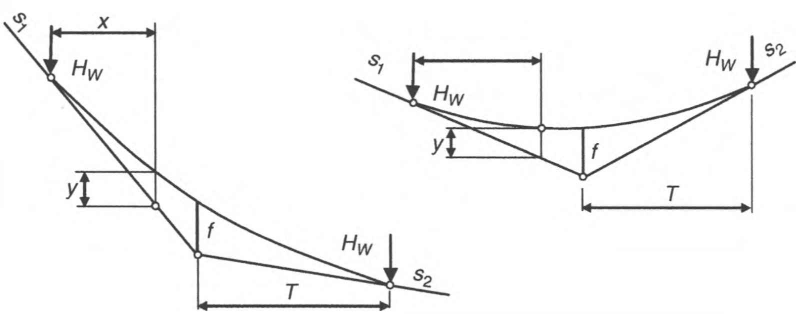


Figure 7.35: Gradient situation with a depression

Crest radius

The most important criteria for selecting the crest radius are safe driving conditions and an esthetically pleasing three-dimensional alignment. The former is achieved by as high a figure as possible so that the stopping sight distance is maintained in the crest area and drivers can recognize sudden obstacles in good time. For economic reasons, however, the crest radius should not be too large either. The calculation can be made using this approach (Figure 7.36):

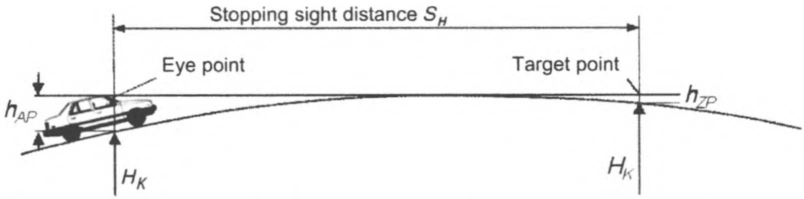


Figure 7.36: Geometric model for determining the crest radius according to RAS-L

$$\min H_K = \frac{S_H^2}{2 * \left(\sqrt{h_{AP}} + \sqrt{h_{ZP}}\right)^2} 1[\mathrm{m}]. \tag{7.85}$$

min H_K: Minimum crest radius, [m]
S_H: Stopping sight distance required, [m]
h_{AP}: Height of driver's eye point, [m]
h_{ZP}: Height of target point, [m].

Sag radius

The esthetic issue is the most important in the case of sag radii.

As the stopping sight distance is normally no problem in daylight, driving with headlights determines whether the sag radius is acceptable. A vehicle must be able to stop safely in front of an obstacle picked up by headlights, if necessary (Figure 7.37).

The threshold for the sag radius is calculated using this approach:

$$\min H_W = \frac{S_H^2}{2 * (h_{AP} + S_H * \sin\alpha)} \tag{7.86}$$

min H_W: minimum sag radius, [m]
S_H: stopping sight distance, [m]
h_{AP}: height of starting point, [m]
α: divergence angle of the headlights ($\alpha \sim 1.11$ gon), [gon].

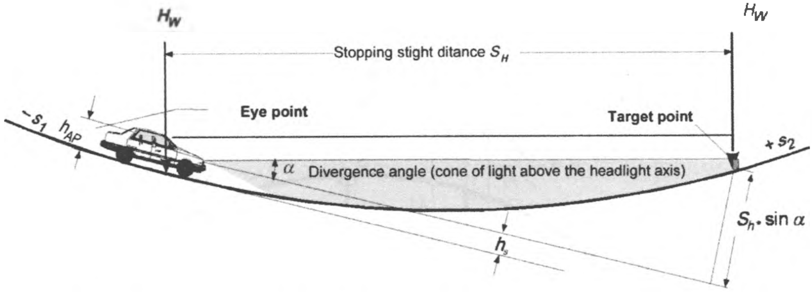


Figure 7.37: Geometric model to determine the minimum sag radius from RAS-L

The sag radius should not be smaller than half the crest radius for esthetic reasons. This provides adequate stopping sight distance when driving with headlights. The minimum sag radii can be taken from Table 7.6.

The crest and sag quadratic parabolas are calculated mathematically using quadratic parabolas, as the differences between the graph for the circular arc and the quadratic parabola are slight in the bordered area. For this reason we talk about quadratic parabola radii and not rounding off radii on the vertical projection. The quadratic parabola calculations are carried out according to Figure 7.38 using approximate equations.

$$s(x) = s_1 + \frac{x}{H} * 100, \quad [\mathrm{m}]. \tag{7.87}$$

$$y(x) = \frac{s_1}{100} * x + \frac{x^2}{2 * H}, \quad [\mathrm{m}]. \tag{7.88}$$

$$T = \frac{H}{2} * \frac{s_2 - s_1}{100}, \quad [\mathrm{m}]. \tag{7.89}$$

Table 7.6: Minimum radii for sags (RAS-L)

v_e [km/h]	min H_W
50	500
60	750
70	1,000
80	1,300
90	2,400
100	3,800
120	8,800

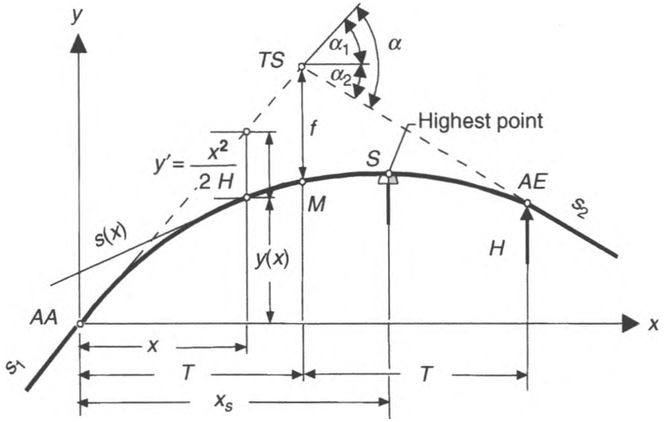


Figure 7.38: Geometric correlations for calculating quadratic parabola radii (Weise and Durth, 1997)

$$f = \frac{T^2}{2 * H} = \frac{T}{4} * \frac{s_2 - s_1}{100} = \frac{H}{8} * \left(\frac{s_2 - s_1}{100}\right)^2, \quad [\text{m}]. \tag{7.90}$$

For the highest point of the crest or the deepest point of the sag, we can assume that $s_2 = 0$ and therefore:

$$x_s = -\frac{s_1}{100} * H, \quad [\text{m}] \tag{7.91}$$

$$y_s = -\frac{x_s^2}{2 * H} = -\left(\frac{s_1}{100}\right)^2 * \frac{H}{2}, \quad [\text{m}]. \tag{7.92}$$

7.3.4 Example: calculations on the vertical projection (Weise, Kuhn, et al., 1980)

Input:

Given that:
- Road category: A II, $\Rightarrow v_e = 80\,\text{km/h}$
- Longitudinal terrain profile

Distance points and heights at the gradient change points (Figure 7.39)
$L_1 = 0 + 000,\ H_1 = 574.00\,\text{m};$
$L_2 = 0 + 750,\ H_2 = 600.00\,\text{m};$
$L_3 = 1 + 620,\ H_3 = 582.60\,\text{m};$

Find:

1. Limits for max s, min s, min H_K, min H_W,
2. Gradients s_i and route lengths L_i
3. Crest radius H_K, tangent length T, vertical gap f
4. Gradient heights at the main and intermediate points (round distance points)

Solution:

- *Limits:* amounting to $v_e = 80$ km/h

max s: 5.0%
min s: 0.7% ($s-\Delta s \geq 0.2\%$)
min H_K: 5,700 m
min H_W: 2,400 m.

- *Gradients*

The gradients are first set from the differences in height and distance points.

$$s_1 = \frac{H_2 - H_1}{L_2 - L_1} * 100 = \frac{600.00 - 574.00}{750.00 - 0.00} * 100 = 3.467\%$$

$$s_2 = \frac{H_3 - H_2}{L_3 - L_2} * 100 = \frac{582.60 - 600.00}{1620.00 - 750.00} * 100 = -2.000\%$$

The negative value for s_2 shows that we are dealing with a descent, while positive figures indicate an ascent.

The next step involves calculating the crest quadratic parabola radius and any factor can be attributed to the three determining values H_K, T, and f. Here $H_K = 12{,}000$ m (>min H_K) has been chosen to provide a generous quadratic parabola.

$$T_K = \frac{H_K}{2} * \frac{s_2 - s_1}{100} = \frac{H_K}{2} * \frac{m}{100} = \frac{12{,}000.00}{2} * \frac{5{,}467}{100} = 328.02 \text{ m}$$

$$f = \frac{T_K^2}{2 * H_K} = \frac{328.02^2}{2 * 12{,}000.00} = 4.48 \text{ m.}$$

The y-values still have to be calculated for the distance points within in order to determine the altitude of the gradients. These heights on the road axis and for the left and right edge of the road are set in table form or in computer programs as they involve calculations that are often repeated (Figure 7.39).

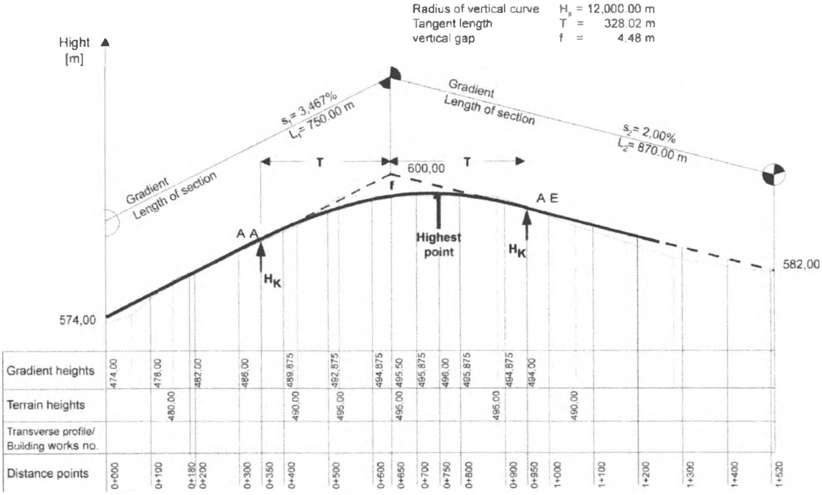


Figure 7.39: Vertical projection

7.4 Three-Dimensional Alignment

7.4.1 General issues

The three-dimensional alignment of a road is formed by using the conventional design methodology of superimposing the horizontal and vertical projections and the cross section.

When driving along a road, drivers perceive the three-dimensional alignment as a sequence of driving images and derive their driving behavior from this. If the driving area can be well understood, drivers select a suitable speed and the road is normally very safe. But if the course of the road cannot be perceived at crests or before bends or if parts of the road cannot be seen, accidents involving oncoming traffic may occur when overtaking on single-lane roads – i.e. these roads are not safe enough. As these kinds of problems do not automatically create accident black spots, they are described as shortcomings and not as errors.

In order to avoid shortcomings in the three-dimensional alignment during the design process, various approaches are possible:

- Using qualitative rules, notes, and criteria
- Using quantitative monitoring checks
- The combined use of qualitative criteria and quantitative monitoring checks.

Extensive research has shown that a qualitative check is always subjective and therefore does not normally provide comparable results. An exclusively quantitative check is not adequate either because of the complexity involved – i.e. a combined check with quantitative monitoring checks and qualitative criteria should be used.

7.4.2 Rules, guidelines, and criteria

The following notes are provided in the relevant design specifications, e.g. RAS-L, for harmonizing the horizontal and vertical projections:

(1) Inflection points should lie at about the same point on the horizontal and vertical projections and their number should be roughly the same (Figure 7.40):
  - Visually:
    Clearly perceptible change of direction before a crest (at least 3.5 gon)

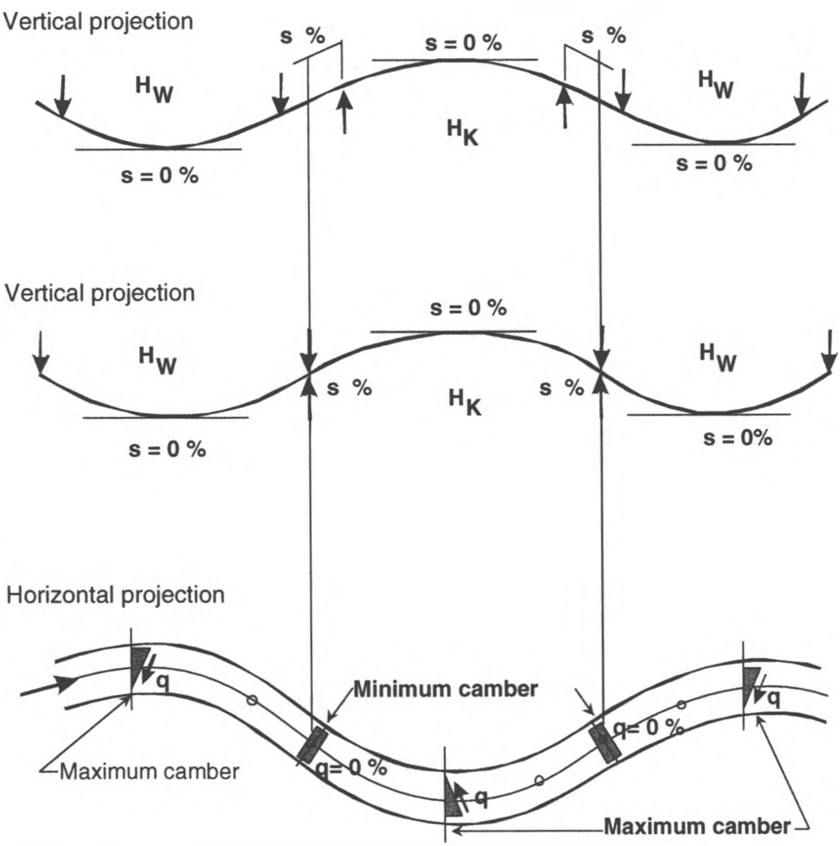


Figure 7.40: Harmonizing the horizontal and vertical projections

- From a drainage point of view:
  Largest longitudinal gradient at the transverse gradient zero crossover
- From a driving point of view:
  Easily recognizable and harmonious alignment

(2) Tangent intersections on the horizontal and vertical projections should be as close to each other, even if the number of inflection points is different.
(3) The ratio of consecutive sag radii H_w and curve radii R must be harmonized.
(4) The ratio between curve radius R and crest radius H_K should not be larger than 1/5–1/10
   - On flat ground: $H_K < H_W \rightarrow$ visually good alignment
   - On hilly ground: $H_K > H_W \rightarrow$ greatest possible sight distances.

Optical illusions or sharp bends should also be avoided:

(1) A short straight between two sags
   → creates the impression of a bulge
(2) A short straight between two straightforward crests
   → creates the impression of a flat stretch
(3) A short sag between long stretches with a constant longitudinal gradient
   → creates an optical kink on the vertical projection
(4) A bend with a small radius and short arc length between two autonomous straights
   → creates an optical kink on the horizontal projection.

Shortcomings can still occur in the three-dimensional alignment even for experienced design engineers despite these rules and guidelines and they are not recognized until the road has been built. Speed restrictions and overtaking bans are then imposed as consequential measures.

7.4.3 Checking methodology using quantitative parameters and qualitative criteria

General issues

A methodology has been developed within a research project (Kühn, et al., 2005) to numerically check the three-dimensional alignment for shortcomings based on perspective images. A distinction is made between:

- shortcomings that affect safety (critical blind spot areas and a concealed start of a bend) and
- design shortcomings (e.g. sharp bends on the horizontal and vertical projections)

While safety shortcomings have to be eliminated by re-planning the horizontal and/or vertical projection, design shortcomings are permissible if the effort to re-plan things is too great or if this creates a clash with other design criteria (Kühn, et al., 2006, 2007, 2009, 2011).

Standardized perspective

In order to be able to use perspective images for checking designs, the following unified input must be used to calculate and prepare them:

- Defined model assumptions
- Degree of detail
- Color and contrast
- Lighting model
- Inserting distance point, gradient, and curvature graph

The assessment of the effect of a traffic situation on a driver should take place from the driver's perspective. Table 7.7 summarizes the defined model assumptions.

A unified degree of detail must be used to calculate the perspective images. The driving area elements to be shown are found in Table 7.8.

Table 7.7: Model assumptions for the driver's perspective (HViSt)

	Height above road	Location in cross section
Eye level	1.00 m (h_{AP})	Middle of own lane (d_{AP})
Target	0.00 m (h_{ZP})	Middle of own lane (d_{ZP})
Target distance	75 m (l_{ZP})	
Focal distance	50 mm (corresponds to 35 mm small image format)	

Table 7.8: Elements in the normal perspective and coloring (HViSt)

Element to the depicted	Coloring	RGB values
Road	Grey	100/100/100
Edge markings	White	255/255/255
Central markings (if present)	White	255/255/255
Distance markers at 50 m intervals	Black-white	(realistic)
Verge	Green	90/180/0
Depression	Green	0/160/0
Shoulder	Light green	0/176/0
Surveyed terrain	Green	0/140/0
Artificial terrain	Dark green	0/80/0
Sky	Grey-blue	220/220/255
Elements dictating the driving area	Realistic	–

The basis for road design – and therefore for visualizing it – is normally a digital model of the terrain. If this is not available in the early stages of the planning work, an artificial terrain should be produced on either side of the roadway to ensure that the optical effect achieved is satisfactory (i.e. the road should not "hover" in the area).

The coloring to be used, including the RGB values for the screen resolution, can also be found in Table 7.8. In principle, an illuminated model should not be used as it may lead to discrepancies with the set coloring.

The standardized perspective (Figure 7.41) that is developed shows the curvature graph, the course of the gradient, and the various distance points on the header bar. This creates a link between the perspective image and the actual site and the existing design elements. Software manufacturers provide the normal perspective and the defined model assumptions as basic settings.

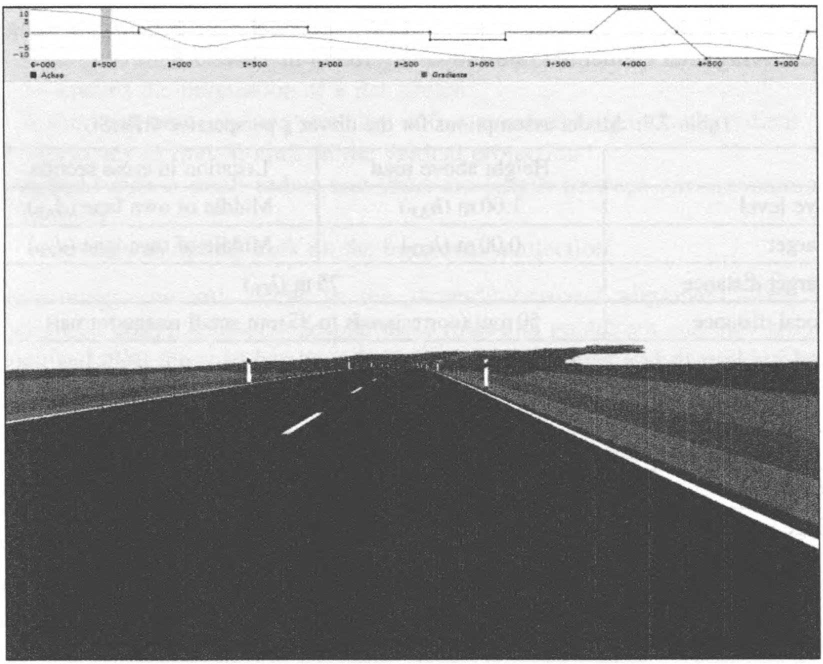


Figure 7.41: Standardized perspective (HViSt)

Three-dimensional elements

When superimposing the horizontal and vertical projections, three-dimensional elements are produced. Standard three-dimensional elements (SRE) are produced if the horizontal and vertical projections are superimposed in a specific manner.

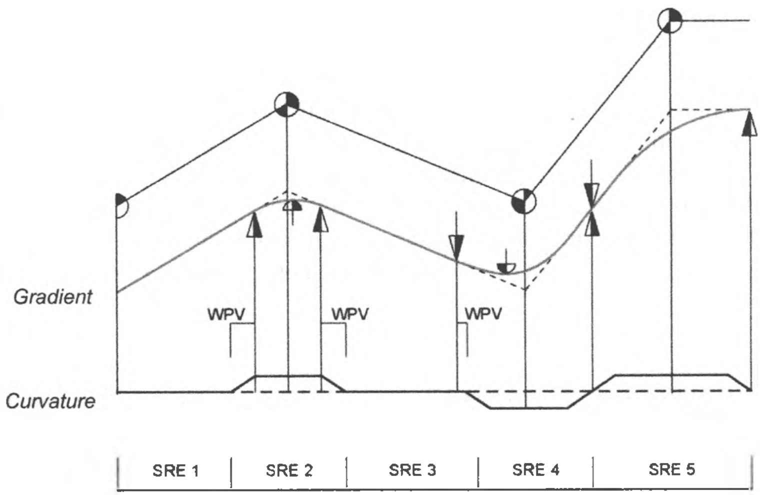


Figure 7.42: Standard sequence of three-dimensional elements (HViSt)

The number of shortcomings in the three-dimensional alignment can be reduced to a minimum from the outset by using standard three-dimensional elements.

Standard three-dimensional elements are produced if the start and end of curves on the horizontal projection coincide with the start and end of crests and sags on the vertical projection (Figure 7.42).

This means that straights on the horizontal projection are treated like bends where $R = \infty$ and constant longitudinal gradients in the vertical projection are treated like vertical radii where $H = \infty$. It is possible to vary things between the start and finish point on the horizontal and vertical projections by up to 20% of the length of the horizontal projection element. A standard three-dimensional element is also produced if the start of a bend has to be moved to a point before the start of the crest when the crest and bend are superimposed in order to ensure that the start of the bend can be recognized by drivers.

The figures required in Table 7.9 take into consideration the most unfavorable case of a sequence consisting of a straight (with constant longitudinal gradient)–clothoid (with crest quadratic parabola)–circular arc.

The following standard three-dimensional elements are possible:

- straight with a constant longitudinal gradient (even straight)
- straight sag
- straight crest
- curve with constant longitudinal gradient (even curve)
- curved sag
- curved crest.

Safety-related shortcomings

Critical blind spots

If drivers are unable to recognize a section of the edge of the road in the area ahead, a blind spot is created. Blind spots (Figure 7.43) are characterized by the depth of the blind spot s_t and its length s_t.

Blind spots affect safety (critical blind spots) if their depth s_t is $\geq$ 0.75 m, they extend $A \geq$ 60 m (three distance markers at an interval of 20 m on the general poor visibility scale) and the surface of the road is visible again from eye level at a maximum distance of 600 m. Critical blind spots must be avoided as a matter of principle.

Dips

Dips in the alignment are produced if critical blind spots occur during a sequence of crests and sags where the direction of the route does not change in the blind spot area. Dips can occur along straights or in bends. They seriously impair the clear view of a road and drivers' ability to recognize the direction that the road takes. Drivers may be misled about the course of the route, oncoming traffic, and the visibility that exists. Dips are shown in Figure 7.44 using a perspective image.

Jumps

Any alignment with jumps occurs if critical blind spots are formed in a sequence of crests and sags, where the direction of the road also changes. This can lead to drivers failing to distinguish between a "disappearing point" and a "reappearance point" along the road. Jumps impair the visibility along a road and prevent drivers from seeing where the course of the road is taking them. Drivers may be misled about the course of the road, oncoming traffic and the sight distance that exists. Jumps are illustrated in Figure 7.45 using a perspective image.

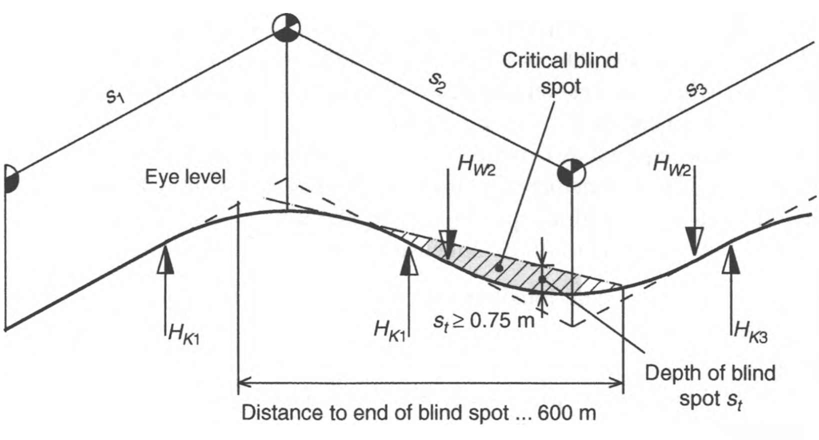


Figure 7.43: Definition of a critical blind spot (Kuhn and Jha, 2011)

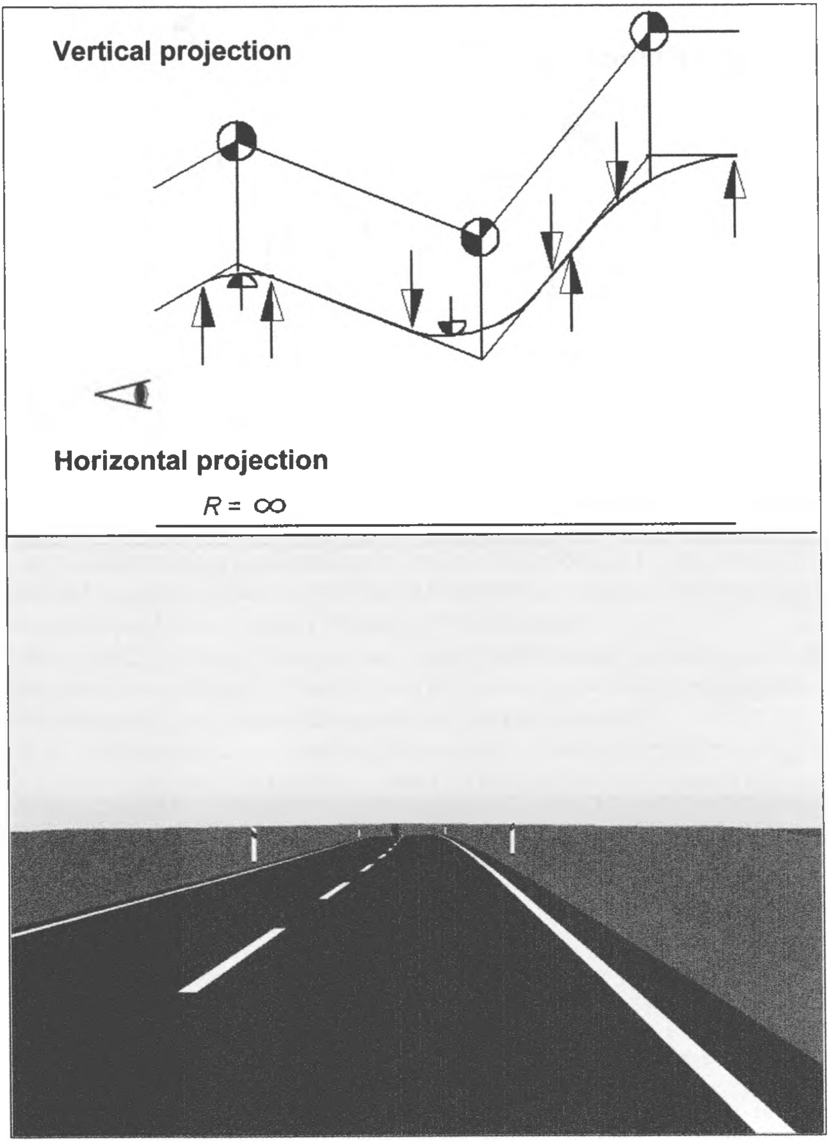


Figure 7.44: Dips (Kühn, et al., 2007)

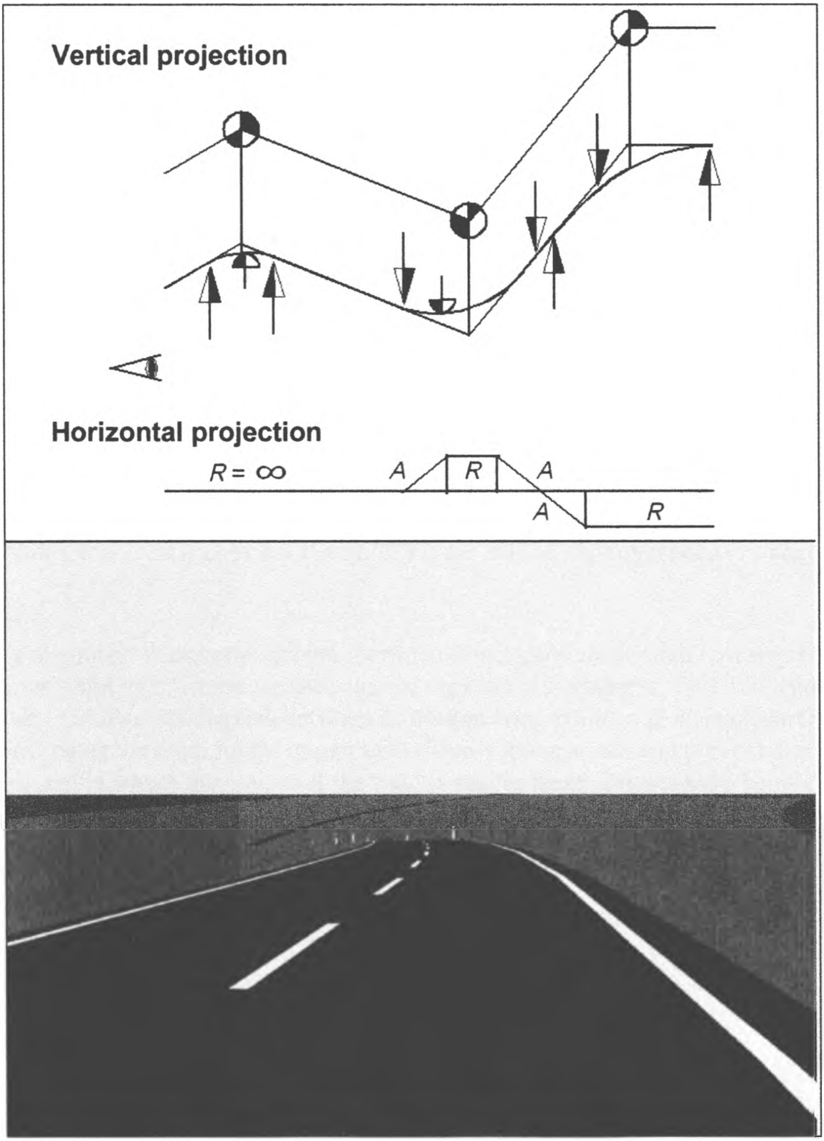


Figure 7.45: Jumps (Kühn, et al., 2007)

Concealed Start of a Bend

If there is an inflection point from the horizontal projection in the area of a crest, the visibility at the start of a bend may be concealed by the crest. Drivers will be surprised by the sudden change in direction (Figure 7.46). The start of a bend is considered to be concealed if the point of the relevant change in direction is invisible 75 m before the start of the bend or change in curvature. This point is reached after a change in direction of at least 3.5 gon or – e.g. with large clothoid parameters – along a bend measuring max 100 m.

Blind spot graph

The quantitative monitoring factors that have been mentioned can be calculated automatically using suitable software modules. In order to be able to assess the safety-related shortcomings in a section of road to a better degree, the test results are shown in a feature graph: the blind spot graph (Figure 7.47).

The blind spot areas are marked black along this feature graph for each eye level. Figure 7.47 illustrates a critical blind spot between distance markers 4 + 620 and 4 + 680 in the opposite direction to the distance points.

Any sections of the road, where the depth of the blind spot is less than 0.75 m, are shown as gray in order to mark the threshold that is relevant for assessing the start of the bend between the visible and invisible areas.

The blind spot graph shows the monitoring factors for journeys in both directions (there and back). The distance from the eye level can be read upwards for the outward journey and downwards for the return journey.

Any concealed start to a bend is shown on the blind spot graph by a diagonal black line, which links the actual distance to the point of the relevant change of direction (3.5 gon) for each eye level marker point. This line starts 75 m before the start of the bend and ends at the point of the relevant change in direction. A concealed start to a bend is shown in Figure 7.47 between distance markers 4 + 840 and 4 + 960 going in the direction of the distance points. The relevant

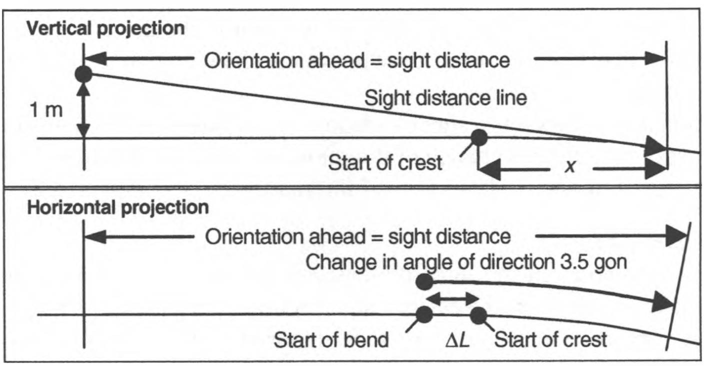


Figure 7.46: Model projection of a concealed start to a bend (HViSt)

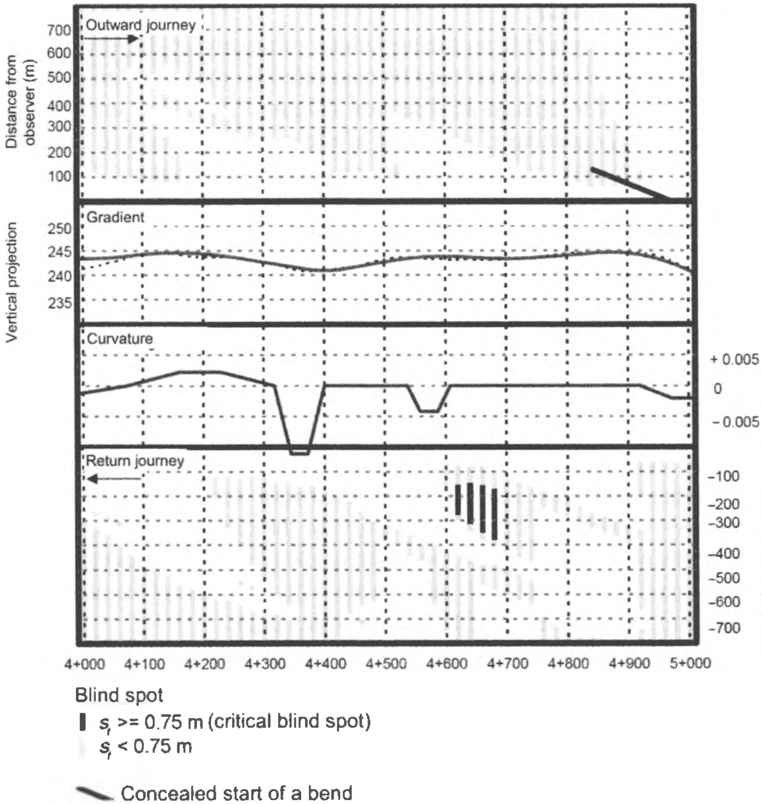


Figure 7.47: Blind spot graph with a critical blind spot and concealed start of a bend (HViSt)

change in direction of 3.5 gon (approx. 4 + 960) should be recognizable 75 m before the start of the clothoid (approx. 4 + 920). But at distance marker 4 + 840, the start of the invisible surface of the road is closer to the driver's eye level than the diagonal line that has been drawn, which shows the real distance to the relevant change in direction.

The line is not shown as a sign of adequate recognition of the start of the bend if the point of the relevant change of direction lies closer to the driver's eye level than the start of the invisible surface of the road.

Design shortcomings

Design shortcomings in the three-dimensional alignment are not safety-related and only take into account the design esthetics for a driving area. They also occur when the various design levels are superimposed and can only be recognized by using perspective images. The important design shortcomings are explained below (Table 7.10).

Judder

Judder occurs if the gradient follows brief elevations in the ground and is characterized by a fairly large number of changes in gradient with short tangent lengths (Figure 7.48). This gives drivers an uneven alignment and poses risks, especially when driving at night.

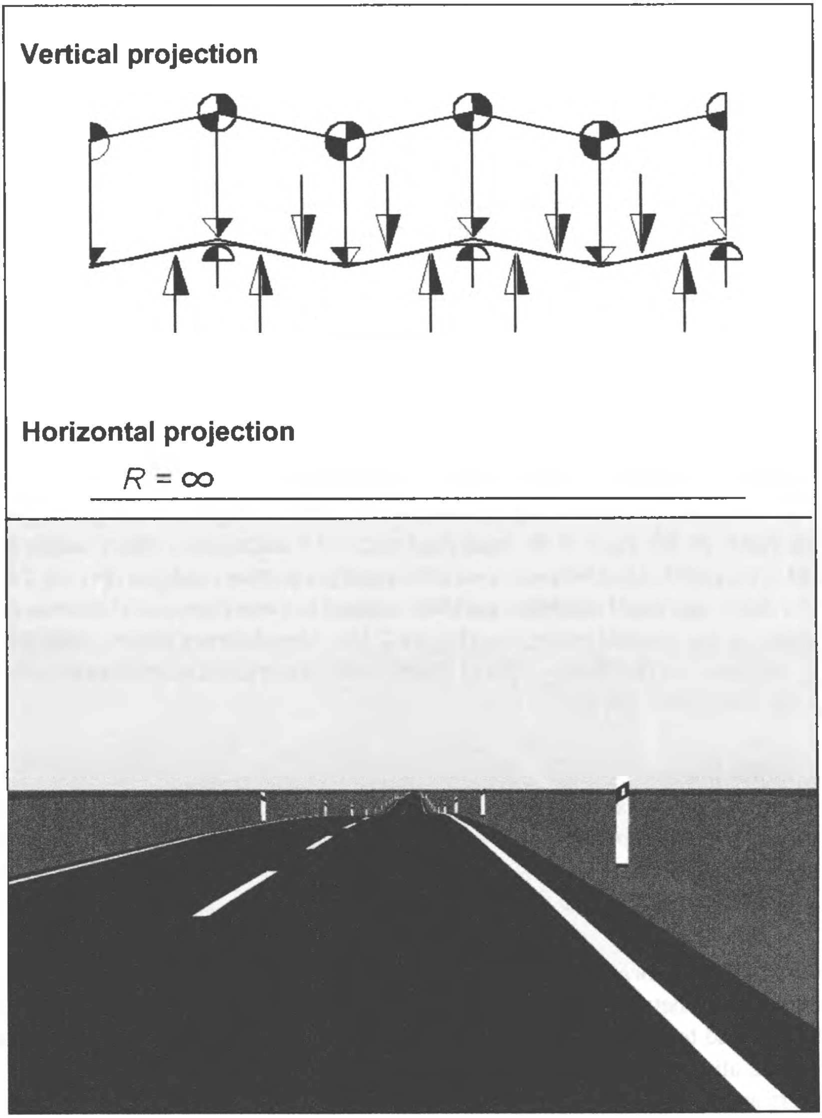


Figure 7.48: Judder (Kühn, et al., 2007)

Table 7.10: Design shortcomings

Design shortcoming	Causes in the alignment	Illustration
Judder	Short quadratic parabolas that follow each other in the vertical projection	
Sharp bend on the horizontal projection	Small rounding off radius between two straights in the horizontal projection	
Sharp bend on the vertical projection	Slight dip radius between two long straights in the vertical projection	
Bulge/flat spots	Short intermediate straight between two quadratic parabolas in the same direction in the vertical projection	

Sharp bend on the horizontal and vertical projection

Sharp bends are uneven changes in direction in the perspective image. Optical sharp bends on the edge of the road may occur if a short curve (short radius and slight arc length) exists between two horizontal projection straights (Figure 7.49) or if a short sag (small quadratic parabola radius) has been designed between two straights on the vertical projection (Figure 7.50). The observer's view point has a huge influence on its effect – optical sharp bends occur particularly where drivers can see a long way ahead.

Bulges/flat spots

Apparent bulges or flat spots occur if quadratic parabolas in the vertical projection (crests or sags) are connected by short intermediate straights. This effect is often seen on bridge-building work with a constant gradient. Figure 7.51 illustrates a flat spot in a crest. Figure 7.52 shows a bulge in a dip.

Contradictions between the road and its surroundings

Contradictions between a road and its surroundings may occur if elements at the side of a road follow a different line to the road itself. This may often be the case when the alignment is changed in various sections, e.g. when a bypass is built. Drivers are misled with regard to the actual course of the road and will possibly recognize a bend too late.

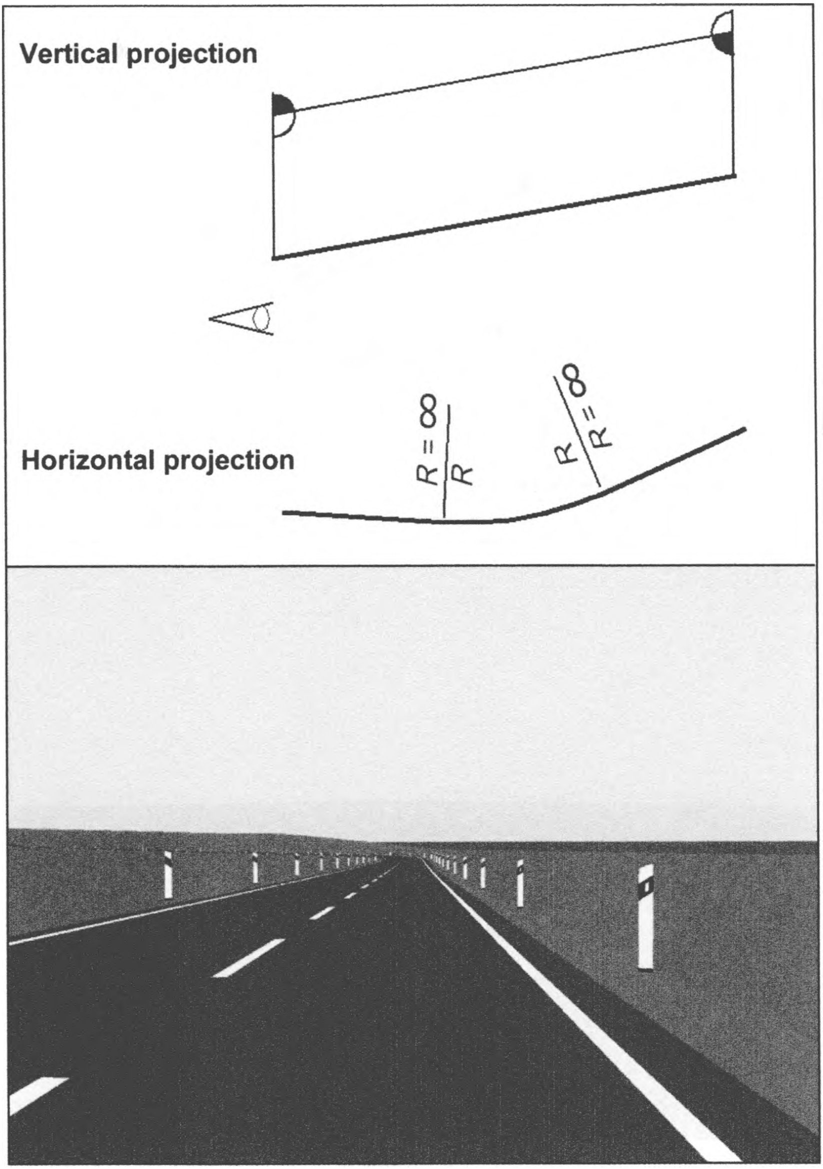


Figure 7.49: Sharp bend on the horizontal projection (Kühn, et al., 2007)

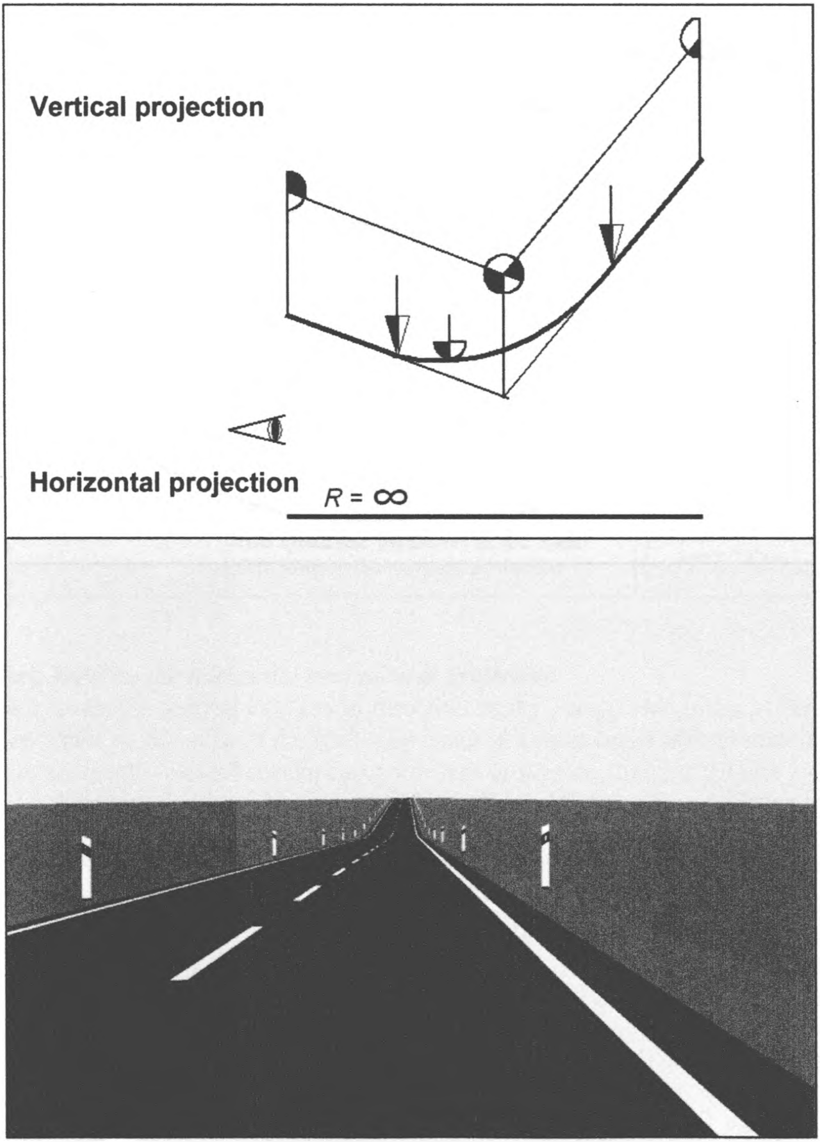


Figure 7.50: Sharp bend on the vertical projection (Kühn, et al., 2007)

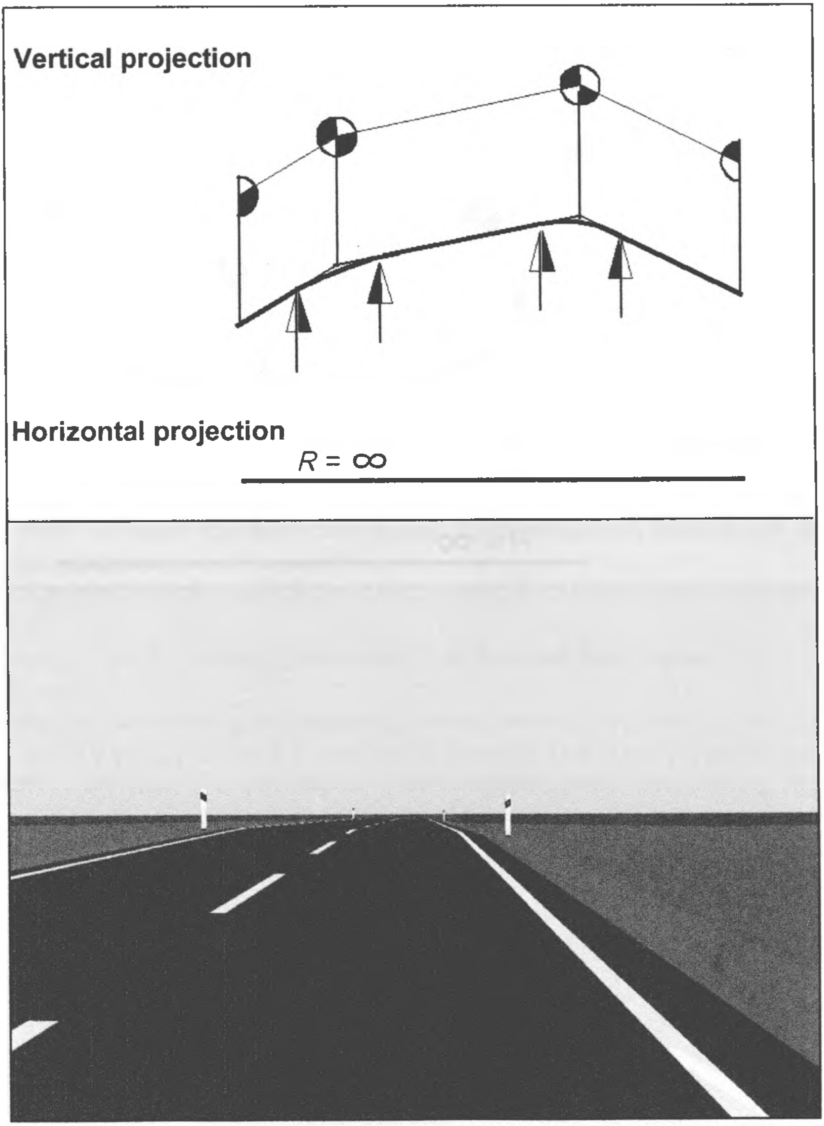


Figure 7.51: Flat spot (Kühn, et al., 2007)

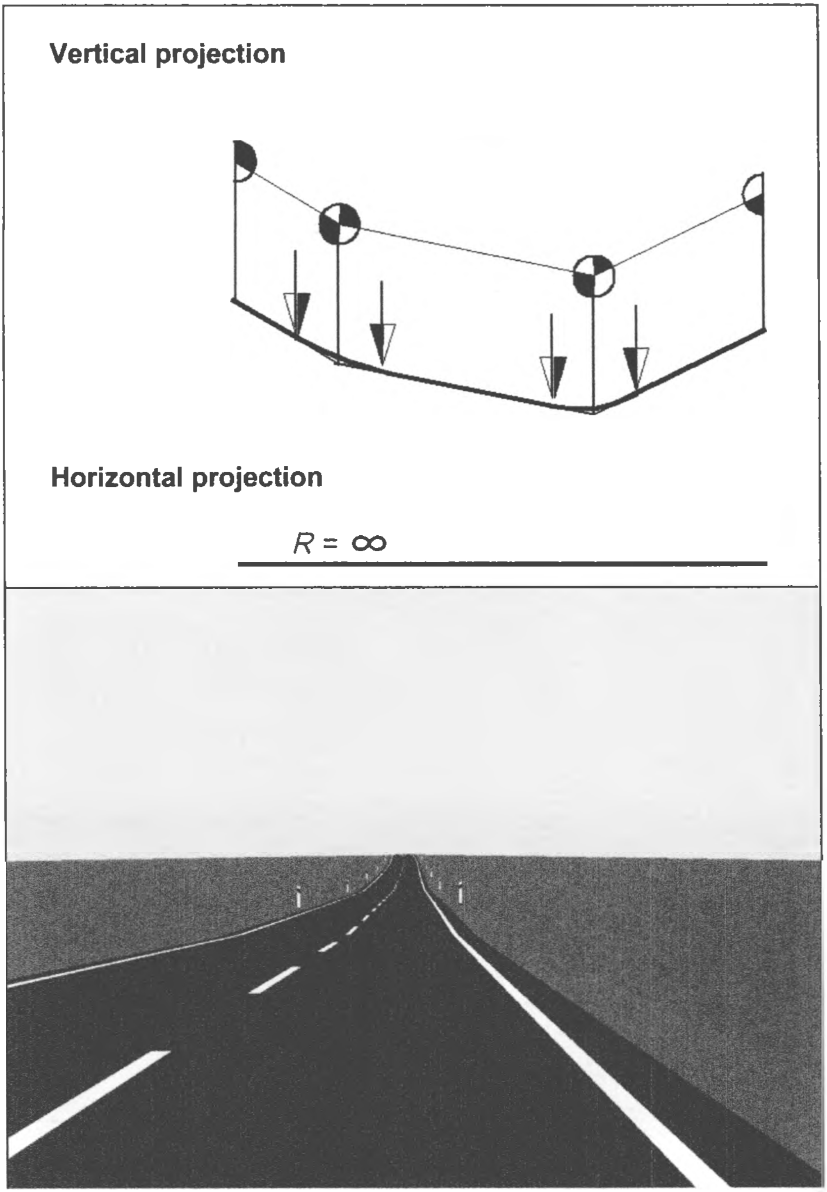


Figure 7.52: Bulge (Kühn, et al., 2007)

Design shortcomings may also be eliminated by altering the horizontal/vertical projection elements. But there is no single approach to achieve this.

7.4.4 Checking procedures

General issues

The starting point for a checking methodology involves calculating the route of the preferred option using a CAD program – that is to say, the axis data for the horizontal and vertical projections and the cross section are already available. Checks are made on the three-dimensional alignment for any shortcomings as part of a three-stage checking procedure. While design engineers normally carry out checks on standard three-dimensional elements (1st section) and design shortcomings (3rd section) manually and qualitatively, the 2nd section may take place numerically or quantitatively (checking for safety-related shortcomings).

Using an iterative process, the design engineer must alter the design elements on the horizontal and vertical projections in the areas concerned until the route no longer has any safety-related shortcomings and none or few of a design nature.

The checking procedure can be found in Figure 7.53 and is explained below.

Working stage 1: checking for sequences of standard three-dimensional elements

During the 1st working stage, design engineers check whether the calculated route can be fully subdivided into a sequence of standard three-dimensional elements. If this is the case, it is possible to proceed directly to the 2nd working stage. But if the engineers discover that sections of the route have an arbitrary sequence of elements, they have to check once again whether a sequence of standard three-dimensional elements can be produced by changing the design elements in the sections concerned. If this second attempt is not successful, the checking methodology used in the 2nd working stage is continued separately for sections with and without standard three-dimensional elements.

Working stage 2: checking for safety-related shortcomings

Checks

As it is impossible to completely rule out any shortcomings in the three-dimensional alignment, even when following a sequence of standard three-dimensional elements, a quantitative check of the whole route takes place during the 2nd working stage to ensure that there are no safety-related shortcomings.

If the complete route consists of nothing but a sequence of standard three-dimensional elements, only critical blind spots have to be calculated. The design elements can normally be corrected. If, on the other hand, there is no evidence

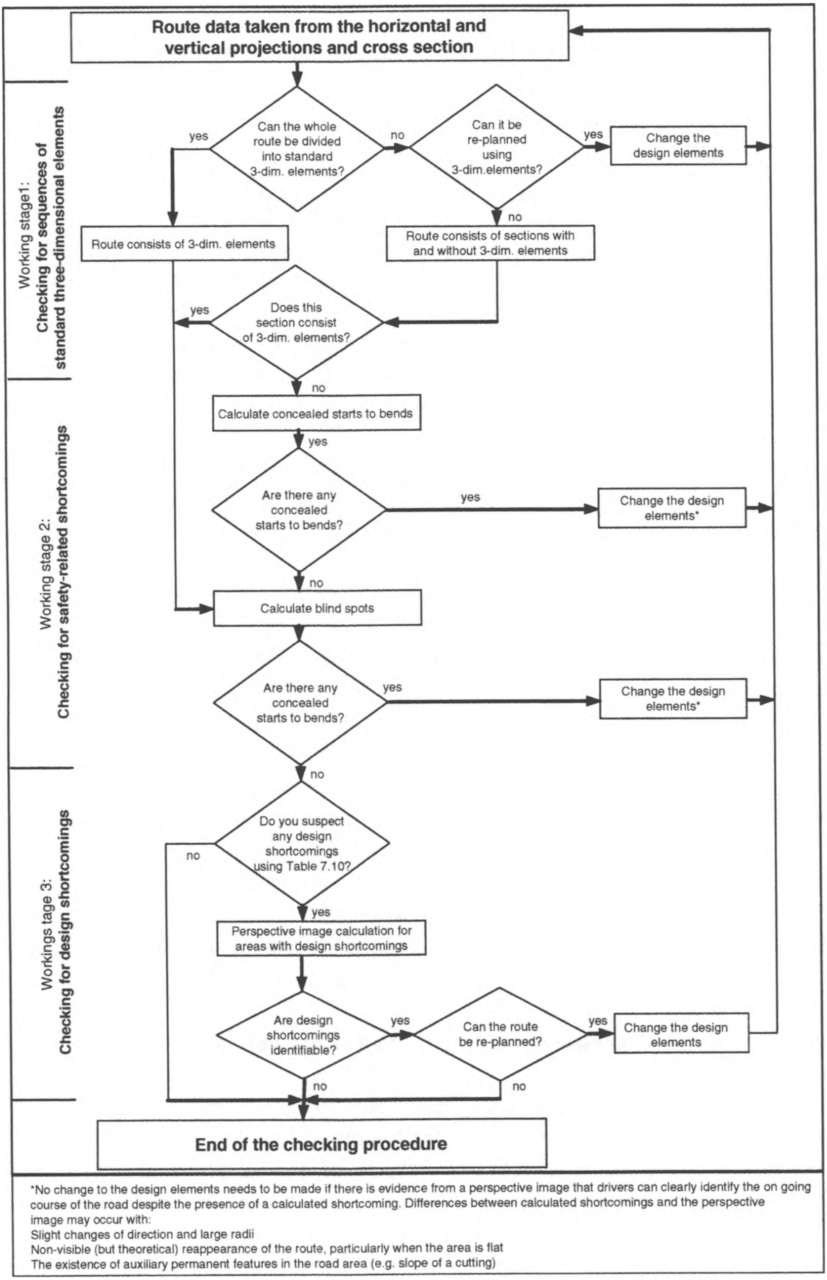


Figure 7.53: Procedure for checking the three-dimensional alignment (Kuhn, et al., 2009)

of critical blind spots as a result of the calculation work, it is possible to proceed immediately to the 3rd working stage.

But if the whole route consists of sections with a sequence of standard three-dimensional elements and sections without standard three-dimensional elements (arbitrary sequences of elements) because of restrictions in the surrounding area, a different approach is necessary. While the checks for critical blind spots take place as described for sections of the route with standard three-dimensional elements, checks have to be made on the concealed start of bends on sections of the route with an arbitrary sequence of elements. If critical blind spots and/or concealed starts to bends occur, the axis design must normally be corrected. Otherwise it is possible to proceed to the 3rd working stage.

Correction work

If safety-related shortcomings are found, it is possible to forego changes to the design elements if evidence is available from a perspective image that drivers can clearly identify the route despite the existence of a calculated shortcoming. Otherwise the shortcoming must be eliminated by changes to the design elements on the vertical/horizontal projections.

Critical blind spots

Critical blind spots can be eliminated by altering the design elements on the vertical or horizontal projections or a combination of both design levels.

As changes to the horizontal projection generally require greater effort and may also affect whether the plans are accepted by the authorities, attempts should first be made to exclude critical blind spots by making changes to the vertical projection.

It is possible to use the following procedures:

(1) Enlarge the quadratic parabola radius H_K of the crest
(2) Enlarge the quadratic parabola radius H_W of the sag
(3) Enlarge the quadratic parabola radius of the crest and sag as part of an iterative process (take care to balance the amount of earth removed)
(4) Alter the longitudinal gradient s
(5) Move the gradient intersection points TS.

A combination of the procedures listed above is normally used.

Concealed starts of bends

Concealed starts of bends can be avoided in principle if the route uses standard three-dimensional elements. In order to guarantee that the crucial change in angle can be identified, it is necessary to move the start of the crest behind the start of the bend (in the direction of travel). When designing a route with an arbitrary sequence of elements, attempts must be made to eliminate the shortcoming by varying the horizontal and vertical projection elements. It has proved effective in the past to enlarge the length of the circular arc and the clothoid parameter or, if possible, replace the circular arc with a straight. As changes to elements on the horizontal projection normally affect elements on the vertical projection, care must be taken to ensure that no new shortcomings are created in adjacent areas.

Working stage 3: checking for design shortcomings

If there are no more safety-related shortcomings along the whole route in the sections with standard three-dimensional elements or an arbitrary sequence of elements, the final check for design shortcomings takes place.

The design engineer now faces the task of checking the whole route for any possible design shortcomings. This qualitative process takes place on the basis of the listed causes with the aid of perspective images. If design shortcomings are identified and can be corrected taking into account the existing guidelines, the course of the route should be changed.

7.4.5 Example

Checking for sequences of standard elements

The checks on the route for shortcomings in the three-dimensional alignment presuppose that the engineer has access to geometrical data regarding the axis, gradient and cross section and data on the surrounding terrain. The selected route was first checked for standard three-dimensional elements (SRE). After re-planning work took place, when the starting and finishing points of design elements in the horizontal and vertical projections were harmonized, the route then consisted of a section of 4,000 m with SRE and a section of 3,700 m without SRE.

Checking for safety-related shortcomings

Six safety-related shortcomings were detected as a result of the quantitative checks and their relevance was checked according to Section 7.4.3. Four of the six shortcomings are relevant and have to be eliminated by re-planning work (Table 7.11). Perspective images were used to prove the irrelevance of the other two shortcomings.

The following text explains how shortcomings in the shape of the concealed start to the bend at distance marker 2 + 700 (return journey) and the critical blind spot from distance marker 3 + 960 (return journey) can be eliminated.

Table 7.11: Numerically detected safety-related shortcomings

Outward journey	Shortcoming	Assessment
2 + 550	Concealed start of bend	Relevant
Return journey	**Shortcoming**	**Assessment**
6 + 920	Blind spot (dip)	Relevant
4 + 800	Blind spot	Irrelevant, as only occurs for short time at one distance marker
3 + 960	Blind spot (jump)	Relevant
2 + 700	Concealed start of bend	Relevant
0 + 950	Blind spot (jump)	Irrelevant, as only occurs for short time at one distance marker

Concealed start to the bend at 2 + 700 (return journey)

The blind spot graph and the relevant perspective image (Figure 7.54) clearly show that a concealed start to a bend is present at marker 2 + 700 or the course of the bend is not clearly visible. The perspective image at distance marker 2 + 550 (Figure 7.55) clearly shows that this is a right-hand bend.

In order to eliminate this shortcoming, the crest radius is enlarged from $H_K = 5{,}000\,\text{m}$ to $H_K = 10{,}000\,\text{m}$ and the gradients are reduced from $s_l = 5.5\%$ and $s_2 = -4.0\%$ to $s_l = 4.5\%$ and $s_2 = -3.2\%$. As these changes were insufficient to eliminate the shortcoming after a further check on the blind spot graph, the radii were also enlarged from $R_l = 550\,\text{m}$ to $R_l = 1.000\,\text{m}$ and $R_2 = 800\,\text{m}$ to $R_2 = 1.000\,\text{m}$ on the horizontal projection. The blind spot line that has been calculated also shows a concealed start to a bend, but the newly calculated perspective image shows that the change in direction is clearly visible to drivers, who follow the course of the shoulder (Figure 7.56). This means that drivers can identify the bend and no shortcoming exists.

Jump at 3 + 960 (return journey)

The blind spot graph shows a critical blind spot from distance marker 3 + 960 to 3 + 880 (return journey) (Figure 7.57). The course of the road cannot be clearly identified according to the appropriate perspective image. So the critical blind spot has to be eliminated by renewed planning work.

By altering the gradients $s_l = -4\%$ to $s_l = -3.2\%$ and from $s_2 = 2\%$ to $s_2 = -1.4\%$ and altering the sag radius from $H_W = 6.500\,\text{m}$ to $H_W = 25.000\,\text{m}$ and $H_K = 6.000\,\text{m}$ to $H_K = 9.000\,\text{m}$, the gradients were harmonized. After further calculation work, the critical blind spot only covered 40 m and was therefore no longer relevant (Figure 7.58).

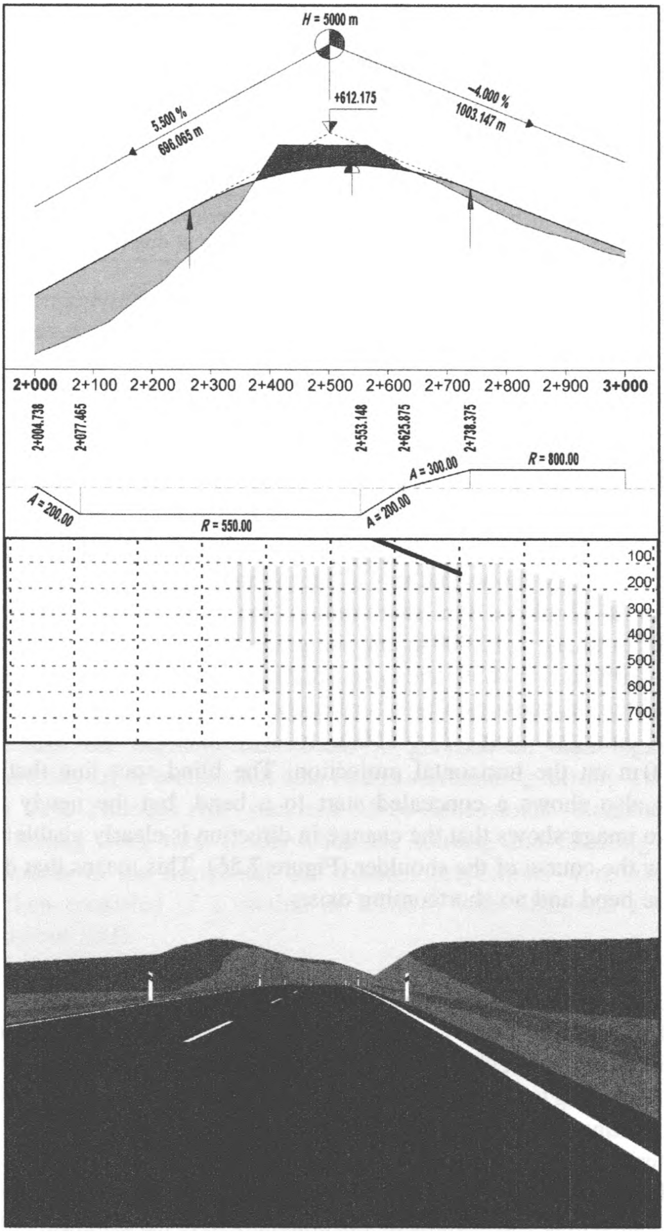


Figure 7.54: Section between distance marker 2 + 000 and 3 + 000 before re-planning work on the concealed start to the bend at distance marker 2 + 700 (return journey)

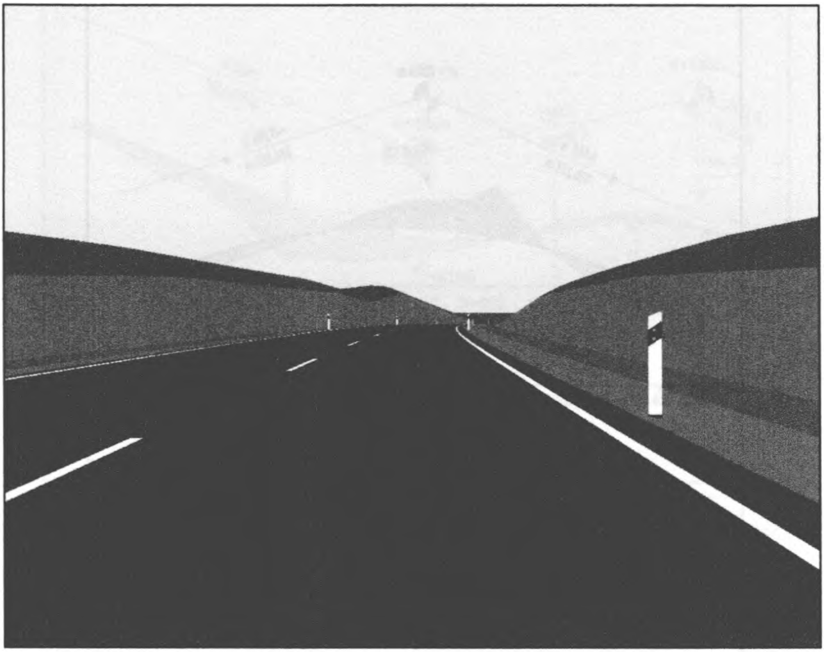


Figure 7.55: Distance marker 2 + 550 (return journey), the course of the bend is visible

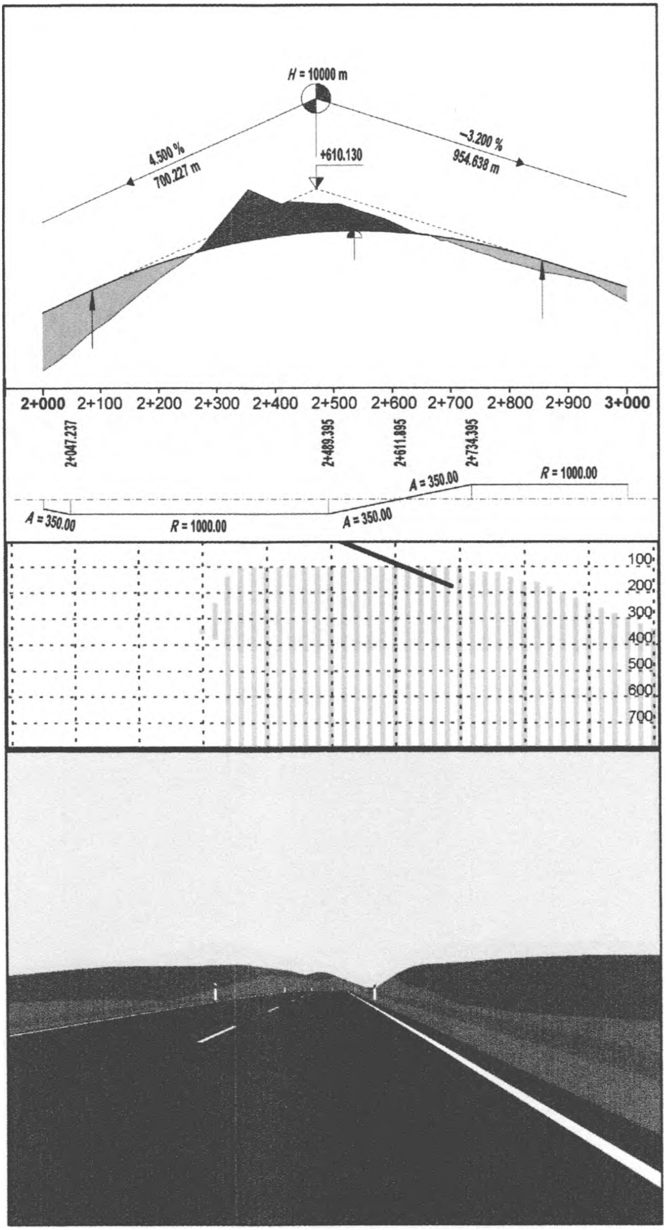


Figure 7.56: Section between distance markers 2 + 000 and 3 + 000 after re-planning work (return journey); the course of the bend is visible using the line of the shoulder

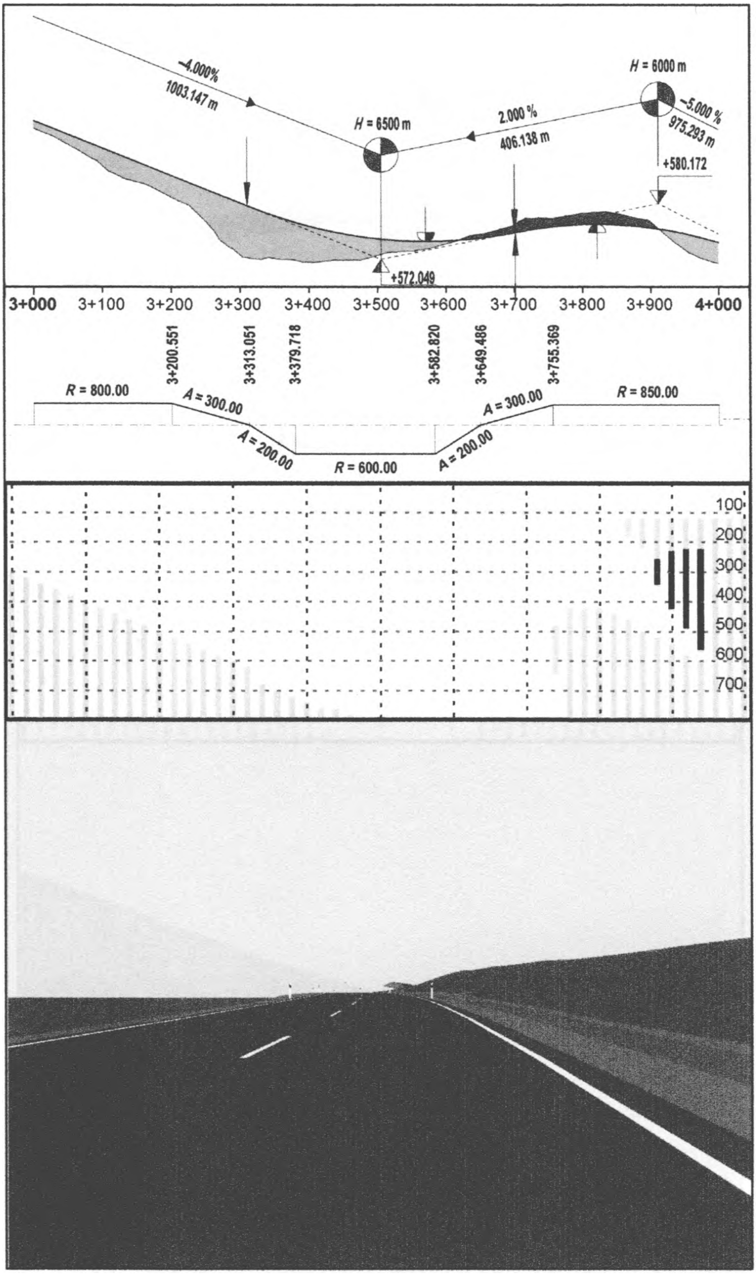


Figure 7.57: Section between 3 + 000 and 4 + 000 before re-planning work, jump at 3 + 950 (return journey)

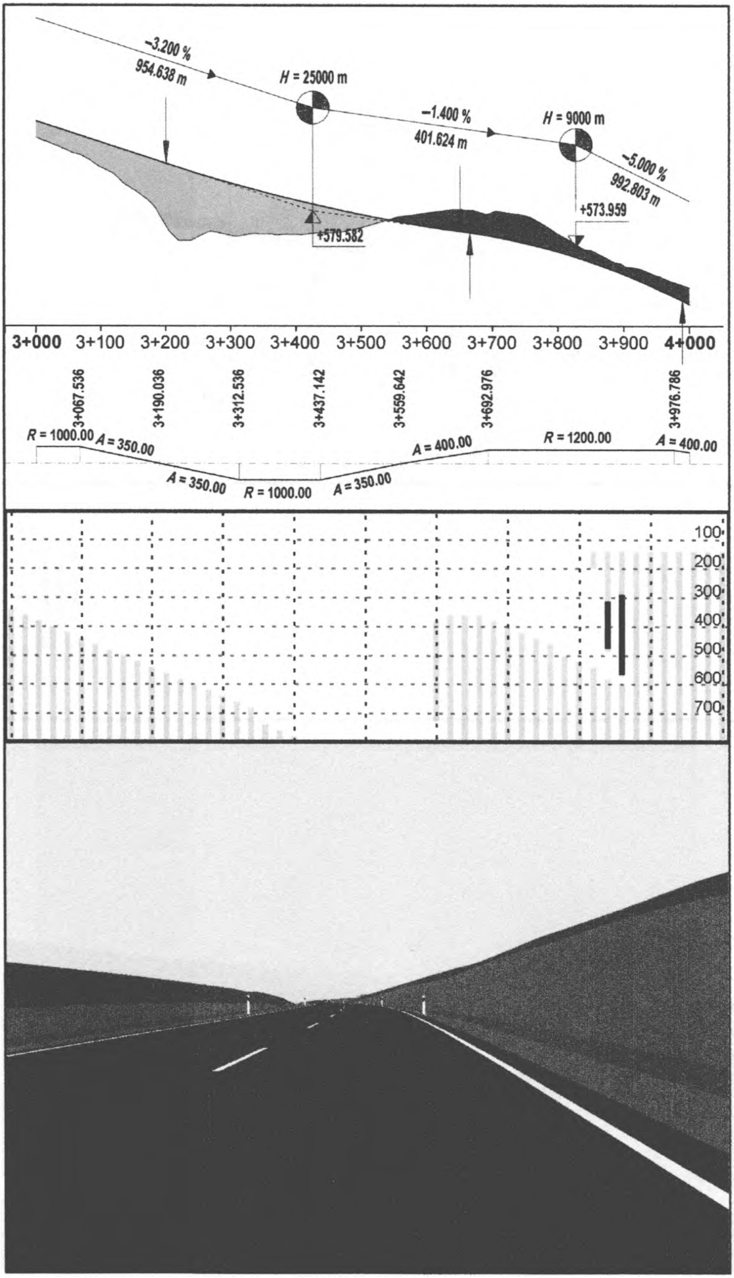


Figure 7.58: Section between 3 + 000 and 4 + 000 after re-planning work (return journey)

7.5 Questions

(1) Which design levels are used to design a road?
(2) Name the design elements on the horizontal and vertical projections.
(3) Why were clothoids introduced as a transitional curve for road design?
(4) What are the functions of a clothoid as a design element on the horizontal projection?
(5) Draw and explain the curvature graph of a combined curve.
(6) Name and justify the criteria for using an apex clothoid.
(7) Explain the differences between a crest, sag, hump, and dip.
(8) What is the gradient of a road and which criteria need to be taken into consideration when shaping the gradient?
(9) Why do we describe a quadratic parabola on the vertical projection as a quadratic parabola and not as a radius?
(10) What do we understand by the three-dimensional alignment of a road?
(11) Name the general criteria for assessing the three-dimensional alignment.
(12) What do you understand by the terms "safety-related" and "design" shortcomings?

Chapter 8

Cross Section

8.1 Constituent Parts

The cross section of a road is a vertical cut at right angles to the road axis. It divides in a transverse direction the driving area, which is used by road users (vehicle drivers, cyclists, pedestrians), and is composed of the following (Figure 8.1):

- the driving area earmarked for traffic,
- the safety areas to the side and above that are to be kept clear of fixed objects.

The basic measurements for an assessment vehicle in Germany consist of a truck that is 2.50 m wide and 4 m high, although the permissible width of an individual vehicle has been raised to 2.55 m (2.60 m in special cases) as part of the European harmonization process.

Depending on the way that selected combinations of vehicles meet, travel alongside and pass each other, the RAS-Q sets the following standard dimensions (Table 8.1).

8.2 Terms

Unified standard cross sections have been introduced to meet the demands of road categorization and provide some comparability:

- Standard cross section:
  The elements of the road cross section are set in a standard cross section with particular geometrical dimensions, technical details, and the appropriate design for the paving. This is typical for sections of a road that belong together.
- Civil engineering cross section
  The engineering cross section shows the geometrical dimensions and the civil engineering details at the selected distance marker.

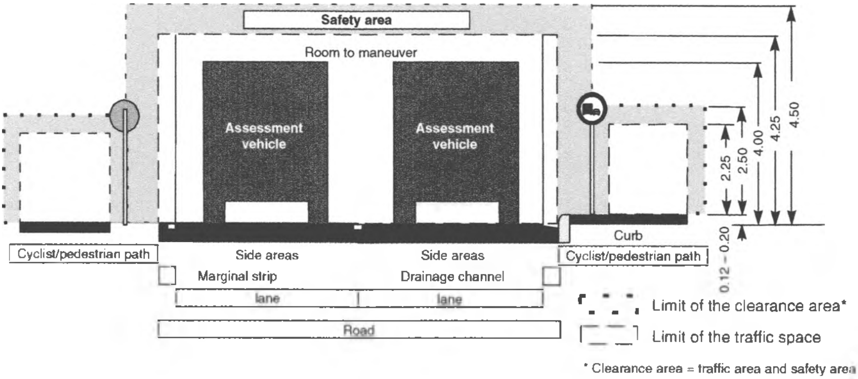


Figure 8.1: Elements in the cross section of a road according to RAS-Q

– Transverse section
  The transverse profile shows the height of the road with territory without the civil engineering details at a particular distance marker.

Table 8.1: Standard dimensions for clearance on roads according to RAS-Q

Parameter	Dimension	Type of traffic		
		Vehicle	Bike	Pedestrian
Initial dimensions – width	[m]	2.50	0.80	0.75
Width of room to maneuver at side	[m]	Depending on standard cross section 0.25–1.25	0.1	–
Width of room to maneuver at side S_S	[m]	1.25* 1.00** 0.75***	0.25	–
Initial dimensions – height	[m]	4.00	2.00	2.00
Height of room to maneuver	[m]	0.25	–	–
Height of traffic area	[m]	4.25	2.25	2.25
Height of upper safety area S_O	[m]	0.25	0.25	0.25
Height of clearance	[m]	4.50 (4.70 with high installations)	2.50	2.50

* For $v_{zul} > 70$ km/h.
** For $50 < v_{zul} \leq 70$ km/h.
*** For $v_{zul} \leq 50$ km/h.

8.3 Determining Factors and Principles

The following key determining factors must be noted when designing the cross section:

- Area planning specifications
  Requirements as a result of travel speeds and traffic quality, taking into consideration a road's network function
- Traffic safety
  Safety of individual vehicles without any hindrances to the traffic flow
- The cost of building, maintaining, and operating the road
- Effects on environmental protection

The following principles should be noted when setting the cross section elements and using the standard cross section approach:

- Taking into account the desirable driving behavior on the road network, roads of the same category should have standard cross sections that are as similar as possible.
- The standard cross section selected must guarantee the desirable assessment speed (average car cruising speed) and therefore the desirable traffic quality.
- Speed restrictions or differences in speed between types of vehicles (trucks, cars) must be taken into account.
- If possible, the different types of traffic should be separated (vehicles, cyclists, and pedestrians).
- For traffic safety reasons, the standard cross section should be maintained for a fairly long section of the road.

8.4 Elements

In principle, a distinction is made between the following cross section elements (Figure 8.2):

- Traffic lanes that are suitable for road traffic and pedestrians, normally paved.
- Side areas not suitable for traffic, normally not paved.

Figure 8.3 illustrates the cross section elements for an undivided highway, a two-lane rural road by way of example.

The cross section elements for a divided highway or freeway, four-lane rural road can be derived from Figure 8.4.

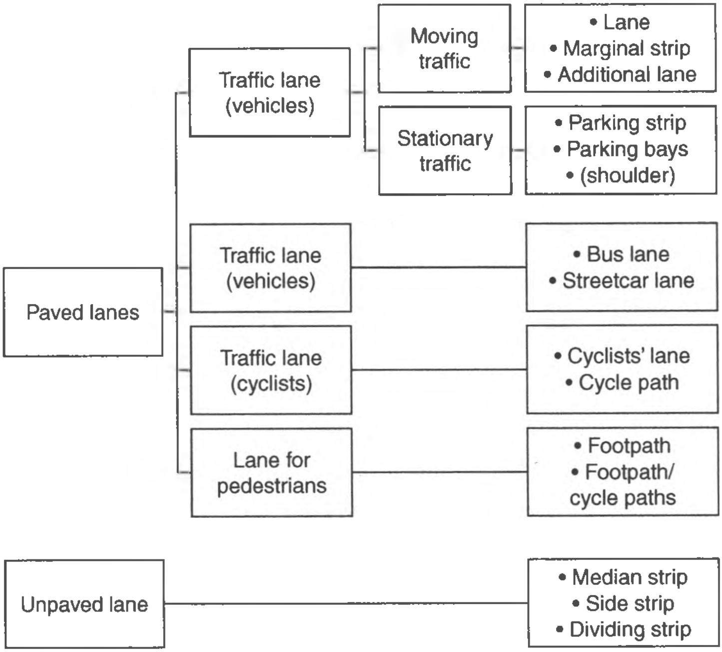


Figure 8.2: Elements of a road cross section

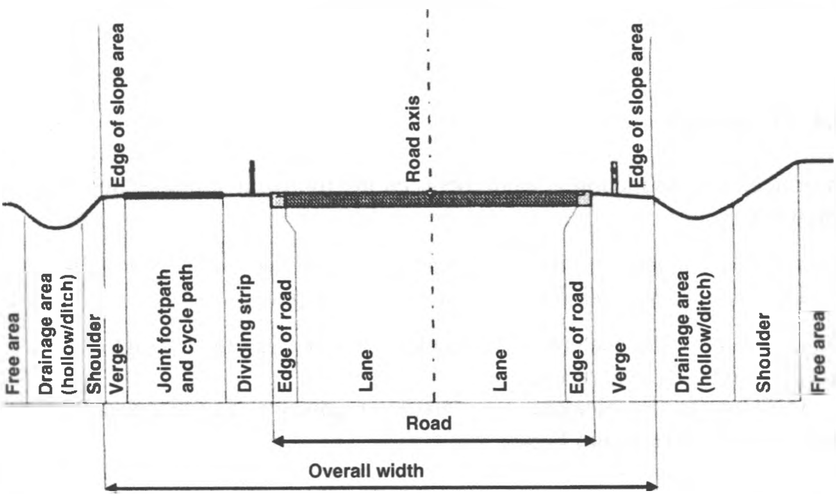


Figure 8.3: Cross section elements of an undivided highway with two lanes (rural road)

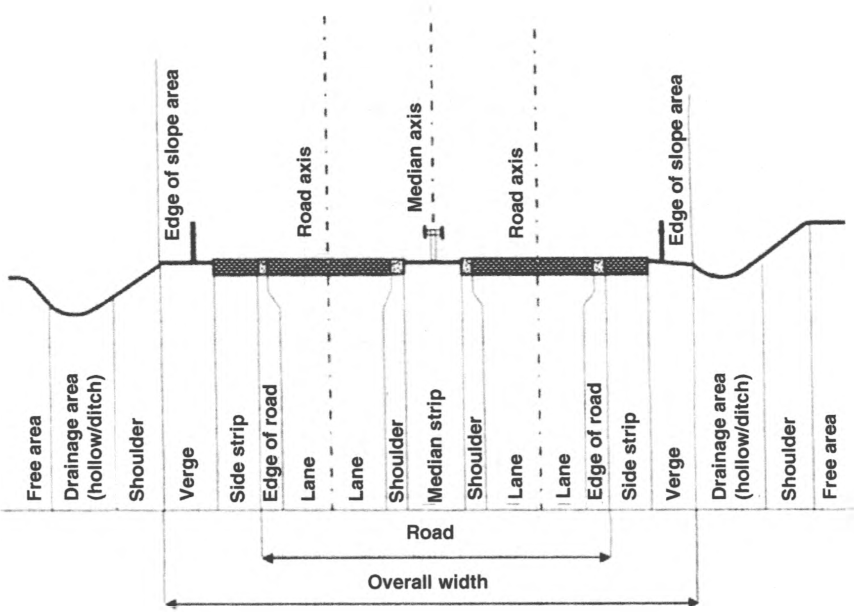


Figure 8.4: Cross section elements of a divided highway or freeway with four lanes (rural road)

8.5 Standard Cross Sections

Depending on its function, the assessment speed and the amount of traffic using the road, the widths of the individual cross section elements (lane, edge of road, separating strip, etc.) are selected and put together to form an overall cross section (standard cross section) (Table 8.2).

In principle, a distinction is made between standard cross sections for undivided highways (Figure 8.5) and divided highways or freeway (Figure 8.6) when designing the cross section. While overtaking is normally carried out by using the opposite road lane on an undivided highway, one-way traffic is the norm on divided highways.

The standard cross section RQ 15.5 of the type “2 + 1” has proved its worth as a so-called intermediate cross section and allows alternating overtaking on the second lane that has been built.

Traffic safety and the quality of traffic flow can be significantly improved as a result of regulated overtaking maneuvers. So for practical applications, there is an intermediate cross section available between the undivided and divided highway or freeway cross sections.

Table 8.2: Standard cross sections according to RAS-Q

	Standard cross section								
	RQ 7.5	RQ 9.5	RQ 10.5	RQ 15.5	RQ 20	RQ 26	RQ 29.5	RQ 33	RQ 35.5
Number of lanes	2	2	2	2 + 1	4	4	4	6	6
Lane width [m]	2.75	3.00	3.50	3.75/ 3.25/ 3.50	3.25	3.50	3.75	3.50	3.75/ 3.50
Width of edge of road [m]	–	0.25	0.25*	0.25	0.50	0.50	0.75	0.50	0.75/ 0.50
Median strip width [m]	–	–	–	**	2.00	3.00	3.50	3.00	3.50
Hard or paved shoulder width [m]	–	–	–	–	–	2.00	2.50	2.00	2.50
Side strip width [m]	1.00	1.50	1.50	2.50***/ 1.50	1.50	1.50	1.50	1.50	1.50
Side strip width [m]°	1.25	1.75	1.75	1.75	1.75	3.00	3.00	3.00	3.00

* The side areas must be designed with a width of 0.50 m if more than 900 trucks/24 h are expected on the road.

** Double line with 0.50 m gap.

*** The verge next to single-lane sections must be stable (e.g. with drainage gravel).

° If footpaths and cycle paths are present.

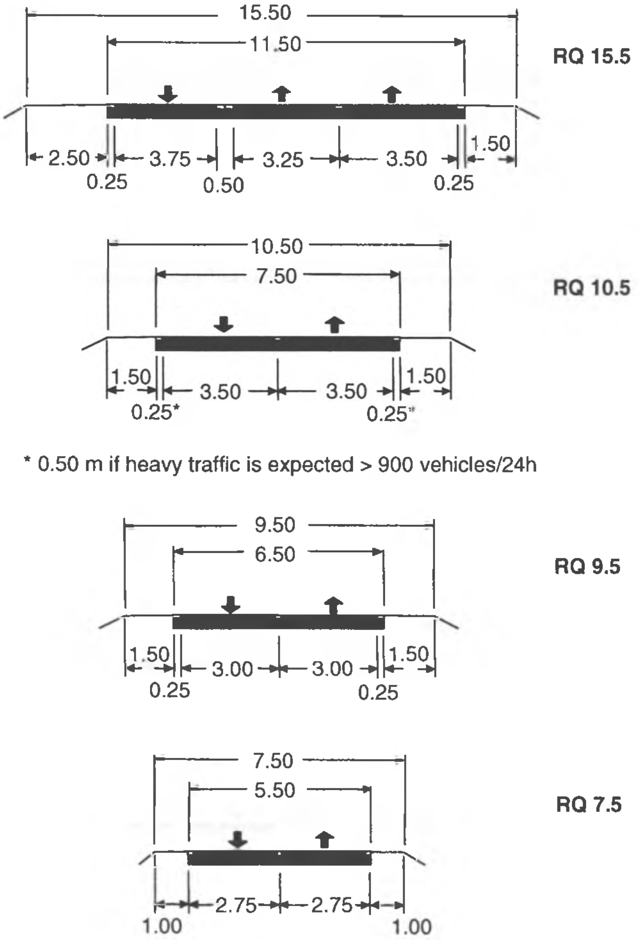


Figure 8.5: Standard cross section for undivided highways without any extra lanes according to RAS-Q [Dimensions in meter]

8.6 Selecting a Standard Cross Section

8.6.1 Working stages

The cross section is selected in three stages:

- **Preselection (working steps 1–3)**
  One or more cross sections, which will probably be suitable for the application, are selected according to Figure 8.7, depending on the road category and the forecast volume of traffic.

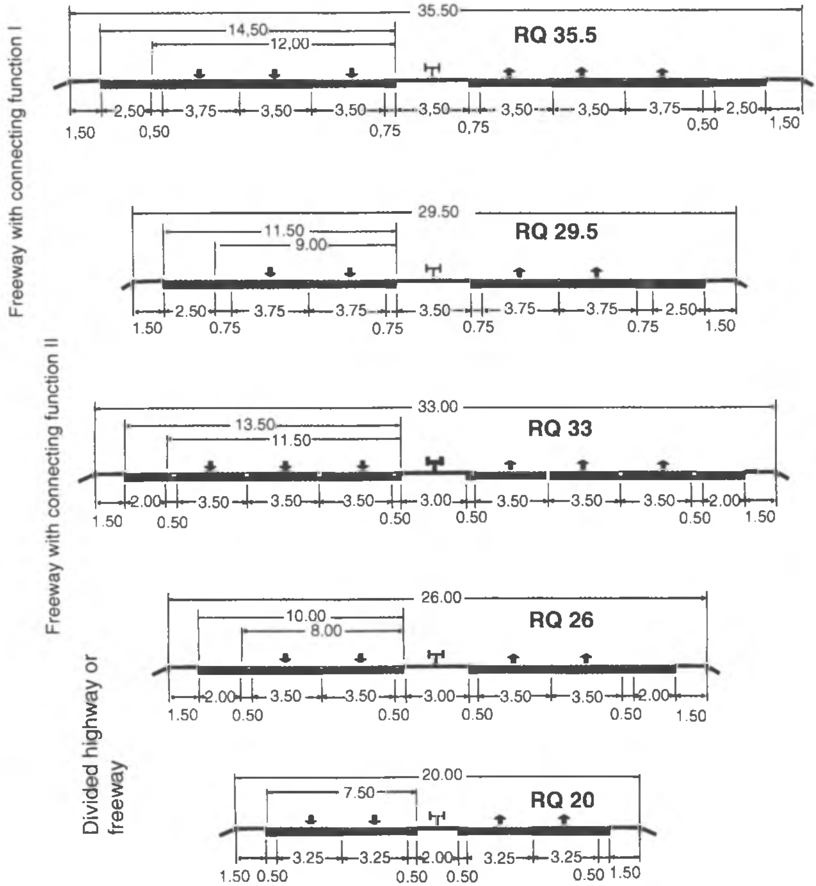


Figure 8.6: Standard cross sections for divided highways or freeway without any lanes according to RAS-Q [Dimension in meter]

- **Evidence of the traffic quality (working steps 4–9)**
  Check whether the set traffic quality level (at least quality level D is required for new building work) can be achieved with the standard cross section that has been selected for the whole route.
- **Evidence of traffic safety**
  If several standard cross sections are feasible, the levels of traffic safety achieved must be compared.

Figure 8.7 illustrates this process.

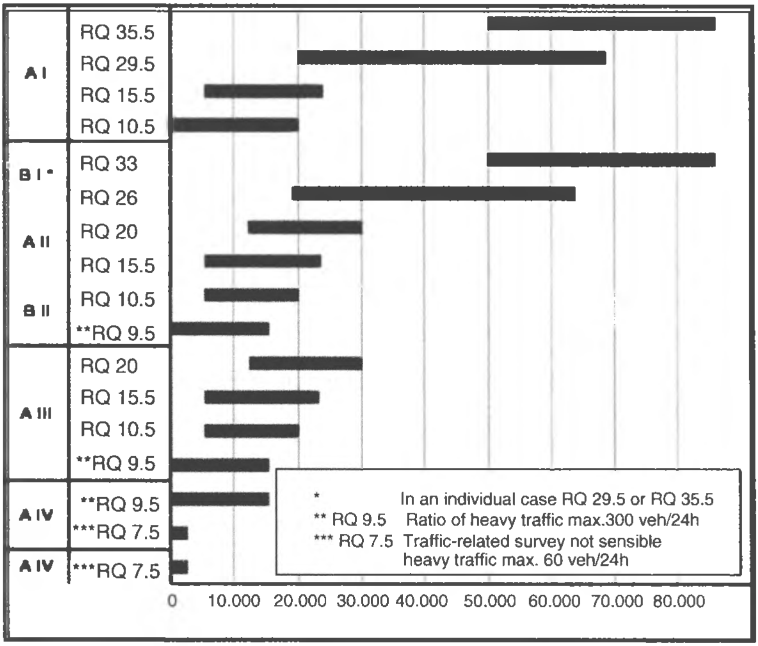


Figure 8.7: Preselection of the standard cross section (RAS-Q)

8.6.2 Example

Once the design work has been completed on the horizontal and vertical projections for a rural road between A and B, a suitable cross section must be selected and evidence of the quality of traffic flows and traffic safety must be provided (HBS, 2001).

Input:

– Given that:

• Alignment of the road (curvature graph, gradient):	horizontal and vertical projections
• Traffic volume:	average daily traffic = 13,000 veh/24h
• Ratio of heavy traffic:	$Sv = 12\%$
• Desirable traffic quality level:	D

– Find: Is traffic quality level D achieved?

Solution:

Preselection

1st working step:

Rural roads with a national/regional connecting function:

→ Road category: A II

→ Crusing speed: $v_B = 60\,\text{km/h}$

2nd working step:
Assess hourly traffic volumes:
$q_B = 0.1$ to $0.13 *$ average daily traffic (DTV) (two-lane rural road)

Selected: $q_B = 0.10 * \text{DTV}$
$q_B = 0.10*\text{DTV} = 0.1*13{,}000$ veh/h
$q_B = 1{,}300$ veh/h

3rd working step:

Road category	:	AII
Traffic volume	:	DTV = 13,000 veh/24h
Ratio of heavy traffic	:	$s_v = 12\%$

Preselection of the standard cross section is given in Figure 8.7.
Selected: RQ = 10.5
Evidence of traffic quality

4th working step:
Subdivide road section into partial areas each with a homogeneous characteristic for the determining factors: assessed traffic volume, longitudinal gradient, general curvature, and passing opportunities (see Figure 8.8).

5th working step:
Determine the gradient classes for each partial section from the speed-path diagram of the heavy assessment vehicle (see Figure 8.9)

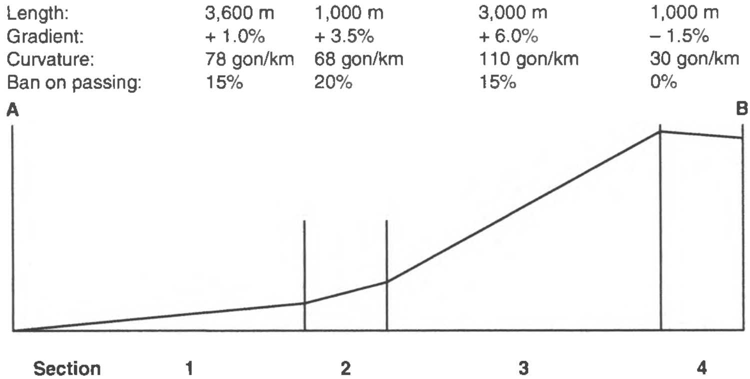


Figure 8.8: Forming section

Section 1: Starting figure: 50 km/h
Final speed after 3.6 km, 80 km/h
Average speed: 65 km/h
Section 2: Starting speed: 80 km/h
Final speed after 1 km, 54 km/h
Average speed: 67 km/h

Section 3: Starting speed: 54 km/h
Final speed after 3 km, 28 km/h
Average speed: 41 km/h
Section 4: Starting speed: 28 km/h
Final speed: 80 km/h
Average speed: 54 km/h

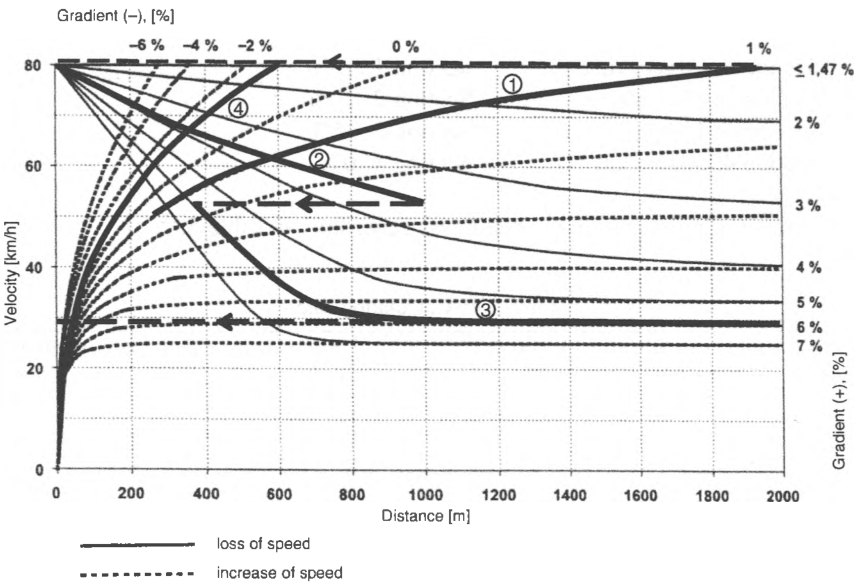


Figure 8.9: Determining the starting and finishing speeds for each section

Determine the gradient classes from the average speed of the assessment vehicle: according to Table 8.3:

Table 8.3: Gradient class

Lowest average speed of the assessment vehicle [km/h]	Gradient class
>70	1
55–70	2
40–55	3
30–40	4
<30	5

Section 1: 65 km/h – > gradient class 2
Section 2: 67 km/h – > gradient class 2
Section 3: 41 km/h – > gradient class 3
Section 4: 54 km/h – > gradient class 3.

6th working step:
Calculate the total from the curvature of the route and the allowance on account of route sections with a ban on passing.
Given that:

Bendiness:

Section 1: 78 gon/km
Section 2: 68 gon/km
Section 3: 110 gon/km
Section 4: 30 gon/km

Parts of route with ban on passing:

Section 1: 15%
Section 2: 20%
Section 3: 15%
Section 4: 0%

Determining the allowance for bendiness according to Table 8.4.

Table 8.4: Calculation rule

Parts of route with overtaking ban [%] $A_{ÜVB} = \frac{L_{ÜVB}}{L} * 100$	Allowance for bendiness [gon/km]
0–30	$5 * A_{ÜVB}$
30–100	$150 + (A_{ÜVB} - 30)/0{,}7$

– Allowance for bendiness: Section 1: $5*15 = 75$ gon/km
Section 2: $5*20 = 100$ gon/km
Section 3: $5*15 = 75$ gon/km
Section 4: $5*0 = 0$ gon/km
– Total = bendiness or route + allowance for curvature
Section 1: $78 + 75 = 153$ gon/km
Section 2: $68 + 100 = 168$ gon/km
Section 3: $110 + 75 = 185$ gon/km
Section 4: $30 + 0 = 30$ gon/km.

7th working step:
Select the appropriate traffic volume/speed diagram and determine the car speed that can be reached:

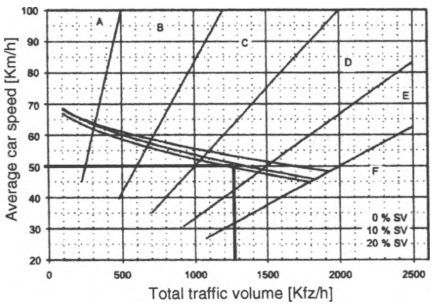


Figure 8.10: Section 1

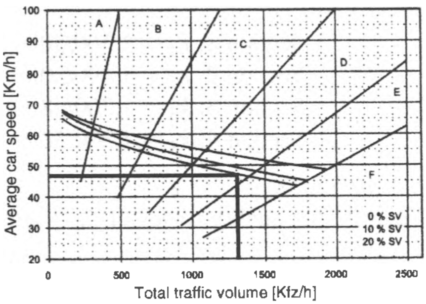


Figure 8.11: Section 2

Section 1: Gradient class 2
Sv ratio: 12%
Traffic volume: 1,300 veh/h
Bendiness: 153 gon/km
$\rightarrow v_{R,1} = 50$ km/h

Section 2: Gradient class 2
Sv ratio: 12%
Traffic volume: 1,300 veh/h
Bendiness: 168 gon/km
$\rightarrow v_{R,2} = 48$ km/h

Section 3: Gradient class 3
Sv ratio: 12%
Traffic volume: 1,300 veh/h
Bendiness: 185 gon/km
$\rightarrow v_{R,3} = 48$ km/h

Section 4: Gradient class 3
Sv ratio: 12%
Traffic volume: 1,300 veh/h
Curvature: 30 gon/km
$\rightarrow v_{R,4} = 58$ km/h

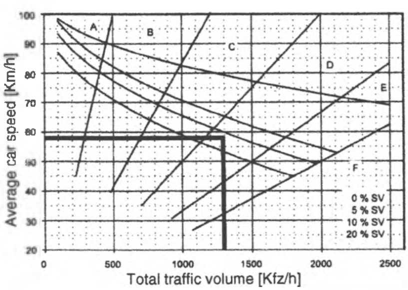


Figure 8.12: Section 4

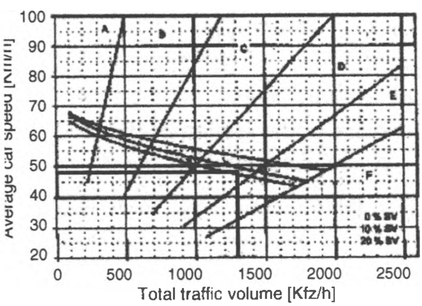


Figure 8.13: Section 3

8th working step:
Calculate the traffic density k_i where $k_i = q_{B,i}/v_{Ri}$

Given that: Section 1: $q_{B,1} = 1{,}300$ veh/h, $v_{R,1} = 50$ km/h
Section 2: $q_{B,1} = 1{,}300$ veh/h, $v_{R,2} = 48$ km/h
Section 3: $q_{B,3} = 1{,}300$ veh/h, $v_{R,3} = 48$ km/h
Section 4: $q_{B,4} = 1{,}300$ veh/h, $v_{R,4} = 58$ km/h

Calculated:

Section 1: $k_1 = 1{,}300/50 = 26.0$ veh/km
Section 2: $k_2 = 1{,}300/50 = 26.0$ veh/km
Section 3: $k_3 = 1{,}300/48 = 27.1$ veh/km
Section 4: $k_4 = 1{,}300/58 = 22.4$ veh/km

9th working step:
Calculate the quality stages of the traffic for the individual part sections:

Traffic density: Section 1: $k_1 = 26.0$ veh/km
Section 2: $k_2 = 26.0$ veh/km
Section 3: $k_3 = 27.1$ veh/km
Section 4: $k_4 = 22.4$ veh/km

Quality level: Section 1 = D
Section 2 = D
Section 3 = D
Section 4 = D

Table 8.5: Calculating the traffic level quality

Level	Traffic density k [veh/km]
A	≤ 5
B	≤ 12
C	≤ 20
D	≤ 30
E	≤ 40
F	> 40

10th working step:
Summary of the section of the road

– Average car cruising speed

Given that: Section 1 $v_{R,1} = 50$ km/h, $L_1 = 3.6$ km
Section 2 $v_{R,2} = 50$ km/h, $L_2 = 1.0$ km
Section 3 $v_{R,3} = 48$ km/h, $L_3 = 3.0$ km
Section 4 $v_{R,4} = 58$ km/h, $L_4 = 1.0$ km

$$v_R = \frac{L}{\sum_{i=1}^{n} \frac{L_i}{v_{R,i}}} = \frac{8.6}{\frac{3.6}{50} + \frac{1.0}{50} + \frac{3.0}{48} + \frac{1.0}{58}} = \frac{8.6}{0.172} = 50 \text{ km/h} \tag{8.1}$$

– Average traffic density

Given that: Section 1: $k_1 = 26.0$ veh/km, $L_1 = 3.6$ km
Section 2: $k_2 = 26.0$ veh/km, $L_2 = 1.0$ km
Section 3: $k_3 = 27.1$ veh/km, $L_3 = 3.0$ km
Section 4: $k_4 = 22.4$ veh/km, $L_4 = 1.0$ km

$$k = \frac{\sum_{i=1}^{n} k_i * L_i}{L} = \frac{(26.0 * 3.6) + (26.0 * 1.0) + (27.1 * 3.0) + (22.4 * 1.0)}{8.6}$$

$$k = \frac{223.3}{8.6} = 26.0 \quad [\text{veh/km}] \tag{8.2}$$

- Quality level: 26.0 veh/km → quality level D
  Maximum permissible traffic density (capacity) according to Table 8.6.

Table 8.6: Maximum permissible traffic densities

Gradient class	Bendiness	Capacity [veh/h] Ratio of heavy traffic [%]					
		0	5	10	15	20	25
1	0–75	2,500	2,490	2,370	2,290	2,255	2,215
	75–150	2,075	2,075	2,065	2,060	2,060	2,060
	150–225	1,935	1,875	1,840	1,815	1,800	1,780
	>225	1,855	1,805	1,770	1,745	1,740	1,720
2	0–75	2,500	2,420	2,295	2,195	2,125	2,100
	75–150	2,070	2,070	2,065	2,060	2,050	2,045
	150–225	1,930	1,870	1,830	1,810	1,795	1,780
	>225	1,855	1,795	1,760	1,735	1,715	1,700
3	0–75	2,500	2,115	1,965	1,865	1,795	1,750
	75–150	2,000	1,975	1,925	1,865	1,795	1,750
	150–225	1,930	1,840	1,795	1,755	1,735	1,720
	>225	1,855	1,780	1,740	1,705	1,680	1,675
4	0–75	2,400	1,735	1,590	1,510	1,445	1,405
	75–150	2,000	1,680	1,580	1,510	1,445	1.405
	150–225	1,930	1,665	1,570	1,510	1,445	1,405
	>225	1,855	1,650	1,570	1,510	1,445	1,405
5	0–75	2,000	1,400	1,230	1,140	1,055	950
	75–150	1,800	1,385	1,230	1,140	1,045	950
	150–225	1,800	1,370	1,230	1,140	1,045	950
	>225	1,795	1,360	1,230	1,140	1,040	950

Given that:
Section 1: Gradient class 2, *Sv* ratio: 12%, curvature: 153 gon/km
Section 2: Gradient class 2, *Sv* ratio: 12%, curvature: 168 gon/km
Section 3: Gradient class 3, *Sv* ratio: 12%, curvature: 185 gon/km
Section 4: Gradient class 3, *Sv* ratio: 12%, curvature: 30 gon/km

Solution:

- Capacities:
  Section 1: $q_{B,1} = 1{,}822$ veh/h
  Section 2: $q_{B.1} = 1{,}822$ veh/h
  Section 3: $q_{B,3} = 1{,}779$ veh/h
  Section 4: $q_{B,4} = 1{,}925$ veh/h
- Degrees of utilization:
  Calculating the degrees of utilization according to formula 8.3

Given that:
Section 1: Average traffic volume $q_B = 1{,}300$ veh/h, capacity $q_i = 1{,}822$ veh/h
Section 2: Average traffic volume $q_B = 1{,}300$ veh/h, capacity $q_i = 1{,}822$ veh/h
Section 3: Average traffic volume $q_B = 1{,}300$ veh/h, capacity $q_i = 1{,}799$ veh/h
Section 4: Average traffic volume $q_B = 1{,}300$ veh/h, capacity $q_i = 1{,}925$ veh/h

$$\text{Degrees of utilization} = \frac{q_B}{\max q_i} * 100 \quad [\%] \tag{8.3}$$

Solution for degree of utilization:
Section 1: 1,300/1,822*100 = 71.4%
Section 2: 1,300/1,822*100 = 71.4%
Section 3: 1,300/1,799*100 = 72.3%
Section 4: 1,300/1,925*100 = 67.5%

The evidence of the standard cross section is summarized on the form (Table 8.7).

In the case of several possible standard cross sections, the various building costs (investments plus running costs) must be contrasted with the accident costs relating to the cross sections.

Evidence of traffic safety for standard cross sections RQ 10.5 and RQ 15.5

- The accident cost rate (UKR) serves as a means of assessing traffic safety
- The annual accident costs (accident cost density or UKD) related to the length of a road can be calculated from the rate of accident costs and the average daily traffic volume (DTV)

$$UKD = 365 * 10^{-6} * UKR(RQ) * DTV, \quad [1000€/(\text{km}*\text{a}).] \tag{8.4}$$

This includes:

DTV: daily traffic volume [veh/h]
UKR (RQ): accident cost rate for the section under consideration €/(1,000*veh*km)
UKR(10.5) = € 33,70/1,000 veh*km
UKR(15.5) = € 24.00/1,000 veh*km
$UKD(10.5) = 365*10^{-6}*33.70*13.00 = 159.91$
$UKD(15.5) = 365*10^{-6}*24.00*13.00 = 113.88$

Table 8.7: Form for calculations

Form 1: Achievable quality for traffic on a section of a main road					
Road section between: the edge of Schönbach and the end of the building work					
Part section no.		1	2	3	4
1	Road category (RAS-N)		A II		
2	Desirable cruising speed v_e [km/h]		60		
3	Average traffic density q_B [veh/h]		1,300		
4	Ratio of heavy traffic b_{sv} [%]		12		
5	Cross section (RAS-Q)		10.5		
6	Desirable quality level QSV_i [−]		D		
7	Length of road section L_i [m]	3,600	1,000	3,000	1,000
8	Longitudinal gradient (vertical projection) s_i [%]	1	3.5	6	−1.5
9	Lowest average speed of the assessment vehicle v [km/h]	65	67	41	54
10	Gradient class [−]	2	2	3	3
11	Curvature KU [gon/km]	78	68	110	30
12	Part of route with passing ban [%]	15	20	15	0
13	Allowance for curvature [gon/km]	75	100	75	0
14	Curvature (total of horizontal projection + allowance) [gon/km]	153	168	185	30
15	Achievable car cruising speed $v_{R,i}$ [km/h]	50	50	48	58
16	Traffic density k_i [veh/km]	26	26	27.1	22.4
17	Quality level on partial section QSV_i[−]	D	D	D	D
18	Average car cruising speed v_R[km/km]		50		
19	Average traffic density k[veh/km]		26		
20	Quality level for traffic QSV_{Ges}[−]		D		
21	Max. permissible traffic density maxQ_i	1,822	1,822	1,779	1,925
22	Degree of utilization (Q_B/maxQ_i) [%]	71.4	71.4	72.3	67.5

– In order to be able to assess the justifiable investment costs or those where savings can be made, the UKD must be converted into cash figures over the planning period (20 years):

$$b_f = \frac{[(1 + (10^{-2})*p)^n] - 1}{(10^{-2})*p(1 + (10^{-2})*p)^n}, \quad [a] \qquad (8.5)$$

$$b_f = 14.9$$

p = updating rate (interest rate), 3%
n = duration of the assessment period in years $n = 20$.

– The cash figures for the accident costs related to the length of a road UKL can be obtained by multiplying the accident cost density UKD or ACD by the cash figure:

$$\text{UKL} = \text{ACD}*\text{b}_\text{f} \quad [\text{€ million}] \qquad (8.6)$$

UKL (10.5) = 159.91*0.0149 = € 2,383 million
UKL (15.5) = 113.88 * 0.0149 = € 1,697 million.

– The cash figures for the accident costs related to the length of the road UKL is higher in the case of RQ 10.5 at EUR 0.686 million than with RQ 15.5, i.e. RQ 15.5 provides a higher level of traffic safety.

8.7 Footpaths and Cycle Paths

For traffic safety reasons, separate or joint footpaths and cycle paths are normally provided, outside a town, parallel to the road on one side of it.

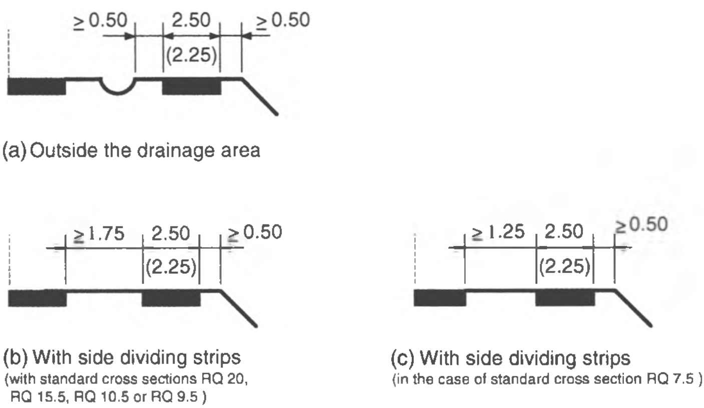


Figure 8.14: Arranging joint footpaths and cycle paths according to RAS-Q [Dimension in meter]

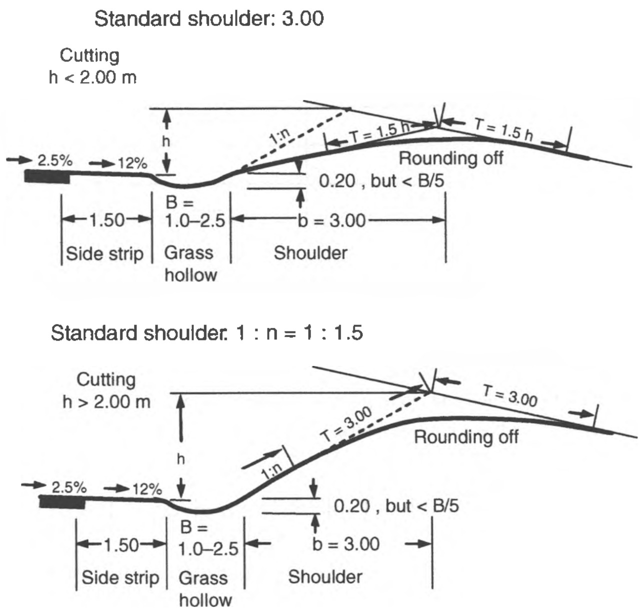


Figure 8.15: Developing the calculations for a cutting according to RAS-Q [Dimension in meter]

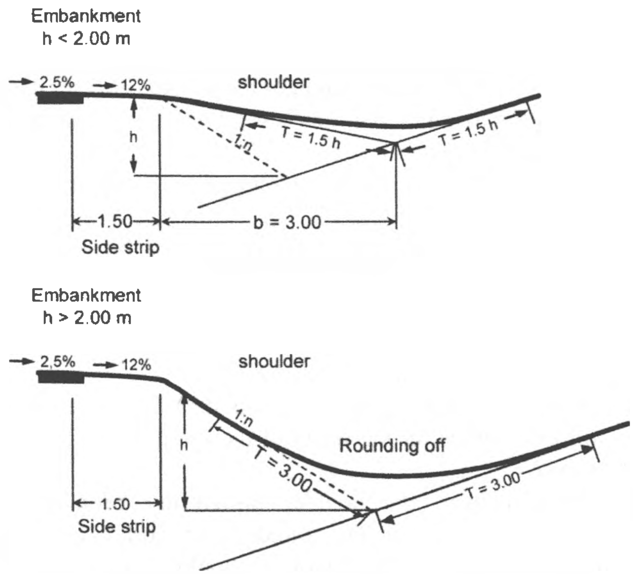


Figure 8.16: Developing the embankment areas according to RAS-Q [Dimension in meter]

Table 8.8: Width of footpaths and cycle paths next to curb including the side safety areas

Permissible speed [km/h]	Joint footpaths and cycle paths [m]	Footpaths [m]	Cycle paths [m]
≤ 50*	3.00	2.00	2.50
	(2.75)**		(2.10)***
≤ 70	3.25	2.25	2.75
	(3.00)**		(2.35)***
> 70	3.50	2.50	3.00
	(3.25)**		(2.60)***

* For bridges too without passive protection.
** When designing joint footpaths and cycle paths with minimum widths.
*** In the case of reduced widths for cycle paths.

The following standard widths (Table 8.8) are given in RAS-Q according to the permissible speed:

In principle, joint footpaths and cycle paths can be designed as follows (Figure 8.14) according to RAS-Q.

8.8 Shoulders

Shoulders are produced as a transition between the artificially created surface of the road and the natural terrain. If the height of the terrain is higher than the height of the road gradient (the road gradient lies beneath the terrain) at the appropriate point, a cutting is created (Figure 8.15).

But if the terrain is lower than the gradient of the road, an embankment is necessary (Figure 8.16).

If the terrain is higher than the gradient on one side of the road and lower than the terrain on the other side, this produces a side cut.

Cuttings or embankments with a shoulder height h of > 2 m are given a standard incline of $1: n = 1: 1.5$. If the height of the shoulder h is < 2 m, an even shoulder width of 3.0 m is maintained.

If different shoulder inclines 1: n are used from the standard incline 1: 1.5, then the following formula applies for the width of the shoulder b if the height of the slope h is < 2.0 m: $b = 2$ n.

Changes in incline between the shoulder and terrain are rounded off to adapt the body of the road to the terrain. The rounding off work is carried out on the basis of a quadratic parabola. If the height of the shoulder h is > 2.0 m, the tangent length for the rounding off line is 3.0 m and if the shoulder height h is < 2.0 m, 1.5

times the shoulder height h should be used. If the shoulder height $h > 6\,\mathrm{m}$, it makes sense to use terraces.

8.9 Developing the Cross Section

The process for developing the cross section of a road for a selected point is shown in the following working steps:

(1) The profile of the terrain is derived from the horizontal projection at a selected point (mark the location and distance marker; see Figure 8.17).
(2) The vertical projection provides the following (Figure 8.18):
  - Height of the terrain
  - Height of the road axis (gradient)

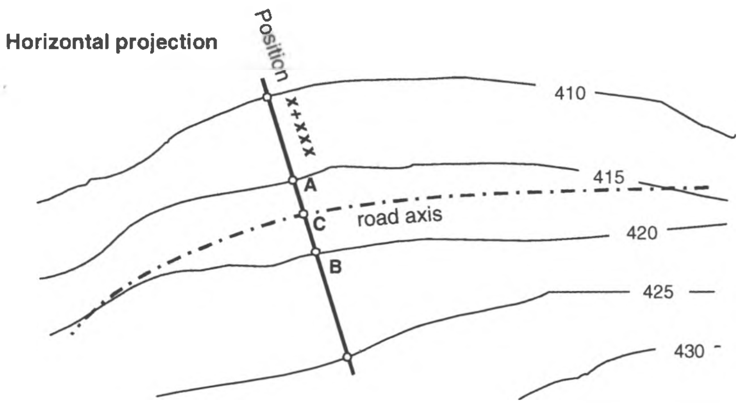


Figure 8.17: Determining the profile of the terrain

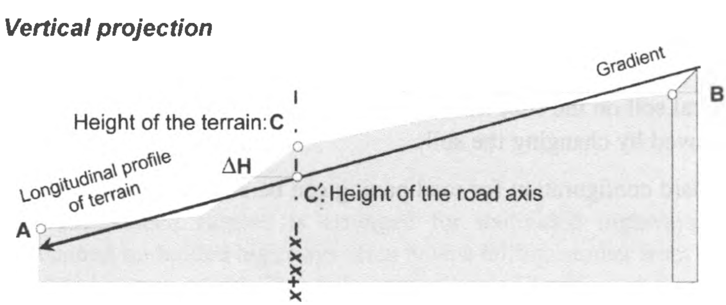


Figure 8.18: Determining the height of the road axis

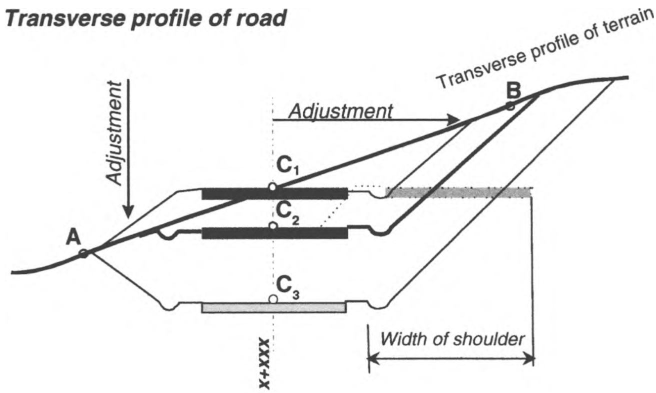


Figure 8.19: Changes in the cross section

(3) The camber for the road can be derived from the camber graph.
(4) After fixing the camber for the further cross section elements and the incline of the shoulder, the appropriate absolute heights are found.

Changes on the vertical projection (correction of the gradient) or on the horizontal projection (moving the axes) automatically lead to an alteration in the formation of the cross section (Figure 8.19).

After determining the cross section elements and arranging matters related to the horizontal and vertical projections, the design of the pavement structure has to be set in line with RSTO 01 (2001) in the light of the traffic volumes.

The street is subdivided as follows:

- Road construction:
  Consists of one or several supporting layers including frost protection (if necessary) and the sealing coat.
- Substratum:
  Artificially created earthworks (embankment or cutting) between the subsoil and the paved area.
- Road bed:
  Natural soil on the site, which may need to be strengthened mechanically or improved by changing the soil.

The standard configuration for road paving can be seen in Figure 8.20.

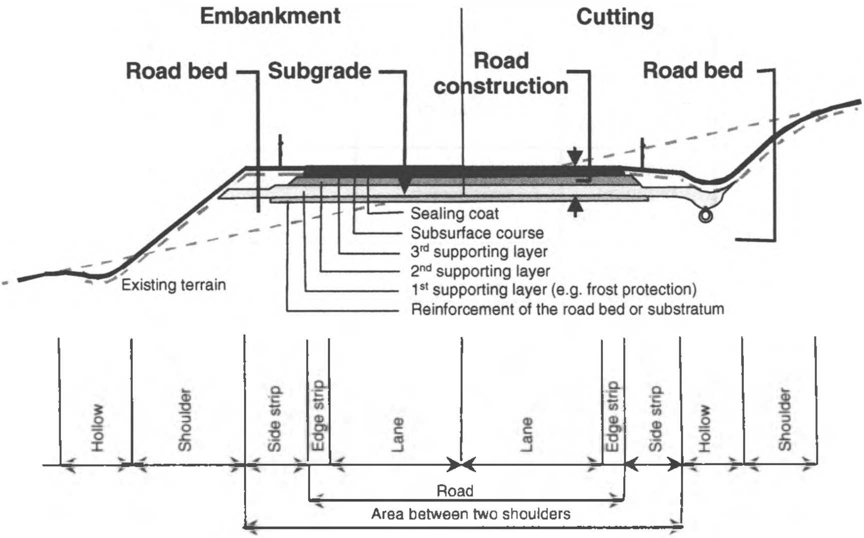


Figure 8.20: Standard configuration for a rural road

8.10 Camber of Cross Section Elements

8.10.1 General matters

The incline on the surface of the road at right angles to the road axis is described as the camber q [%]. Depending on the design element, the camber fulfils the three following tasks:

(1) Draining the surface of the road (straights and bends)
(2) Proportionately accommodates centrifugal forces (bends)
(3) Supports the optical line (bends).

While the camber in straights ($q_G \geq 2.5\%$) and in circular arcs ($2.5\% \leq q_B \leq 8\%$) is constant, it changes constantly in the clothoid depending on the latter's length.

8.10.2 Camber in straights

Normally single-sided camber is arranged for undivided highways that are straight. Existing undivided highways often have a falling camber from the center of the road to the two edges. This is known as a roof profile and is suitable for reconstruction or upgrading work or when circumstances impose constraints – for example, as a result of intersections or access roads to real estate.

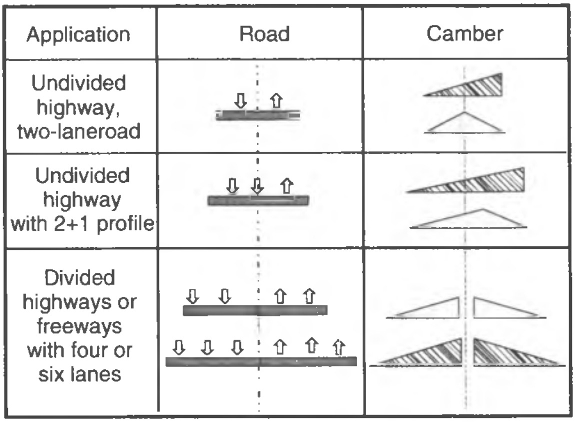


Figure 8.21: Camber for straights

Table 8.9: Maximum camber in circular arcs (RAS-Q)

Road category	A, B I, B II	B III, B IV	C
max q_B [%]	8.0	6.0 (7.0)	2.5 (5.0)

On divided highways with four lanes without a median strip, the roof profile is used in principle on straight stretches. The lanes on divided highways with a median strip are always given camber drops towards the outside of the lane on straights.

The paved edges of roads and additional strips should have the same camber as the road. Side strips, which are used to drain the road, should have a 12% incline. Dividing strips at the sides should have camber of 4%. Footpaths and cycle paths should have camber of 2.5%.

Figure 8.21 illustrates the camber on straights.

8.10.3 Camber in circular arcs

The same minimum camber applies to circular arcs as to straights (min $q_B = 2.5\%$) because of the need to drain away water. In a circular arc, the road is normally angled towards the inner side of the bend for driving reasons. The maximum camber is directly dependent on the category of road (Table 8.9).

The figures in brackets may be used if it is necessary to fall below the minimum figures for bends that are set for individual design speeds. The camber can then be determined depending on the radius of curvature R and the speed v_{85} and v_e according to Figure 8.22.

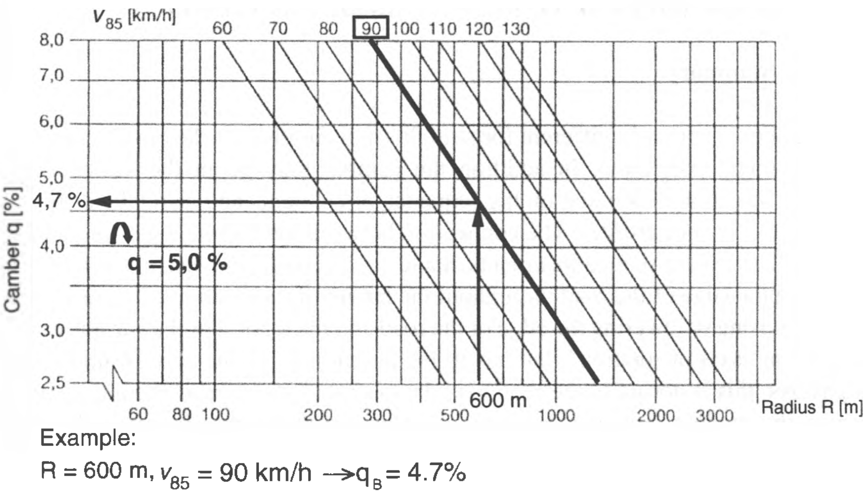


Figure 8.22: Determining the camber on roads in categories A, BI, BII (RAS-L)

Table 8.10: Arranging negative camber in a circular arc

v_e	km/h	70	80	90	100	110	120	130
min R (if $q = -2.5\%$, $n = 30\%$)	M	600	950	1,400	2,100	3,000	4,100	5,500
min R (if $q = -2.0\%$, $n = 30\%$)	M	550	850	1,300	1,900	2,600	3,500	4,600

In order to prevent areas where it is hard for water to drain away in areas of torsion with an insufficient longitudinal gradient or at intersections, the camber may be arranged to maintain minimum radii on the outside of the circular arc (Table 8.10).

8.11 Diagonal Gradient

The diagonal gradient p on a road results from the longitudinal gradient s and the camber q. It always runs in the direction of the greater drop on the road surface and is the direction in which the water drains away. The diagonal gradient can be calculated as follows:

$$p = \sqrt{s^2 + q^2}, \quad [\%]. \tag{8.7}$$

In order to prevent vehicles sliding towards the inside of the bend in icy conditions, the diagonal gradient must always be $p \leq 10\%$.

8.12 Raised and Lowered Edges of Roads and Torsion

8.12.1 Summary

The camber is normally different for straights and circular arc design elements on the horizontal projection. In addition the camber may go in the same or the opposite direction depending on the sequence of elements and also take different forms (desk or roof profile). The connection between the various types of camber is provided by constant adjustments made to the road and usually raising or lowering the edge of the road throughout the length of the clothoid. The reference line for raising or lowering the edge of the road and the torsion of the surface of the road is an "axis of rotation" that has to be determined. In the case of undivided highways, this is normally the road axis. In special cases (narrow median strip or where the median strip can be crossed, intersections on bends, etc.), the axis of rotation on divided highways may lie at the edge of a lane or on the road axis. The torsion of the road surface should always take place within the clothoid, i.e. avoid any use of straight or circular arc sections.

The gradient for raising or lowering the edge of the road Δs is described as the relative inclination of the edges of the road in relation to the assumed axis of rotation. It is the difference between the longitudinal gradient of the axis of rotation and that at the edge of the road (see Figure 8.23). For driving and optical reasons, the gradient for raising or lowering the edge of a road in category groups A and B should comply with minimum and maximum figures depending on the design speed (Table 8.11).

When torsioning one-sided camber into the opposite camber, the camber runs through the horizontal plane. Regardless of the minimum longitudinal gradient, the gradient for raising or lowering the edge of the road may not fall below the lower threshold (min Δs) for drainage reasons. This gradient for raising or lowering the edge of the road must be maintained until the minimum camber, min $q = 2.5\%$. The remaining raising or lowering of the edge of the road along the rest of the transition curve is carried out until the required camber level is achieved in the circular arc.

The minimum length of the torsion stretch is the result of taking into consideration the maximum figure for raising or lowering the edge of the road, max Δs, or the length of torsion that is required for drainage engineering reasons, min Δs, and this is:

$$\min L_v = \frac{q_e + q_a}{\max \Delta s} * a \quad [\mathrm{m}]. \tag{8.8}$$

The representation of the differences in height between the edges of the road and the torsion axis is shown on the edge graph (characteristic graph). During practical design work, the edge graph is arranged using the same scale of length beneath the curvature and camber graphs.

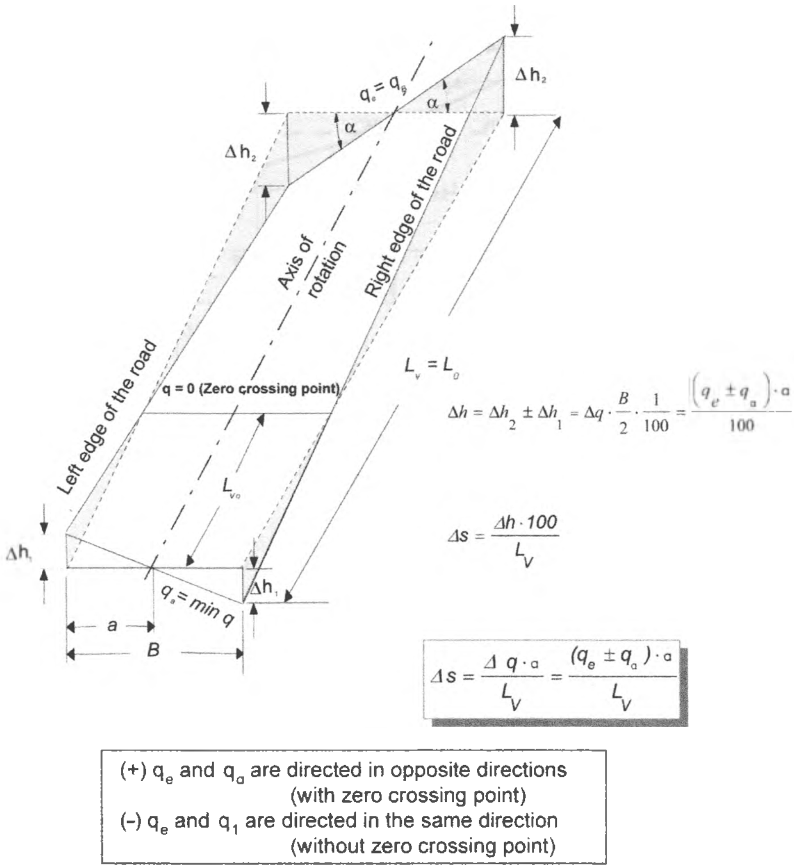


Figure 8.23: Gradient for raising or lowering the edge of the road – determining Δs (Weise and Durth, 1997)

Table 8.11: Limits for the gradient for raising or lowering the edge of a road

v_e	max Δs [%]		min Δs [%]
[km/h]	$a < 4.00$ m	$a \geq 4.00$ m	
50	0.50 * a	2.00	
60–70	0.40 * a	1.60	0.10 * a
80–90	0.25 * a	1.00	(≤max Δs)
100–120	0.225 * a	0.90	

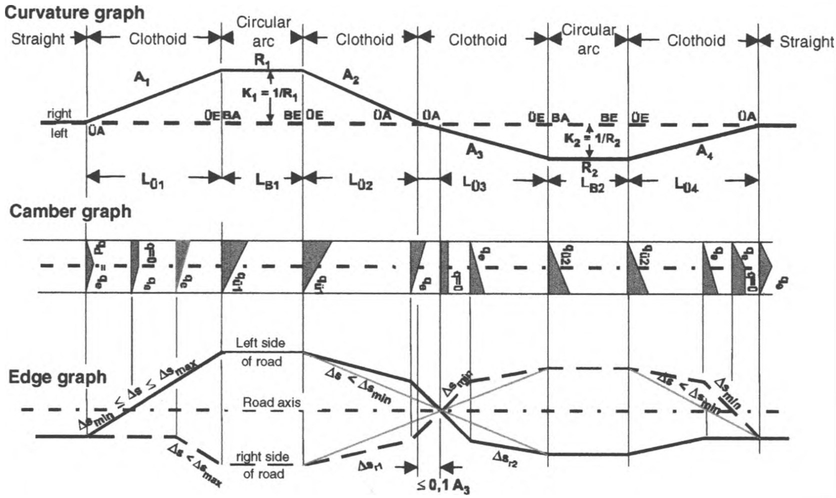


Figure 8.24: Curvature and edge area in an inflection line

Areas of torsion with zero crossover (Figure 8.24) often do not meet the requirements for draining away surface water. In order to prevent any possible aquaplaning, the flow of water from the zone round the zero crossing should be improved by arranging a minimum longitudinal gradient.

$$\Delta s \geq 0.7\% \quad \text{better} \quad \Delta s \geq 1.0\% \tag{8.9}$$

A safety problem may also occur if relatively small gradients for raising or lowering the edges of roads Δs are created as a result of long areas of torsion (large clothoids) and/or slight differences in camber. Minimum figures are needed for gradients for raising or lowering the edges of roads when designing the torsion for drainage areas.

Types of road torsion

The following illustrations demonstrate possible types of road torsion (see Figure 8.25 to 8.31).

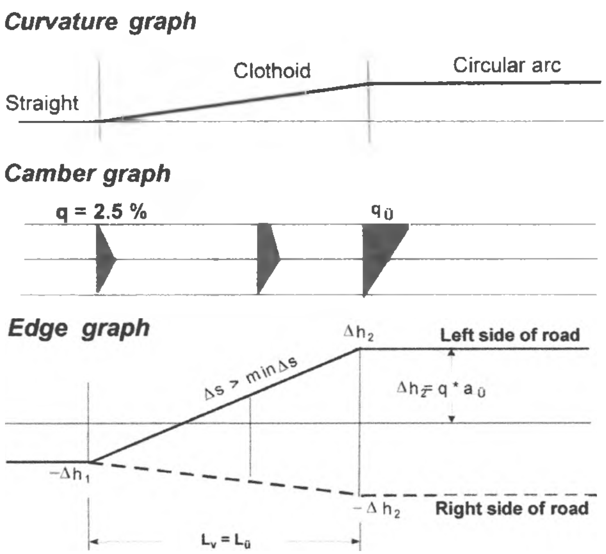


Figure 8.25: Road torsion between roof profile and desk profile in clothoids where $\Delta s >$ min Δs

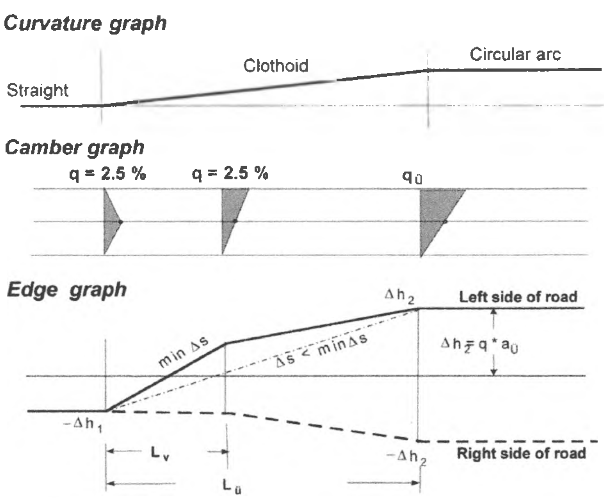


Figure 8.26: Road torsion between a roof profile and desk profile for clothoids where $\Delta s <$ min Δs

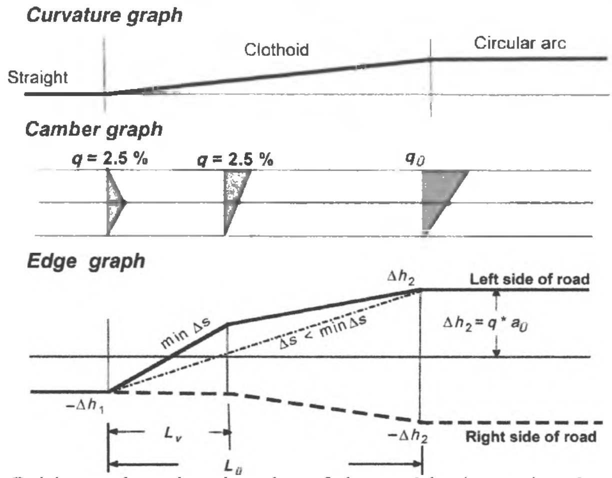


Figure 8.27: Raising or lowering the edge of the road by increasing the camber in the transition curve

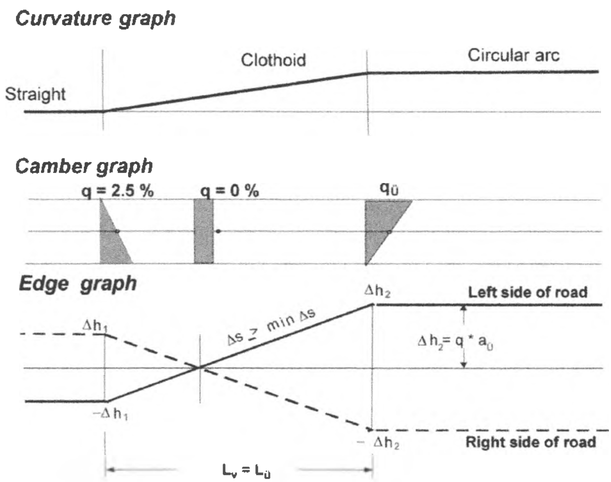


Figure 8.28: Road torsion in the clothoid with opposing camber where $\Delta s > \min \Delta s$

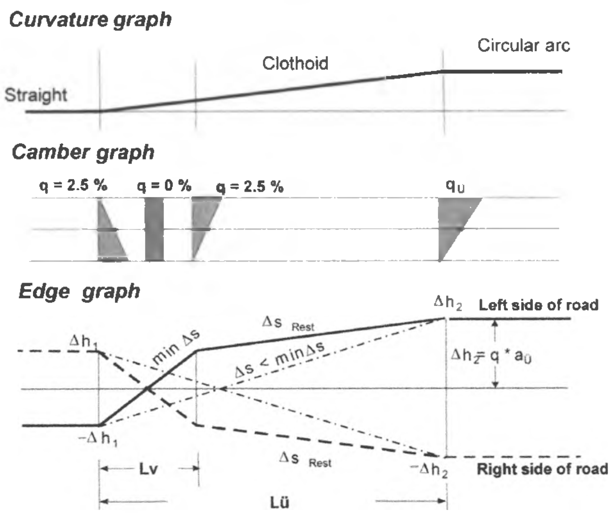


Figure 8.29: Road torsion in the clothoid with opposing camber where $\Delta s < \min \Delta s$

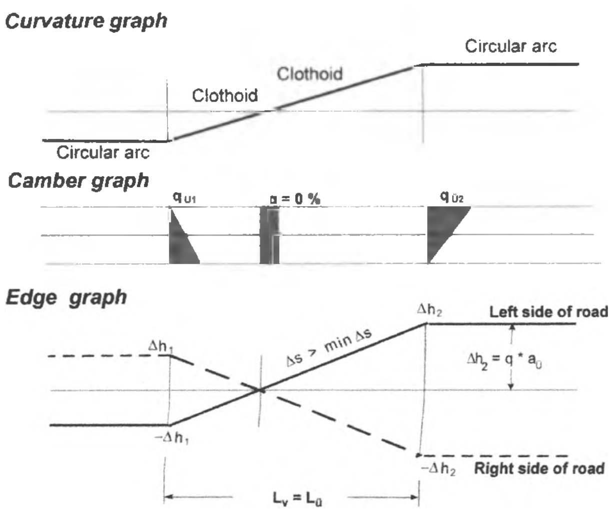


Figure 8.30: Road torsion in the clothoid where $\Delta s > \min \Delta s$

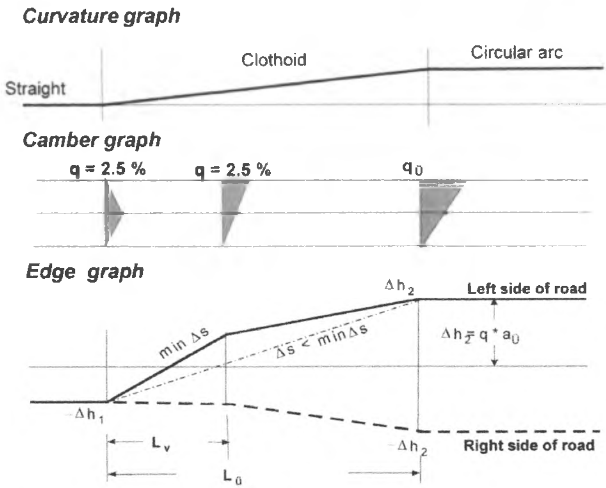


Figure 8.31: Road torsion in the clothoid where $\Delta s < \min \Delta s$

8.13 Questions

(1) Name the essential elements in the cross section of a road.
(2) Explain the differences between a standard cross section, a civil engineering cross Section, and a transverse section.
(3) Sketch and explain the RQ 10.5 standard cross section.
(4) What advantages are obtained by using RQ 15.5?
(5) How is it possible to obtain the necessary standard cross section?
(6) Explain the connection between a longitudinal gradient, camber, and a diagonal gradient.
(7) What do you understand by the term torsion when designing the cross section of a road?

Chapter 9

Intersections

9.1 Principles

Intersections enable roads to become a continuous network – i.e. intersections are a vital element in networks alongside roads. The area where two or more roads are linked is described as an intersection. If a road ends at another continuous road, what is known as a T-intersection is created (with three arms). A crossing, however, is an intersection with more than three arms, which normally involves two continuous roads.

The following issues are crucial when designing an intersection:

- the volume of traffic
- route features
- the topographical situation
- the urban development situation
- environmental requirements.

In principle, the following subdivision can be made:

- intersections at the same level with a free flow of traffic
- intersections at the same level regulated by stop lights
- graded or grade-separated interchanges
- interchanges, where some roads are grade-separated (a combination of systems at the same or different levels).

The design of the intersection (at-grade, grade-separated, or a mixture of both) has a key influence on the quality of traffic and the safety levels in the whole network. This is partly based on decisions made to classify a road in the relevant road category (Chapter 2).

As both at-grade intersections and graded interchanges could be selected for some road categories, the decision taken in the final resort depends on the road's individual characteristics. No at-grade intersections should be used along a section of road with grade-separated links if possible and conversely.

The basic requirements at an intersection are dictated by the specific goals of a road network:

(1) Quality of the traffic flow
The quality of the traffic flow describes more than its pure efficiency. It is dictated by the network function, the road area, and the use of the intersection arms. The quality of the traffic flow and the suitability of a given type of intersection can be substantiated using the existing calculation processes. In principle, intersections should be designed in such a way that no unreasonably long waiting times occur at the subsequent access routes to the intersection or for flows of traffic trying to turn off. If several intersections are controlled by stoplights, the flow of traffic can be improved and the pollution of the environment reduced at the same time.

(2) Traffic safety
Safe traffic flows at an intersection are guaranteed if all the components at an intersection and the intersection itself can be recognized in good time and if it is clear, comprehensible, and accessible to pedestrians.

- Recognition
  - The location of the intersection or at least the secondary road is in a sag
  - The access roads to the intersection are widened in good time
  - Traffic islands are built in the secondary road to indicate the need to wait
  - Identification of the intersection and yield instructions must be given in good time
- Clarity
  - The intersection is located in a sag
  - The arms of the intersection meet at 90 degrees, if possible
  - Adequate visibility
- Comprehensibility
  - Using unified and generally familiar types of intersections
  - Building design, which underlines the yield rules
  - Good optical management of individual traffic flows by road markings and control equipment
  - Clear management of the routes provided in the intersection for all road users
- Accessibility for vehicles and pedestrians
  - Adequate lanes, which manage traffic flows appropriately and continue beyond the intersections
  - Clearly marked restrictions on lanes
  - Clearly managed and highly visible crossings at the intersection for pedestrians and cyclists
  - Good drainage facilities

(3) Environmental compatibility
- Need for a small area
- Minimum interference with the landscape or cityscape
- Minimizing emissions (noise, pollutants)

(4) Cost effectiveness
An intersection is only cost-effective if minimum investment and operating costs are generated while meeting the requirements related to traffic quality, traffic safety, and environmental compatibility. But the investment costs are normally what influence the planning permission process, as the time, operating and accident costs are harder to forecast and no overall economic analysis takes place. But the costs of road users should also be included in the decision in any foresighted planning.

9.2 Traffic Procedures and Conflict Points

Conflict points emerge from traffic procedures and they are described as crossing, merging in or out, and interweaving points for flows of traffic (Figure 9.1).

Figure 9.2 illustrates conflict points at different types of intersections. The number of conflict points becomes larger if the intersections have more than four arms, if traffic flows are divided into several lanes, or if there are lanes for various types of traffic (cyclists and pedestrians, local public transportation services).

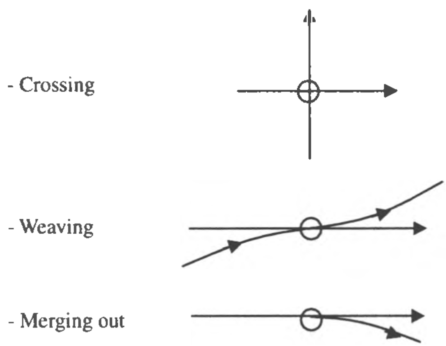


Figure 9.1: Traffic procedures

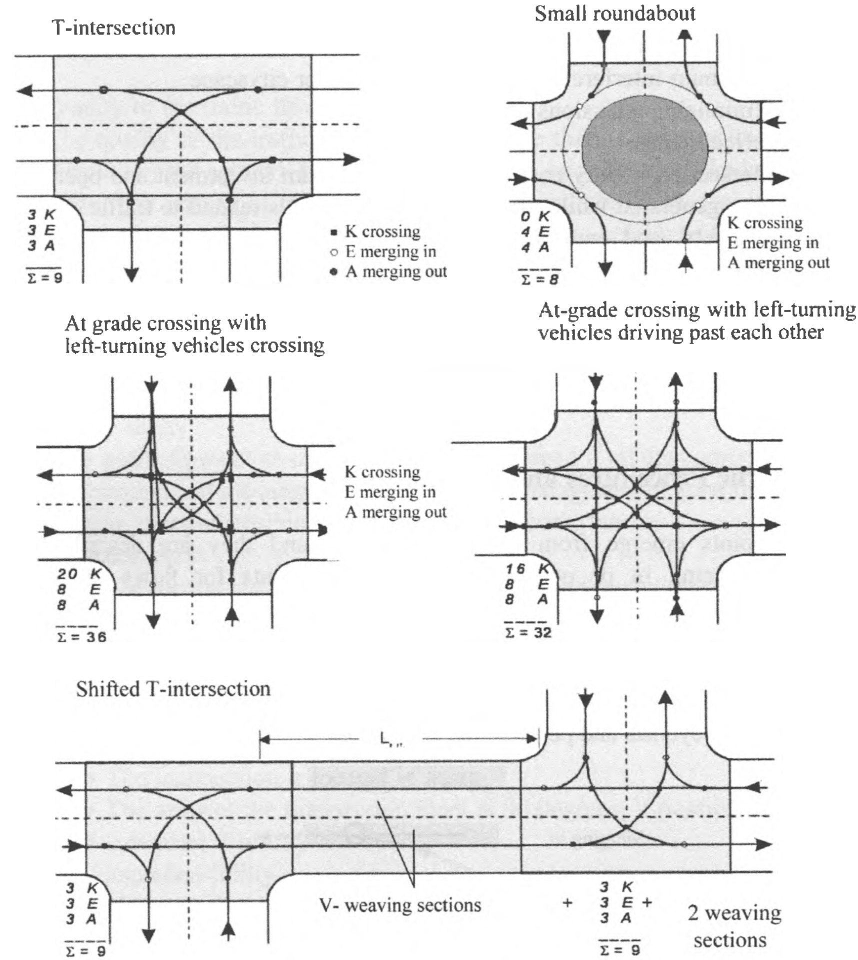


Figure 9.2: Conflict points at various types of intersections (RAS-K-1)

Conflict points have to be minimized to increase traffic safety:

- Reduce the number of intersections
- Separate traffic flows at several levels
- Have traffic flows cross at right angles as far as possible
- Have small inlet and outlet radii
- Channel traffic flows

The conflict areas should be kept as small as possible and be clearly visible to drivers.

9.3 Speeds at Intersections

The speed at intersections is used according to Table 9.1 for measuring the design elements in the major arm of the intersection.

A decision must first be made on whether the permissible maximum speed should be generally restricted by a traffic sign or not. If a general restriction on the permissible speed is used, the following equation applies:

$$v_K = v_{\text{zul}} \tag{9.1}$$

If no description of the required speed is planned, v_K is selected according to Table 9.1 v_K should be in a balanced relationship to v_{85} for road traffic reasons:

$$v_{85} - v_K \leq 20\,\text{km/h}. \tag{9.2}$$

Determining sight distance can only take place at v_{85}. All the other intersection elements dependent on speed are determined by v_K except the exit radii.

Table 9.1: Speeds at intersections (RAS-K-1)

Category group		Cross section	Intersection speed	
			v_{zul} in km/h	v_k in km/h
I	Spacious link	Four lane	(No at-grade intersections)	
		Two lane	100 (80)	90 (80)
II	Regional link	Four lane Two lane	70 (100) 70	70 (90) 80
III	Inter-municipal link	Four-lane Two lane	70 (100) 70	70 70
IV	Connecting link	Two lane	70	70
V	Subordinate link	Two lane	60	60 (50)
II	Fast road	Four lane	70	70
III	Main road	Four lane Two lane	70 70	70 70
IV	Main distributor road	Two lane	(60) 50	50
III	Main road	Four-lane Two lane	50 50	50 50
IV	Main distributor road	Two lane	50	50
Figures in brackets are exceptions				

9.4 Intersection Intervals

Intersections can have a negative influence on the homogeneous flow of traffic in a network. So they should be located as far away from each other as possible. The minimum distances between intersections largely depend on the following criteria:

- Location of the intersection inside or outside built-up areas
- Type of intersection (at-grade, grade-separated, mixture of both with or without stoplights)
- The design speeds v_K and v_{85}
- Arrangement of intersection elements
- Distance requirements for signposts

The distance between intersections required for planning work is determined by their network function, the flow of traffic, and signposts (Table 9.2).

Table 9.2: Minimum intersection distances as a result of signposts according to RAS-K-1

Intersection speed v_K in km/h	50	60	70	80	90	100
Intersection distance in m	140	170	205	235	270	300

Outside built-up areas the intervals between intersections – if they can be freely selected – should be set in such a way that the necessary minimum overtaking sight distances required by RAS-L exist between as many intersections as possible.

In built-up areas, the intervals between intersections from a traffic point of view are determined by the required space to accommodate traffic jams, areas for road users to change lanes, and the requirements for stoplights.

Small roundabouts can be arranged at shorter distances from each other because generally fairly long jams do not occur on access roads because the flow of traffic is harmonized (in cities: 140–200 m, outside cities: 200–300 m).

9.5 At-Grade Intersections

9.5.1 Basic forms

Intersections along two-lane roads are normally formed at-grade or with partial grading. A standardization of types of intersections and the elements related to intersections make it easier for drivers to recognize the set-up and this generally increases the level of traffic safety because of the repetition effect. There are also planning and engineering advantages.

RAS-K-1 sets out the basic types I to VII (Figure 9.3). The selection of the basic type is made according to the specifications of the network planning and according to the number of lanes on the roads being linked.

The link between two-lane roads at a T-intersection or crossing is described as basic type I. Depending on the significance of the traffic and traffic volumes on the

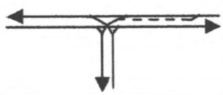
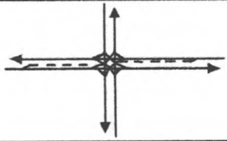
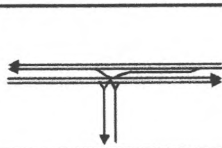
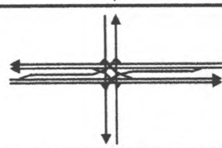
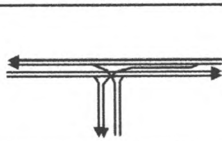
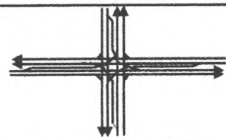
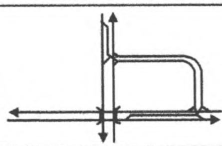
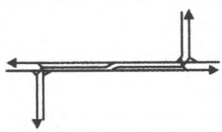
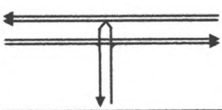
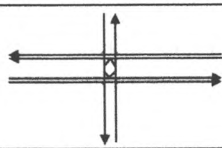
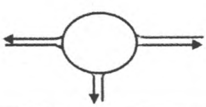
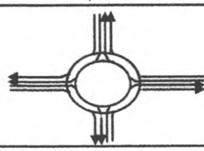
Basic type	T-intersections	Crossings
I T-intersection or crossing of 2-lane roads possibly with lane for traffic turning left. Traffic divider		
II T-intersection or crossing of highway with 2-lane road With stoplights, left lane, island		
III T-intersection or crossing of 2 highways With stoplights, left lane, island		
IV Partially graded crossing of 2-lane or roads or highways With stoplights, left lane, island		
V Crossing of 2-lane roads (offset T-intersection) With stoplights, left lane, island		
VI Enlarged T-intersection or crossing with at least one highway		
VII Roundabout on 2-lane road or highway Normally without stoplights		

Figure 9.3: Basic types of non-graded intersections (RAS-K-1)

more important road, the latter may have a lane for traffic turning left. The less important intersection arms outside urban areas should be given traffic guidance equipment (drop-shaped islands) to make it clear to drivers that they have to yield to other traffic.

The basic type II arises if a less important two-lane road and a more important highway with four or more continuous lanes meet at an at-grade intersection or crossing in the form of a T-intersection or crossing. These kinds of intersections normally require stoplights with restrictions on permissible speeds of $v_{zul} \leq 70$ km/h. Within urban areas ($v_K \leq 50$ km/h), it is possible to forego stoplights in exceptional cases. Outside urban areas, it is normally necessary to have left-turning lanes on the more important road and traffic islands on the less important arms of the intersection.

Basic type III arises when two highways intersect or cross, each with four or more continuous lanes. This is particularly the case on urban main routes (category groups B and C), as at-grade or partially graded interchanges are normally used when two roads with four or more lanes cross outside urban areas. Intersections with the basic type III require stoplights with restrictions on permissible speeds of $v_{zul} \leq 70$ km/h and lanes for traffic turning left and traffic islands on all the intersection arms.

Basic type IV arises if two two-lane roads cross at two levels. The link between the two crossing roads is provided by a loop ramp. The T-intersections that are created normally have a lane for traffic turning left and are designed according to basic type I.

As this solution is a combination of an at-grade crossing and two T-intersections of basic type I, it is no longer an at-grade interchange. This intersection solution does away with all the conflict points within the crossing and this increases traffic safety to an enormous degree. An increase in performance is significant if the volume of traffic from the crossing traffic is more important in terms of the total traffic volume.

Basic type V arises if a crossing is designed as an offset with the result that the less important intersection arms meet the more important road at a small distance from each other from different sides. An offset intersection consists of two intersections of basic type I. Depending on the arrangement, a distinction can be made between right and left offsets. A left offset is the less beneficial solution from a traffic engineering point of view, because the flow of traffic from the less important roads first has to turn left and therefore has to pay attention to two flows of traffic on the more important road at the same time.

Basic type VI implies a continuous highway with priority, which is crossed by another road or has a T-intersection with it. A middle lane on the main road is expanded to such a degree that space is created for traffic turning left. If the lanes for the different directions are directly next to each other in the subsequent areas, the expansion provides a central traffic island.

Basic form VII arises if three of more intersection arms are normally linked to each other without stoplights using a roundabout without any road having priority.

9.5.2 Small roundabouts

Criteria for their use

Small roundabouts provide a number of advantages in comparison with other basic types of intersections up to an average traffic volume (total of all intersection approaches from 15,000 vehicles/24 hours up to max 25,000 vehicles/24 hours).

- Capability: The capability of roundabouts is normally greater than comparable intersections and crossings without any stoplights, because traffic flows are not dependent on any higher level of rank. At intersections with stoplights, efficiency checks have to be made – i.e. the efficiency of a roundabout is not necessarily high.
- Traffic safety: Roundabouts provide a high degree of traffic safety because the number of possible conflict points with other road users is much reduced in comparison to T-intersections and crossings. The accident rate is relatively low on account of speed levels (20–40 km/h). Roundabouts along non-upgraded roads reduce the average speed and provide some interruption in the road features. They can be arranged to good advantage in the crossover between rural and urban roads.
- Environmental compatibility: Roundabouts normally occupy a smaller paved area than comparable efficient at-grade intersections. By harmonizing the flow of traffic, noise and pollution emissions are normally lower. Roundabouts can be integrated in urban areas without any problems.
- Cost-effectiveness: Investment costs for roundabouts are normally lower than for comparable, at-grade intersections (small paved area, no need for stoplights). The operating costs are also much lower, for there is no expenditure on operating and maintaining stoplights.

Design principles

The body of rules and regulations introduced in Germany is dictated by single-lane roundabouts. In principle, roundabouts can be described according to the size of the outer diameter, as follows:

- Large: $D > 40\,\text{m}$ (35 m)
- Small: $22\,\text{m} < D < 40\,\text{m}$ (35 m)
- Mini: $13\,\text{m} < D < 22\,\text{m}$.

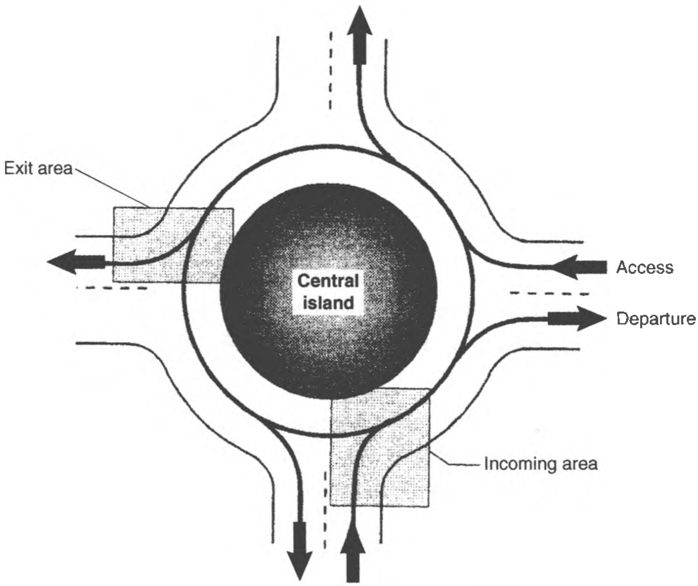


Figure 9.4: Elements of a roundabout (Weise and Durth, 1997)

Figure 9.4 illustrates the key elements of a roundabout. The key design elements are found in Table 9.3.

9.6 Graded Interchanges

9.6.1 Planning principles

The decision on whether an intersection should be grade-separated or at-grade largely depends on the road's function and the associated road category. As a result, graded interchanges are normally a link between national roads in road categories A I and A II. The link between highways takes place at highway intersections and the link with less important roads takes place at normal exits.

Grade-separated interchanges normally consist of:

- continuous lanes
- entrance and exit areas
- on and off ramps.

The RAS-L-K 2 is the body of rules and regulations in Germany. In principle, the following factors should be taken into consideration in any planning work:

- Continuous lanes must be planned in terms of the network available in line with key traffic relations.

Table 9.3: Design elements for (fairly small) roundabouts (Weise and Durth, 1997)

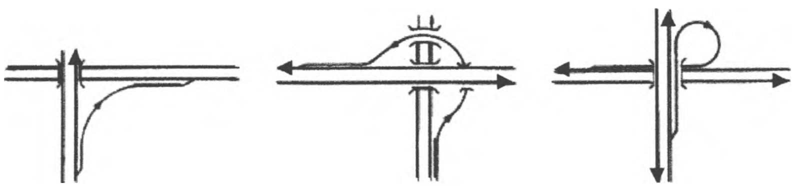
Design element	Min. figure	Standard figure	Max. figure
Outer diameter – Outside urban areas – At city limit – Within urban areas	30 m 30 m 20–22 m	40 m 36–40 m 35 m 25–28 m 26–35 m	45 m 40 m 35 m
Width of roundabout lane – Outside urban areas – (non-divided) Within urban areas – Non-divided – Divided Figures in brackets refers to the outer diameter	5.80 m (45 m) 5.30 m 4.00 m 3.50 m	6.00 m (40 m) 6.00 (35 m) 5.00 m outer ring 2.5 m inner ring	7.20 m (30 m) 7.50 m (26 m) 6.00 m 1.50 m
Width of inlet and outlet lanes Entrance single-lane two-lane Exit single-lane	3.00 m 3.50 m	3.25–3.50 m^2 6.00 m 3.50–3.75 m^2	4.00 5.00 m
Width of funnel-shaped traffic islands – Without any pedestrian crossing – With pedestrian crossing opportunities	1.60 m (at the start) 2.00 m	2.50 m 2.50 m	5.00 m (at circle) 5.00 m
Camber on roundabout lane – Non-divided – Divided		2.5% 2.5/3–5%	
Circular radii at edge arc Entrance Exit		14.00–16.00 m 16.00–18.00 m	

- The ramps must be arranged taking into consideration the important flows of traffic turning off.
- The entrances and exits on highways and roads in category group A should always be on the right, but it is possible to split these up between right and left. On highway lanes in category group B, on and off ramps can also be placed on the left, if the design work is appropriate.
- On ramps may be located before off ramps if weaving lanes with the necessary efficiency have been set next to the main lane. Weaving lanes are the normal solution for highways in road categories A I and A II.

- The flows of traffic approaching an intersection should be united within the linking ramp and be jointly fed into the continuous lane.
- After bringing together the lanes or link ramps, the number of lanes in the cross section should be no more than one less than the total number of lanes before they joined.
- The same design speeds normally apply on continuous lanes as on open roads. The geometrical requirements within the ramps are lower for reasons of cost and containing the overall dimensions.

9.6.2 Linking ramps

The road links between the various levels of a grade-separated interchange are called ramps. In principle, a distinction can be made between direct, semi-direct, and indirect linking ramps (Figure 9.5).



(a) Direct linking ramp (tangential ramp, tangent)

(b) Semi-direct linking ramp

(c) Indirect linking ramp (loop ramp, loop)

Figure 9.5: Basic types of linking ramps

Figure 9.6 shows the types of linking ramps and their recommended design speeds.

The degree of comprehensibility and the flow of traffic decrease down the sequence shown above.

The limits on the design elements for the linking ramps are shown in Table 9.4.

9.6.3 Intersection areas

Grade-separated interchanges consist of the following main parts:

- Continuous lanes
- Exit areas from the continuous lane
- Linking ramps
- Exits and entrances to or from ramps
- Weaving areas
- Entrance areas into a continuous lane.

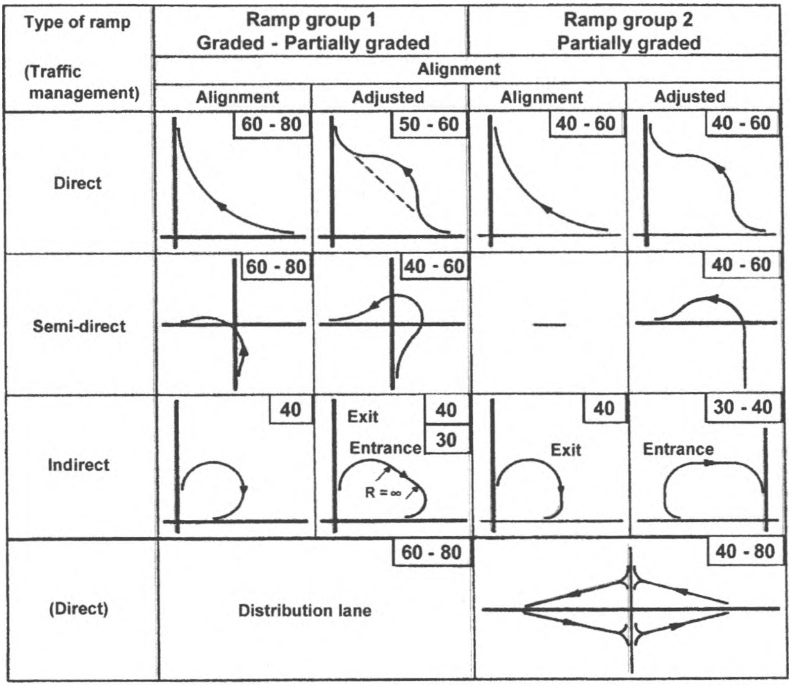


Figure 9.6: Types of linking ramps (RAS-K-1)

The following example of a four-arm graded interchange illustrates the parts (Figure 9.7).

9.6.4 Intersection solutions

Three-armed intersections

The trumpet is also the normal solution for a three-arm intersection. If a trumpet is used on a highway with traffic moving in two directions, it is possible to prevent any traffic from travelling in the wrong direction: the exits can be arranged in front of the entrance on the opposite side (Figure 9.8).

The trumpet turns into a semi cloverleaf if one of the tangential ramps is laid on the other side of the less important road (Figure 9.9) and becomes a semi-direct ramp.

The trumpet used to be one of the most frequently encountered types of three-armed intersection. The merging highway moves from an open road directly into a ramp and conversely. So this application should be restricted to situations where it

Table 9.4: Limits on design elements (RAS-K-1)

Design elements	Abbreviation	Limits on the design elements for the design speed V_e[km/h]					
		30	40	50	60	70	80
Minimum radius of arc	R_{min} [m]	25 (20)[1]	45 (40)[2]	75 (70)[2]	120 (110)[2]	175	250
Max. uphill longitudinal incline downhill	$+s_{max}$ [%] $-s_{max}$ [%]	6(10)[3] 7(10)[3]					
Min. diameter of crest	$H_{K\min}$	500 (125)[1]	1,000 (325)[2]	1,500 (750)[2]	2,000 (1,600)[2]	2,800	4,000
Min. diameter of sag	$H_{w\min}$	250 (100)[1]	500 (200)[2]	750 (400)[2]	1,000 (800)[2]	1,400	2,000
Min. camber	q_{min}	2.5					
Max. camber in bends	$q_{K\max}$	6.0					
Min. incline for raising or lowering edges	Δs_{min}	0.1[a] a = Distance from the edge of the lane from the roundabout axis					
Min. stopping distance	$S_{h\min}$	25 (15)[1]	30 (25)[2]	40 (40)[2]	60 (60)[2]	85	115

[1]Exceptional figures, category group B; [2]Category group B; [3]Only if the layout is extended, category group B.

is possible to reduce the speed of traffic. In the textbook solution (trumpet pointing left), it is best to start the main bend of the ramp before the crossing by entering a reversed curve and crossing over the highway with which the road merges (Figure 9.10).

The left leaning trumpet is preferable for traffic safety reasons, as the fast road user turning left from the merging highway is led into a left bend with constant radii. In the case of the right leaning trumpet, the road user is confronted with an oval line with an ever decreasing radius. The pear (Figure 9.11) avoids the disadvantage of the right leaning trumpet as the radii in the overall solution do not diminish. This intersection is normally used if there is little space on the side of the merging highway.

At the triangle there are only direct or semi-direct ramps and they should not be designed on too large a scale, as the route features of the open road have to be interrupted at the points where they join the continuous road. The space required and building costs are normally greater than with a trumpet or pear solution. In principle, solutions are possible with two or three structures or a single one, but the latter is a three-level structure (Figure 9.12).

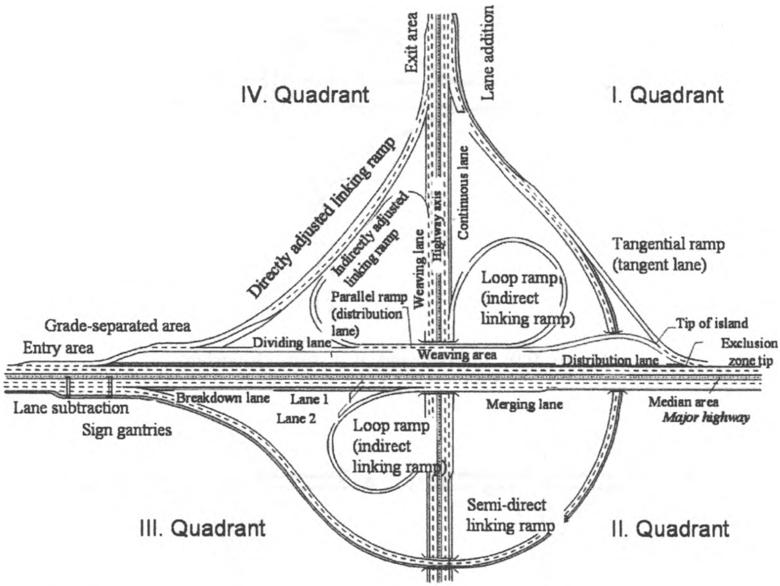


Figure 9.7: Four-arm grade-separated interchange (Weise and Durth, 1997)

(a) Triangle with a three-level structure
(b) Triangle with two crossing structures
(c) Triangle with two-level crossing structure

Four-arm intersections

The cloverleaf is normally the most frequently encountered and most economical basic type of four-arm intersection. Tangential and loop ramps are arranged in all four quadrants (Figure 9.13).

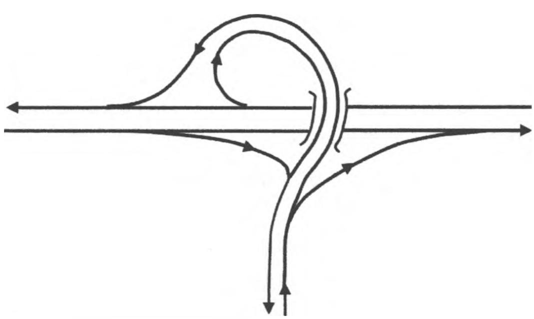


Figure 9.8: Trumpet as an intersection

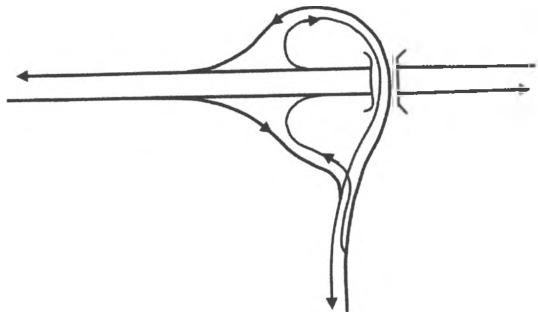
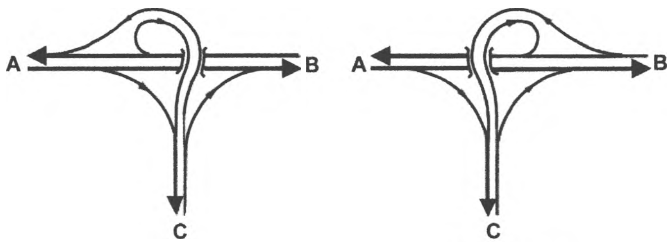


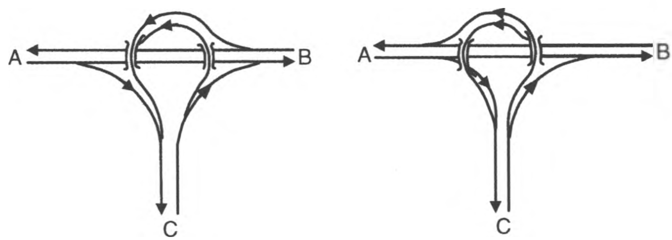
Figure 9.9: Intersection as a semi cloverleaf



(a) Left trumpet
(textbook solution)

(b) Right trumpet
(exception)

Figure 9.10: Trumpet as a three-armed intersection



(a) Preferable line of
B-C link

(b) Preferable line of
A-C link

Figure 9.11: Pear as a three-armed intersection

Only one bridge structure is required and the ramps are relatively short. All the traffic turning left is fed indirectly through loop ramps. The weaving sections that emerge are between 230 and 300 m long. Distribution lanes are normally arranged in the cloverleaf solution and they are fed parallel to the main lane behind a separating strip.

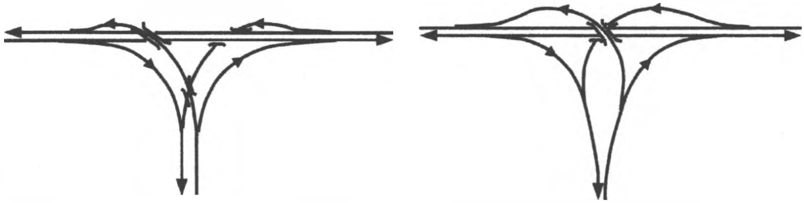


Figure 9.12: Triangles as three-arm intersection

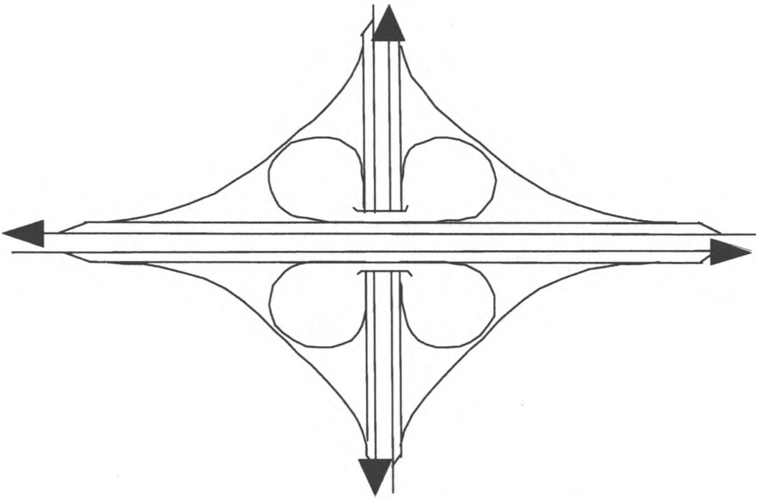


Figure 9.13: Cloverleaf solution

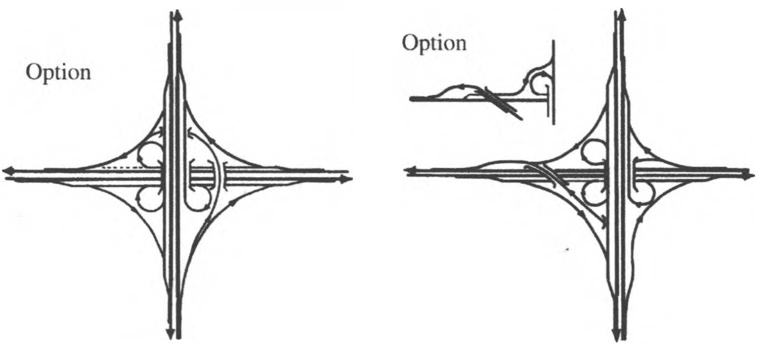


Figure 9.14: Modified cloverleaf solution with semi-direct management of lane for traffic turning left

Modified cloverleafs with semi-direct ramps are useful if individual flows of traffic turning off are very high (Figure 9.14).

Particular care must be taken with regard to maintaining road safety where the distribution lane passes directly into the loop lane.

In the case of the turbine and windmill intersection types, the four weaving lanes of the cloverleaf are replaced by semi-direct lanes for flows of traffic turning off left. Four additional peripheral bridges are required in addition to the central crossing structure. It is not possible for road users to turn back on the linking ramps. There are often fairly large longitudinal cross sections and unfavorable visibility at the crests of the linking ramps (Figures 9.15 and 9.16).

The Maltese Cross, also known as the Los Angeles solution, only has direct or semi-direct ramps. There are no weaving areas and it is not possible for road users to turn back. A four-level structure is required and it consists of five partial areas (see Figure 9.17).

Four-arm partially graded intersection

Textbook solutions for these four-arm intersections are the semi-cloverleaf and the diamond. At-grade intersections are used on the less important roads. In the case of semi-cloverleafs, the location and type of ramps depend on the size and direction of the flows of traffic, local conditions, and the height of the roads being connected. In principle, a difference is made between symmetrically and

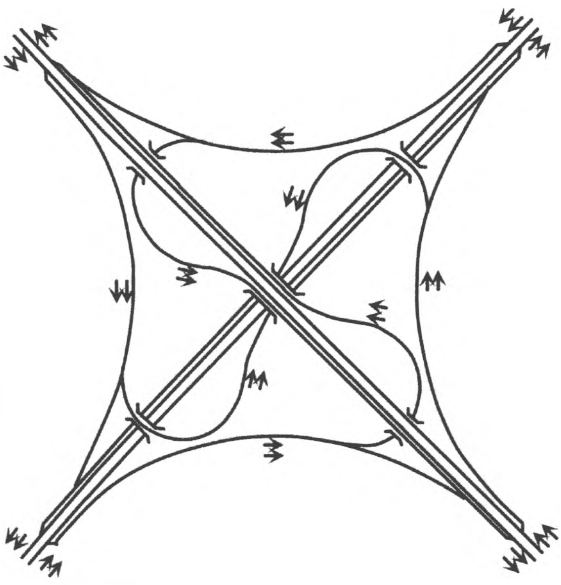


Figure 9.15: Windmill

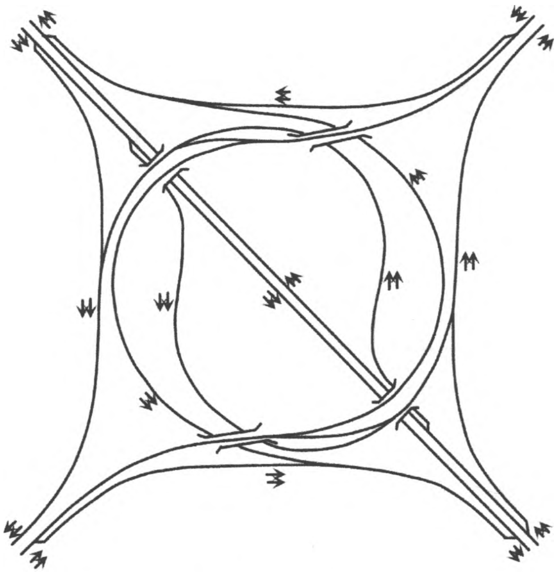


Figure 9.16: Turbine

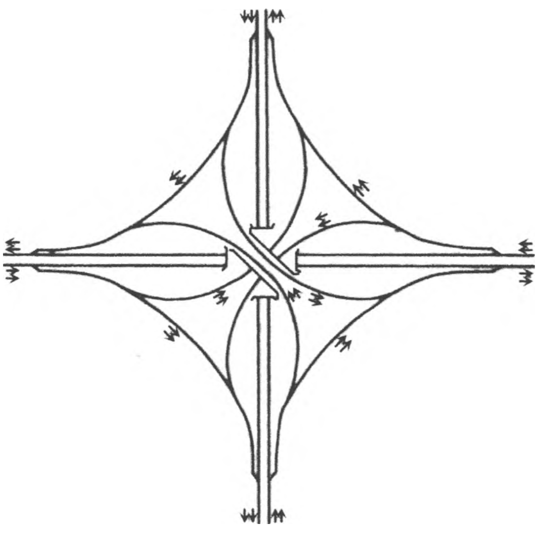


Figure 9.17: Maltese cross

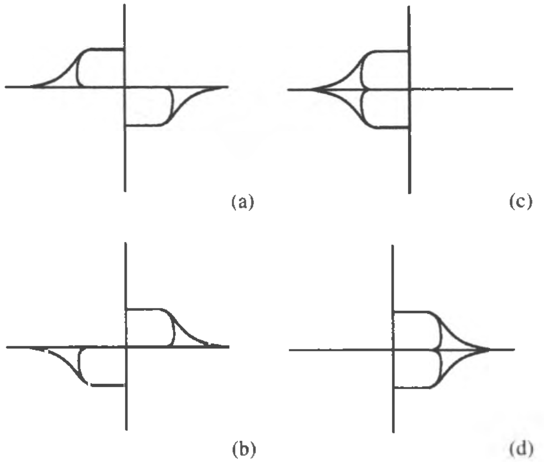


Figure 9.18: Four-arm intersection as a semi-cloverleaf

asymmetrically arranged ramps with regard to more important roads and between the location of lanes for traffic turning off left on less important roads.

Figure 9.18 illustrates a textbook solution. The dominant flows of traffic can be managed on a large scale and the lane for traffic turning off left can be lengthened if necessary. The ramps have to be extended for the lanes for traffic turning off left located within the intersection, behind each other or next to each other or a large structure is required (Figure 9.18 (b) and (c)).

There is no preferable direction for traffic turning off in a symmetrically semi-cloverleaf. It is arranged parallel to the less important road if obstacles exist.

9.7 Questions

(1) How are intersections arranged in principle?
(2) What essential criteria are used to select a type of intersection?
(3) Name the elements that determine the flows of traffic at an intersection.
(4) Name and explain the basic shapes used for intersections.
(5) Explain the advantages of roundabouts.
(6) Explain the make-up of a roundabout by using a sketch.
(7) What do you understand by a highway junction and how is it formed?
(8) What is the difference between a highway exit and a highway junction?

Chapter 10

Road Design Procedures

10.1 Basic Principles

Road design procedures are generally understood to be the development of the three-dimensional alignment of a road, taking into account the surrounding restrictions that apply. The preferred solution is chosen after a complex comparison of various options taking into consideration the following crucial design criteria:

(1) Design efficiency
(2) Traffic safety
(3) Environmental impact
(4) Cost effectiveness

The fundamental course of the road design can be seen in Figure 10.1.

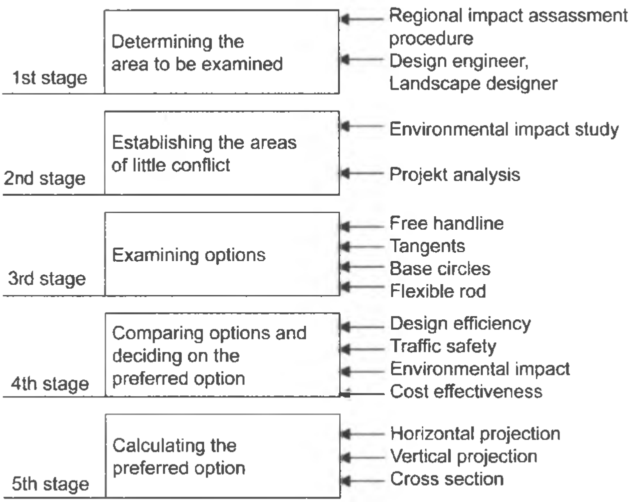


Figure 10.1: Route planning procedures

10.2 Process

10.2.1 The area under examination

The basic principle of the actual road design procedure involves an area that is being examined; normally a topographical map is used for this purpose (Figure 10.2). The boundaries of the area being examined and therefore the extent of the area are determined by a regional impact assessment procedure or agreement reached by all those involved in the planning work. The following principle applies: "The area under examination should be as large as necessary."

10.2.2 The area of little conflict

Normally landscape planners and design engineers are able to establish the areas of little conflict in the area under examination on the basis of a map showing probable resistance to the route from various sources.

The map showing probable resistance to the route is produced by superimposing the subject of protection maps for the various subjects requiring protection.

(1) Animals
(2) Plants
(3) People
(4) Soil
(5) Water
(6) Air
(7) Climate
(8) Agriculture
(9) Cultural and material treasures

The greatest resistance to the construction of a road is shown using the dark colors. Slight resistance is shown by the light sections (Figure 10.2).

If a map outlining the probable resistance to the route has not been produced as a result of an environmental impact study, the design engineer must draw up and assess the restrictions in the area using existing maps, data, and information, and establish the most suitable route (Figure 10.3).

Limits of the area under examination

- Very high
  e.g.residential area, conservation, bird protection, fauna/flora habitat or drinking water areas I and II, stretches of water and ponds
- High with high level of sensibility
  e.g. fairly large connected forest areas, recreation areas with regional meadows with an effect on living space
- High
  e.g. residential area, fairly small structural elements (hedges, fields, simple wooded areas) are important
- Medium with increased level of sensibility
  Water and soil are very important. Archaeological soilmarks, grazing areas, wild geese, planned residential area
- Medium
  Water and/or soil and/or climate are very important, protected agricultural area
- Low-ranking
  Industrial areas, space where buildings have been demolished
- Route where relatively little conflicts is likely

Figure 10.2: Map outlining probable resistance in the area under examination

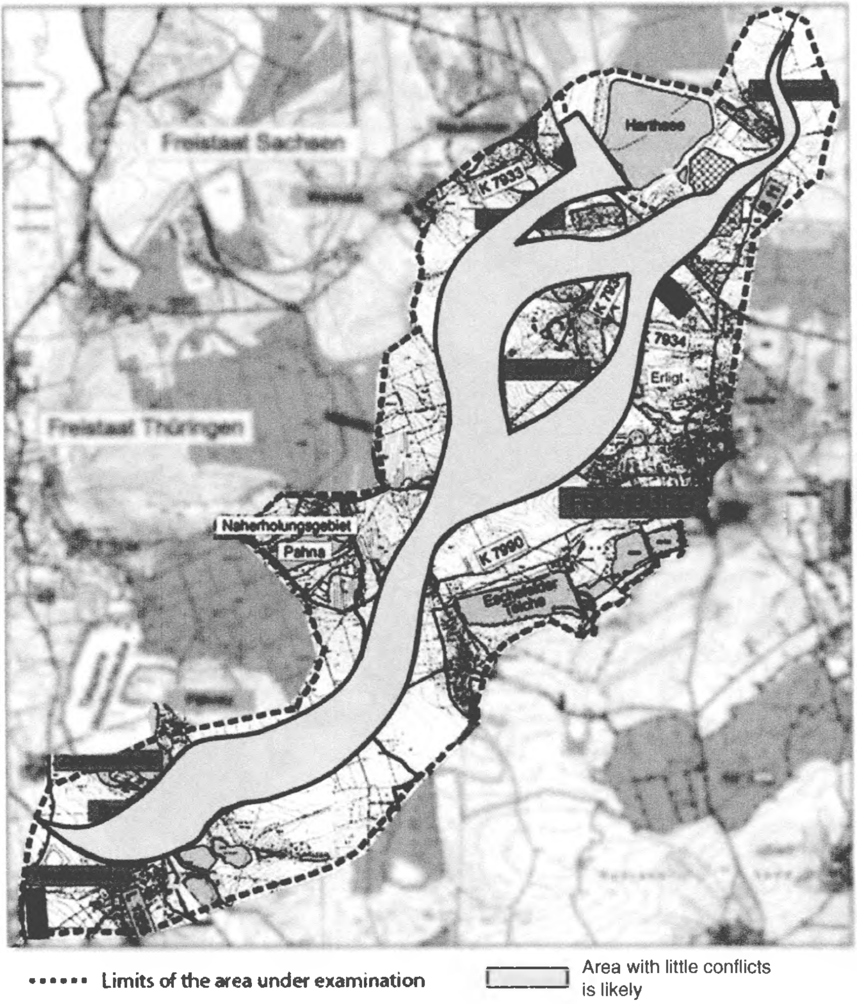


Figure 10.3: Route planning corridor established using an environmental impact study

10.2.3 Examining options

Options are then examined in the road design corridor – i.e. the design engineer must develop at least three main options, which may be linked to each other using sub-options.

The following conditions have to be met to ensure that the options can be compared to each other:

(1) same starting point and finishing point (horizontally and vertically);
(2) same gradient (location of the tangent) at the starting and finishing point; and
(3) The minimum and maximum figures for the design elements on the horizontal and vertical projections must be followed (R, A, H_K, H_W, s).

10.2.4 Route design methods

Various route design methods may be used in conjunction with different aids to develop the options on the horizontal projection, the preliminary graphical draft:

Zero line process (Figure 10.4)

The graphical design line can be determined on undulating territory using a compass. An arc is drawn with a compass between the contour lines to mark the route L_{zi}, which emerges from the ratio of the difference between the contour line Δh and the desired longitudinal slope s_n. The result of the zero line process is a compass line, which represents a succession of tangent lines on a constant longitudinal slope.

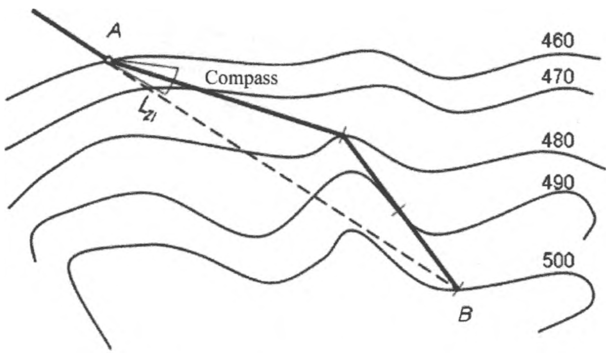


Figure 10.4: Zero line process

$$L_{zi} = 100 * \frac{\Delta h}{s_n}. \qquad (10.1)$$

Base circles (Figure 10.5)

The location of base circles can also be determined depending on the topographical circumstances and other constraints in the road design corridor. By linking the base circles with tangent lines, the preliminary graphical line can be once again obtained.

Tangent polygon (Figure 10.6)

If the design engineer decides on various tangent lines in different sections as a result of the surrounding restrictions, a tangent polygon is created, which has to be rounded off at the intersection points.

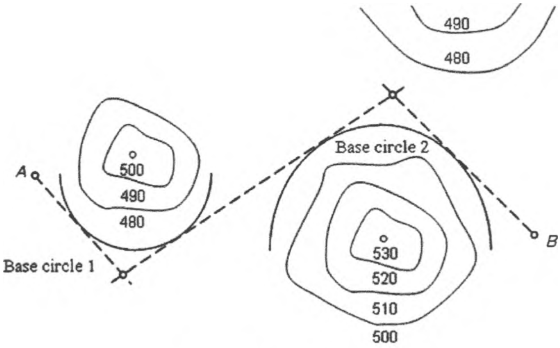


Figure 10.5: Base circles

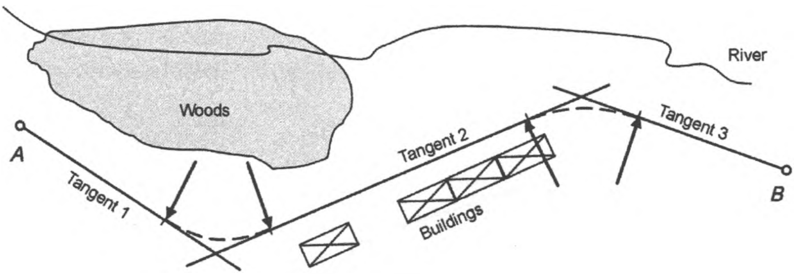


Figure 10.6: Tangent polygon

Freehand line (Figure 10.7)

However, the design engineer can also enter his design idea using a freehand line in the design corridor. This method normally requires a high degree of skill and good technical capability. This rarely provides a well-balanced preliminary graphical draft.

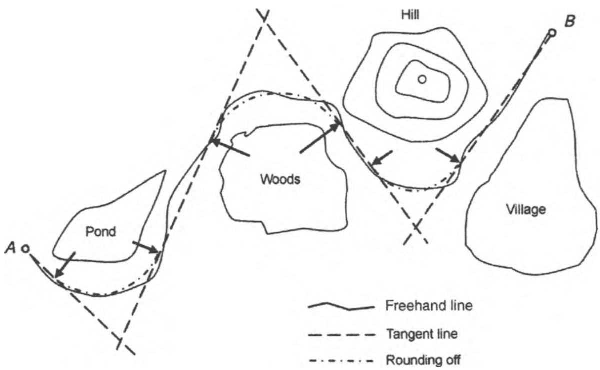


Figure 10.7: Freehand line

10.2.5 Preliminary graphical draft with a flexible rod

The flexible rod technique not only involves flexible rods with different cross sections, but also weights and radius patterns (Figure 10.8). The weights are used to mark the fixed points on the flexible rod line in the road design corridor. If the flexible road is laid between the weights, it automatically assumes the shape of a continuously curved line on account of its elasticity. It can only serve as a ruler for transferring the flexible rod line on to the horizontal projection when the flexible rod marks out the desired route and corrections are no longer necessary by moving the weights. Special radius patterns are used to check the radius figures and later break down the flexible rod line into the traditional road design elements.

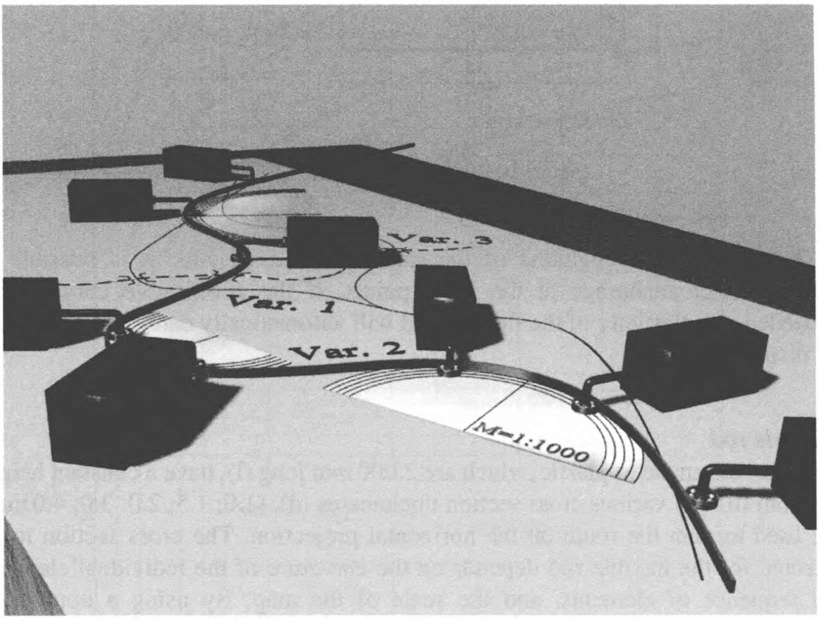


Figure 10.8: Flexible rod with gadgets and aids

Kühn (1976) carried out research work into the theoretical principles and interrelations between the weights and flexible rod and a flexible rod model was produced for practical applications.

Weights

The ball bearing weights made of steel with a mass of 1,000 g and standardized dimensions (Figure 10.9) guarantee that the flexible rod can be turned freely at the support points and this ensures that the flexible rod line is continuous.

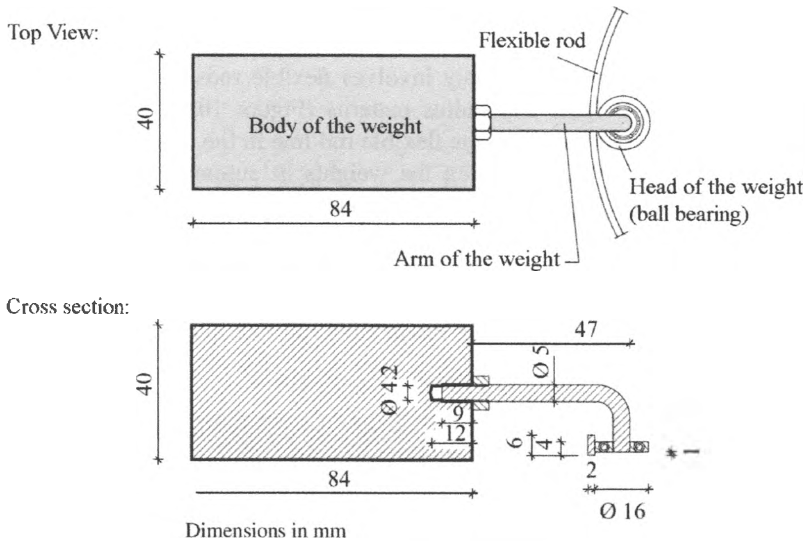


Figure 10.9: Weights (Kühn, 1982a)

Because of the roughness of the surface of the weights, it is possible to achieve exact anchorage at the fixed points. If the permissible curvature is exceeded, the elasticity of the flexible rod will automatically cause the weights to be displaced.

Flexible rod

Flexible rods made of plastic, which are 2,000 mm long (l), have a constant height of 6 mm (h) and various cross section thicknesses (d), (1.0; 1.5; 2.0; 3.0; 4.0 mm) are used to plan the route on the horizontal projection. The cross section to be selected for the flexible rod depends on the curvature of the individual element, the sequence of elements, and the scale of the map. By using a nomogram (Figure 10.10), the suitable cross section for the flexible rod can be selected by setting the minimum radius of a circular arc R_{min} within a sequence of curves. If the smallest radius is not known from the outset, the cross section of the rod is determined by the design specifications (Figure 10.11).

Radius patterns

The radius patterns can be pushed under the head of the weights to roughly monitor the radius figures within the preliminary graphical design. They are also used to determine the radii and break down the flexible rod line as part of the subsequent axis calculations.

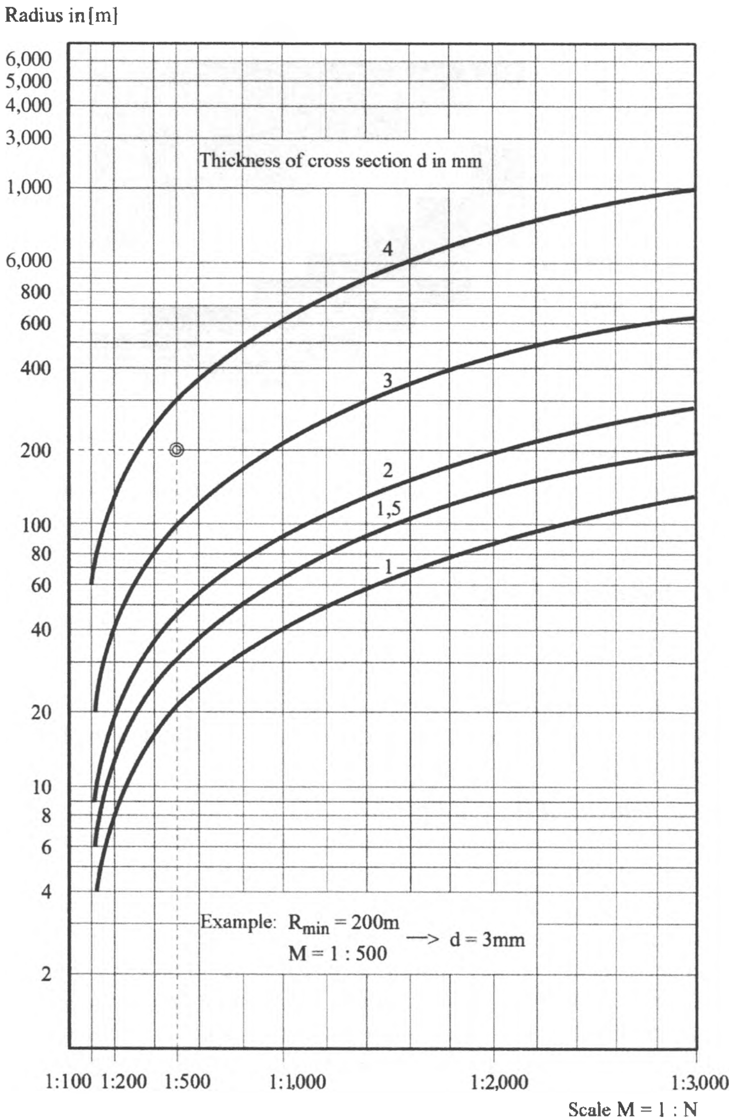
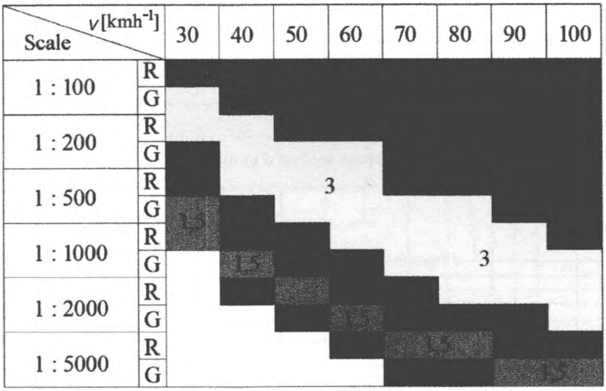


Figure 10.10: Nomogram for determining the cross section of the flexible rod



R - radius guideline
G - radius limit

$v = 50\ \text{kmh}^{-1}$, M = 1 : 500 → d = 3 mm

Figure 10.11: Determining the cross section of the flexible rod from the design guidelines (Kühn, 1976)

Using and calculating the flexible rod line

The following general procedures are used to accurately break down the flexible rod line:

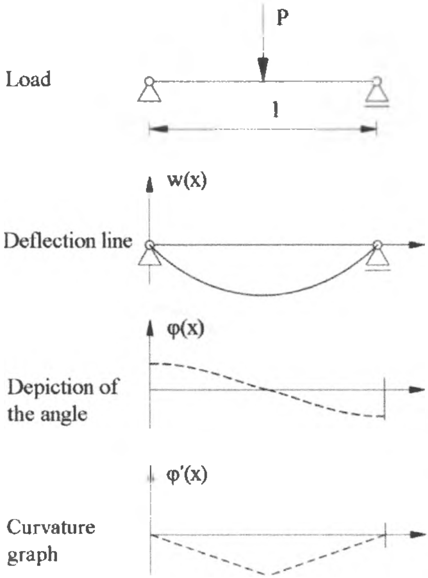
– ***Staking out the graphically determined coordinates***

This process is not used in practice for reasons of accuracy and in the light of the staking out techniques that are available.

– ***Static calculations of the rod line***

When using ball bearing weights, the flexible rod represents a continuous beam with huge fixed elevations or depressions at the support points, viewed from a purely static point of view. Only lateral forces, but no longitudinal forces or bending moments, are registered at the support points. Theoretical and practical experiments have demonstrated that it is possible to create the road design elements (Kühn, 1977) of straights, clothoids, and circular arcs and combinations of these using the flexible rod, which is mounted on bearings from a static point of view. Depending on the forces exerted, characteristic curvature images are created which enable a design engineer to recognize the existing road design elements (Figure 10.12). Apex clothoids (two clothoids moving in the same direction) are created if the forces present are centric or eccentric as a result of the pure lateral forces. This type of curvature is only acceptable if the change of direction is slight and if the abutting radii are large. A curvature image of a combined curve is created if two weights exert forces in the area of the supposed circular arc.

If forces are exerted on a straight rod and its shape is altered, it initially assumes a position, which concords with its bending line. If the displacement of the flexible rod is large, the BERNOULLI theory about the uniformity of cross



$$E * I * w(x) = M_{max} * l^2 * c$$

$$c = \frac{1}{12}(3\xi - 4\xi^3), \quad (0 \leq \xi \leq 0{,}5)$$

$$c = \frac{1}{12}[3(1-\xi) - 4(1-\xi)^3], \quad (0{,}5 \leq \xi \leq 1)$$

$$E * I * \varphi(x) = M_{max} * l * c'$$

$$c' = \frac{1}{4} - \xi^2, \quad (0 \leq \xi \leq 0{,}5)$$

$$c' = \frac{3}{4} - 2\xi + \xi^2, \quad (0{,}5 \leq \xi \leq 1)$$

$$E * I * \varphi'(x) = M_{max} * c''$$

$$c'' = -2\xi, \quad (0 \leq \xi \leq 0{,}5)$$

$$c'' = -2 + 2\xi, \quad (0{,}5 \leq \xi \leq 1)$$

Figure 10.12: Deflection line, depiction of the angle and curvature if the load is centric (Kühn, 1977)

sectional areas when bent no longer applies because of the large degree of curvature. In addition, the direction and amounts of the applicable forces change if the deflection is great. In the end, the familiar differential equation for the deflection line is no longer valid in the case of major deflections

$$W''(x) = -\frac{M(x)}{E*I(x)}, \tag{10.2}$$

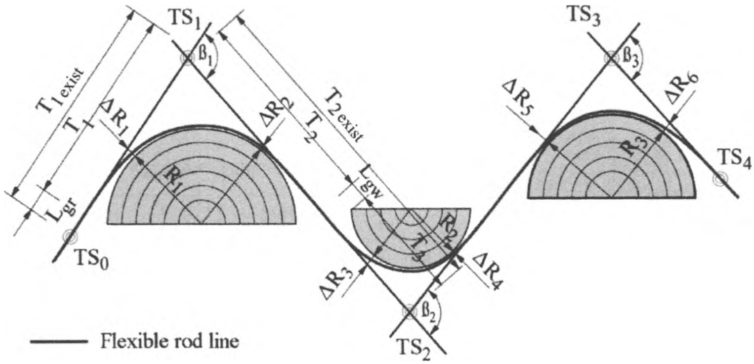
and the distortion has to be determined using a nonlinear differential equation of the second degree:

$$-\frac{W''(x)}{(1+[W'(x)]^2)^{\frac{2}{3}}} = \frac{M(x)}{E*I(x)}. \tag{10.3}$$

Experiments (Kühn, 1977) have shown that there is a difference between the course followed by the flexible rod line and the deflection line calculated using the differential equations because of the large deflections of the flexible rod, i.e. the static calculation approaches to the deflection line are not suitable for accurately calculating the flexible rod line, taking into consideration the particular road design conditions.

– ***Breaking down the flexible rod line into traditional design elements***

When breaking down and calculating the flexible rod line using the road design elements of straights, clothoids, and circular arcs, the design engineer has to follow the preliminary graphical design line as closely as possible in order to reduce to a minimum any discrepancies between the calculated axis and the



Operating procedure:

1. Lay the tangents on the flexible rod line.
2. Determine the radius figures and the points at the center of circles
3. Define the tangent retraction ΔR
4. Calculate the clothoid parameter and the relevant defining parts

– $\frac{\Delta R}{R} \xrightarrow{\text{E-Tafel}} l \longrightarrow A = R * l$ (A runden)

– multiply the remaining figures outside the angle with A

5. Graphically define the intersection angle of the tangent and the length of the existing tangents (β,Tvorh)
6. Calculate the lengths of the tangents, the middle point angle and the length of the circular arc:

– $T = (R + \Delta R) * \tan\frac{ß}{2} + x_M \pm d,\ d = \frac{\Delta R_1 - \Delta R_2}{\sin ß}$

– $\alpha = ß - (\tau_1 + \tau_2),\ L_B = \frac{R * \pi * \alpha}{200}$

7. Check the tangents

– Boundary tangent: $L_{gr} = \overline{TS_0\ TS_1} - T_1$

– Tangent through point of inflection: $L_{gw} = \overline{TS_1\ TS_2} - (T_2 + T_3) < 0{,}08\,(A_2 + A_3)$

Figure 10.13: Breaking down the flexible rod line into curves (Kühn, 1977)

flexible rod line. It is necessary for this purpose to accurately define characteristic definitions from the flexible rod line so that they can be used to calculate the parameters and staking out values. The breaking down and calculation process used will determine which definitions should be inferred graphically. Generally speaking, it is possible to use two different procedures:

(1) A breaking down and calculation of the flexible rod line by curves on the basis of the combined curve design.
(2) A breaking down and calculation of the flexible rod line using the inflection line and the closed C-arc at different sections.

When using the flexible rod road design system in practical applications, the breakdown of the flexible rod line by curves into traditional road design elements has proved to be particularly worthwhile. The procedure is explained in Figure 10.13.

When breaking down the flexible rod line into the traditional road design elements, discrepancies will emerge between the flexible rod line and the axis calculated:

(1) The discrepancies between the flexible rod line and the traditional road design elements that have been calculated are slight.
(2) The size of the discrepancies directly depends on the type of flexible rod line and the road design elements that have been calculated. If the flexible rod line has not been broken down into a sequence of apex clothoids when the weights are centric or eccentric, but a sequence of elements involving a clothoid, a circular arc, and a clothoid, discrepancies will emerge between the two lines because of the different curvature ratio. The greatest discrepancies will occur in the circular arc areas (Figure 10.14a). On the other hand, the two lines do not differ from each other or only marginally within the clothoid. In the case of minor changes of direction and slight circular arc lengths, the discrepancies are also minor and can be disregarded.
(3) But if the weights are located at the beginning and end points of the supposed circular arc, only slight discrepancies emerge when breaking down the flexible rod line into the traditional sequence of elements (Figure 10.14b).
(4) When breaking down the flexible rod line, irregular figures are normally found for parameters A and R. If rounded parameters or standard figures are to be used, checks must be made to determine whether the discrepancies that arise are acceptable. Rounded parameters should normally be used.

Summary

The flexible rod has proved its worth as an aid for road planning when examining different options. Depending on the scale of the map and the minimum radius that applies, the rod should have a cross section that guarantees elastic distortion.

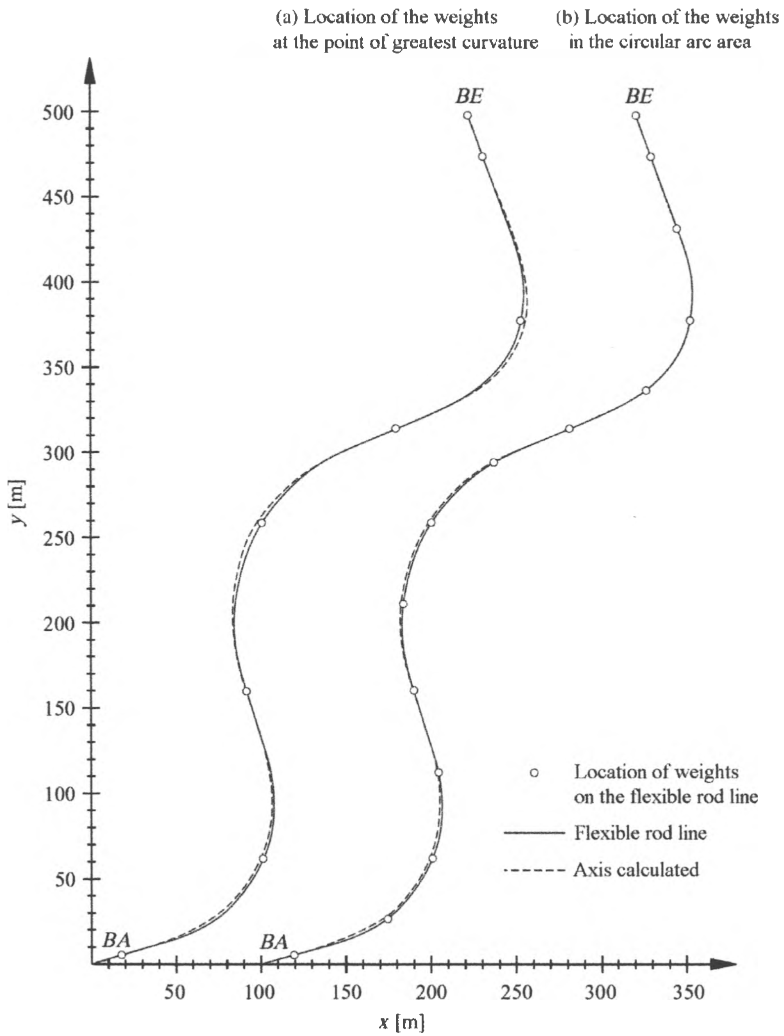


Figure 10.14: Approximation of the flexible rod line using the traditional road design elements of straights, clothoids, and circular arcs (Kühn, 1982b)

The design engineer can examine various options with little manual effort as part of the preliminary graphical design phase within the set road design corridor.

Rough checks are made to ensure that the standard values and limits are followed during the search for the route by using special patterns and a scale of length on the flexible rod.

Static checks have demonstrated that the traditional road design elements can be generated by using ball bearing weights with a flexible rod.

Traditional road design elements are used to break down and calculate the flexible rod line. If the preliminary design is good in graphical terms and the breakdown process is accurate, there will only be slight discrepancies between the flexible rod line and the axis that has been calculated, i.e. the flexible rod line will largely concur with the course established by the road design elements of straights, clothoids, and circular arcs.

Carrying out road design work with a flexible rod is a design methodology, which was specially developed for difficult situations and upgrading roads. A good preliminary graphical design with the flexible rod forms the basis of this methodology. Characteristic determining factors for calculating the ensuing axis are taken from the flexible rod line. Familiar CAD systems are used to calculate the parameters, the major and minor points. The procedure when using this design methodology is shown in Figure 10.15.

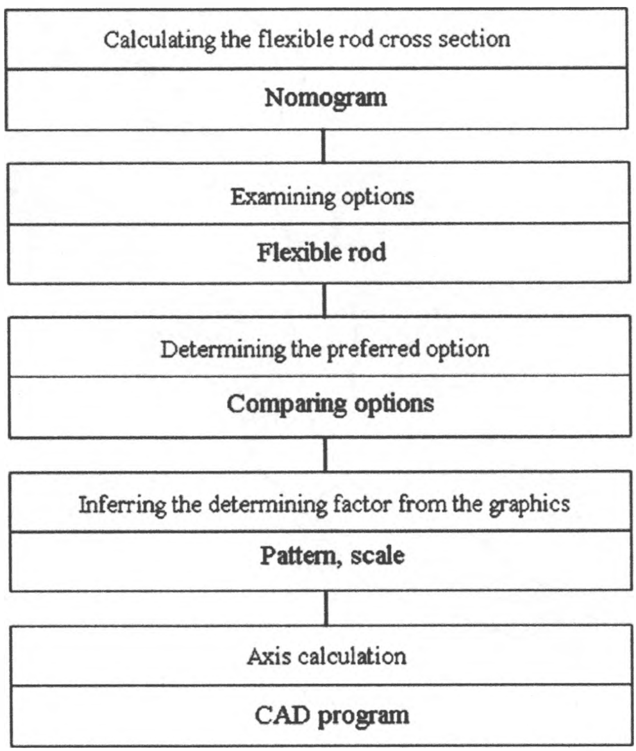
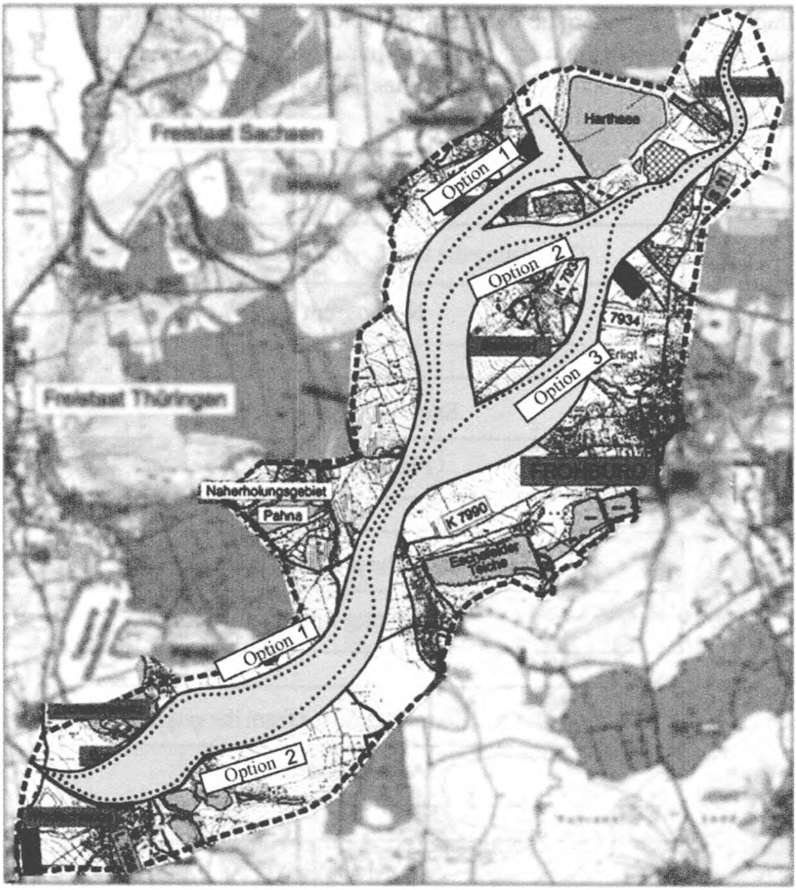


Figure 10.15: Procedure when designing a route with a flexible rod (Kühn, 1982c)

10.2.6 Comparing options

The zero option (existing situation), zero plus one option (upgrading based on the existing road design), and the main options drawn up (Figure 10.16) are calculated on the basis of the four standard design criteria and are compared to each other as part of a broad comparison (Figure 10.16). Various features are normally set for the individual design criteria, which allow formal assessments to be made in tabular form (Tables 10.1 to 10.4).



—— Limits of the area being examined   ···· Main options

- - - Limits of area arousing little dispute

Figure 10.16: Main options

Table 10.1: Effectiveness

Goals	Evaluation Parameters
Appropriate traffic quality for motorized traffic	– Average traffic speed – Traffic density [vehicles/km] – Average waiting time at intersections at peak periods – Degree of utilization of intersections and sections of route
Good links and development for cyclists and pedestrians	– Length of diversions [km] – Difference in altitude [m] – Number of crossings [no.] – Seperation from motorized traffic – Direction and waiting times at intersections [s]
Good local public transport quality	– Special lanes – Priority at intersections
Adequate development of neighboring areas	– Number of business access roads [no.] – Lengths of diversions [km] – Traffic moved to surrounding area

Table 10.2: Traffic safety

Goals	Evaluation parameters
Allowing safe maneuvers	– Balance of radii – Three-dimensional alignment – Camber – Diagonal gradient – Drainage
Creating safe overtaking areas	– Visibility for overtaking – Parts of the route where overtaking lane switches – Additional lanes
Three-dimensional separation of the types of traffic	– Cycle traffic lanes on open roads and at intersections – Special lanes for agricultural traffic
Minimizing conflicts at intersections	– Standard upgrading (at-grade, partly at-grade at the same level, roundabout) – Protection for traffic turning left at stoplights – Adequate room for traffic turning off
Minimizing the consequences of accidents	– Frequency of obstacles – Distance of obstacles from edge of road – Quality of protection against obstacles

Table 10.3: Environmental impact

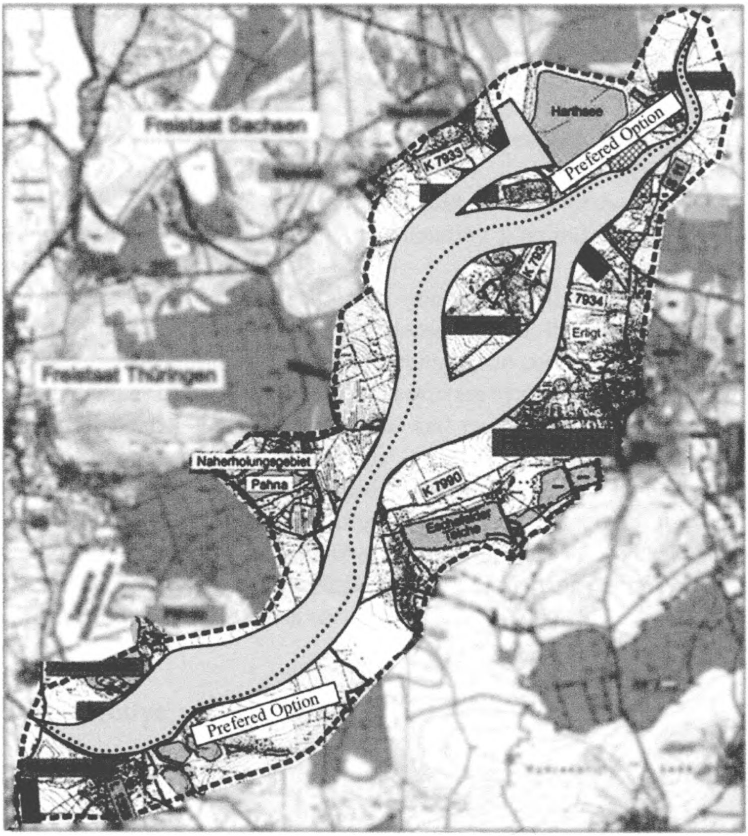
Goals	Evaluation parameters
Little noise or air pollution	– Assessment level [db(A)] for noise output – Concentration of air pollution: CO, CO_2, NOx, HC, Pb, particles, dust
Little use of areas requiring protection	Size [m²] and quality of – traffic areas, additional areas, residual areas, soil quality (soil score)
Little dissection of important area functions and minimizing negative influences on: – Functional separation – Types of use – Animal movements – Network of habitats (biotope network) – Variety of species – Drainage above and below ground – Quality of soil	Size of area [m²] – Distance [m] – Length [m] – Density of traffic [vehicles/h] – Width of lane and auxiliary equipment [m] – Number of accidents with animals [no.] – Use to which surrounding area is put – Isolation of partial areas – Embankment height and depth of cutting [m] – Soil score
Good microclimate	– Degree of sealing [%] – Ratio of seepage [%] – Volume of green space [m³]
Road design taking into account landscaping	– Embankment height and depth of cutting [m] – Degree of harmonization of design elements with the topography – Perceptibility of direction and distance
Minimizing separation and optimizing quality of time spent here	– Lane width [m] – Crossing places – Density of traffic [vehicles/h] – Size of area being used
Road space design to fit area	– Dimensions of road space – In proportion to neighboring buildings – Degree of separation created by design

Table 10.4: Cost effectiveness

Goals	Evaluation parameters
Investment expenditure	– Planning costs [€] – Land purchase costs [€] – Building costs [€] – Equipment costs [€] – Cost of compensation measures [€]
Conservation, maintenance and operating costs	– Buildings conservation and maintenance costs per annum [€/annum] – Operating costs [€/annum]
Total expenditure	– Total expenditure in estimate period [€]

While the efficiency of the individual options is proven by traffic surveys, an environment impact study is used to check the environmental impact of the project. In the case of cost effectiveness, the operating costs are often taken into account as well as the investment costs. Traffic safety is assessed objectively and in the long term using safety audits.

Once the analysis of the various options has been completed, the design engineer must draw up a priority list and sequence and finally determine the preferred option, which will then be confirmed by the public authorities as part of a planning permission procedure (Figure 10.21). Once a preferred option has been confirmed, the road design work is completed.



Limits of the area being examined

Prefered option

Limits of the area arousing little dispute

Figure 10.21: Preferred option

10.3 Questions

(1) Name the essential criteria for comparing different options.
(2) What is a map showing probable resistance to a route and how can you obtain a corridor that provokes the least conflict?
(3) Which methods are you familiar with when it comes to determining the location of a road?
(4) Explain the elements and procedure for selecting a route using a flexible rod.

Chapter 11

Visualization

11.1 Theoretical Principles

11.1.1 Preface

The process of obtaining pictorial images with different information is generally known as visualization.

McCormick, et al. (1987) defines it as an aid, tool, or method for interpreting visual data entered into a computer or generating pictorial images from multi-dimensional data.

The development of visualization in general and the use of visualization techniques in particular have directly depended on the power of computer technology. The transition from manual techniques to computer visualization was only possible when suitable hardware and software systems became available and when pictorial images could be processed and exported using input and output devices that changed the information.

Pictorial images of different qualities are increasingly being used in road design to better illustrate planning intentions, assess concerns objectively, and provide more realistic three-dimensional (3D) planning results and assess them in a more realistic way because of the separate work carried out on the horizontal and vertical projections and the cross section (Kühn, 2002a).

11.2 Perspective

11.2.1 Definition

Perspective (*Latin: perspectare, to see through*) summarizes the opportunities of mapping 3D objects on a two-dimensional (2D) plane in such a way that a pseudo-3D (p3D) effect is achieved. For this purpose, depending on the case at hand, various image methods are used – central projection, parallel projection, vanishing point projection, and similar applications. Central perspective is the primary method used to generate perspective images for road design using computer visualization.

11.2.2 Central perspective

The process of seeing in the human eye corresponds to an optical image development process in a lens system with triple refraction (cornea, lens, and vitreous body). In simplified terms, the image process in photography in a traditional camera can be viewed as a single-lens system (lenses portrayed as a line in Figure 11.1). Here a realistic reverse image is focused by beams of light from an object to an image point.

If we consider the case of small apertures (where only center point rays pass through the lens), it is possible to use the mathematical principle of central perspective.

A ray of light running between the projection center and an object point passes through an image plane that is vertical to the visual axis and forms an image point. Figure 11.1 makes it clear that the image process in optics (single-lens system) and the central perspective produce the same images of the object using this simplified process. This means that if the assumptions and simplifications that have been made are taken into account, a driver's optical perception can be described accurately enough by using the correlations of central perspective.

Uniform parameters have to be defined to enable people to use the laws of central perspective in road design in a meaningful manner (Figure 11.2):

Eye level: Stylized point in space, which corresponds to the position of the driver's eyes.

Target point: Point in space which corresponds to the driver's fixation point.

Target distance: Distance between eye level and the target. The target distance corresponds to the driver's forward orientation.

Visual orientation: Direction vector between the eye level and target point.

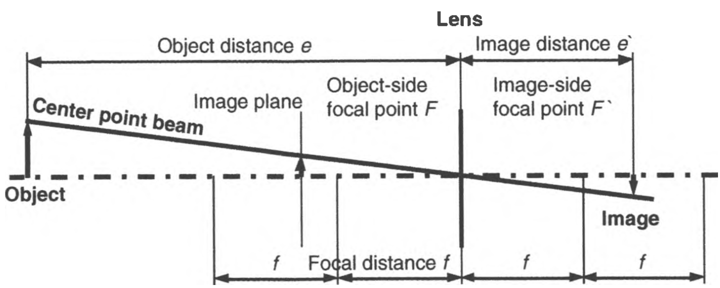


Figure 11.1: Single-lens system with restrictions on center point beams (Zimmermann, 2001)

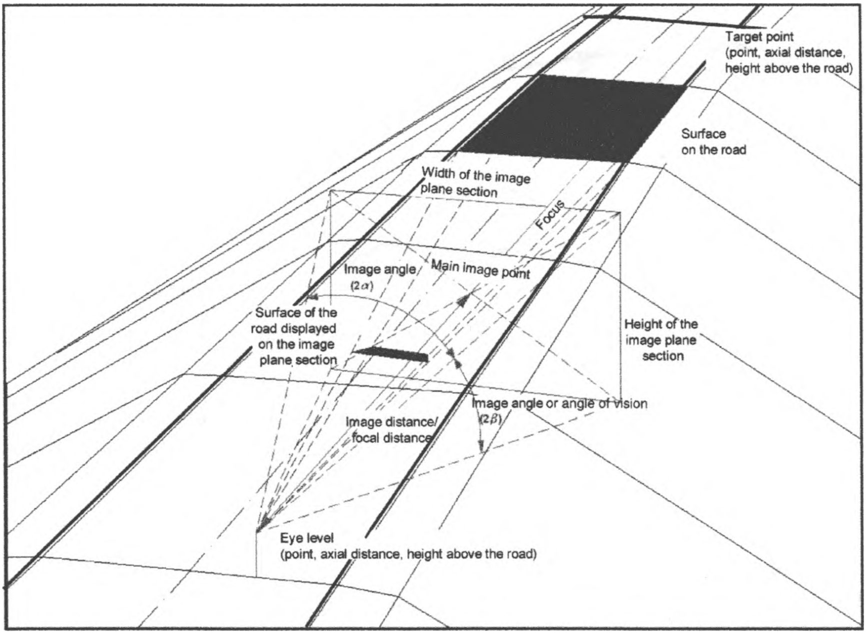


Figure 11.2: Central perspective representation (Zimmermann, 2001)

Visual axis: A straight line, which runs through the eye level and target point.

Image plane: Plane, which stands vertically on the visual axis and has a set distance (image distance) from the eye level.

Image section: Rectangular surface on the image plane.

Image distance: Distance between the eye level and image plane.

Visual ray: Ray, which goes from the eye level to a point in space being mapped.

Image point: Created when a visual ray cuts through the image plane.

Main point: The image point which is created when the visual axis is penetrated by the image plane. This represents the center point of the image.

Angle of vision: Angles which arise between the eye level and the restriction on the image section. They are called α, β, and ψ.

11.2.3 Model assumptions

Various parameters related to central perspective have to be taken into account when generating perspective images. They describe drivers' field of vision, from which they absorb essential information about the course of the road.

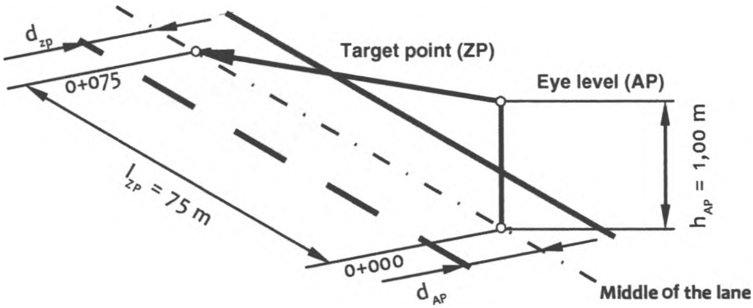


Figure 11.3: Model assumptions for the eye level and target point for calculating perspective images (HViSt, 2008)

The parameters that can be deduced from the central perspective are used as model assumptions for calculating the perspective image. The model assumptions for the eye level and target point for calculating the perspective image are shown graphically in Figure 11.3.

Eye level height h_{AP}

The eye level height depends on the driver's seat height in the vehicle and therefore on the height of the vehicle. HViSt (2008) sets this as 1.00 m for passenger cars.

Eye level axial distance d_{AP}

The eye level axial distance cannot be set exactly as it changes depending on the alignment. In straight areas, the vehicle is in the middle of its own lane, while in bends the position moves to the inside of the bend. Therefore, a simplification was necessary in order to specify a generally valid figure, which is given as $d_{AP} =$ middle of own lane in HViSt (2008).

Target point height h_{ZP}

The driver receives the necessary information about possible obstacles or moving vehicles ahead if the height of the target point is fixed as $h_{ZP} = 0.00$ m. This also means that it is easy to recognize the course of the road.

Target point axial distance d_{ZP}

The target point distance from the central axis cannot be uniformly measured or determined, as is the case with the eye level axial distance. The driver's fixation moves toward the inside of the bend in bends, as is the case with the eye level axial distance. The middle of the driver's lane is set as the target point axial distance d_{ZP}.

Target point distance l_{ZP}

The target point distance corresponds to the forward orientation and mainly depends on the road characteristics. As a simplification, the distance is uniformly set at $l_{ZP} = 75$ m. This figure may fall, e.g. in narrow bends with small radii of curvature or be exceeded on long straights.

Focal distance f
The focal distance or the visible image section for the driver is an important model assumption. This involves focusing on a certain target area or the size of the image area that can be seen. This parameter is especially critical for driving behavior in bends. The focal distance is $f = 50$ mm for normal sight when the human eye is resting (equivalent to a 35-mm small picture format).

Table 11.1 contains a summary of the standard model assumptions for calculating perspective images.

11.2.4 Degree of detail

The perspective images can be subdivided as follows depending on the degree of detail:

Grid perspective (GNP)
Only the key edges of objects are shown on a grid perspective in a 3D terrain model (grid model) (Figure 11.4).

Shaded perspective (FGP)
If the line of the road and the terrain in a grid perspective are given different colors, this produces a shaded perspective (Figure 11.5).

Photo-like perspective (FRP)
If the line of the road, the terrain, and other objects are provided with textures and shadow is marked to match the daylight, a photo-like perspective is created, which can hardly be distinguished from a real photo (Figure 11.6).

Table 11.1: Model assumptions for calculating perspective images from the driver's point of view (HViSt, 2008)

Parameter	Height above roadway	Position in cross section
Eye level	1.00 m (h_{AP})	Middle of the driver's lane (d_{AP})
Target point	0.00 m (h_{ZP})	Middle of the driver's lane (d_{ZP})
Target point distance	75 m (l_{ZP})	
Focal distance	50 mm (matches 35 mm small picture format) or horizontal picture angle $2\alpha = 40°$	
Aspect ratio	4:3 (width:height)	

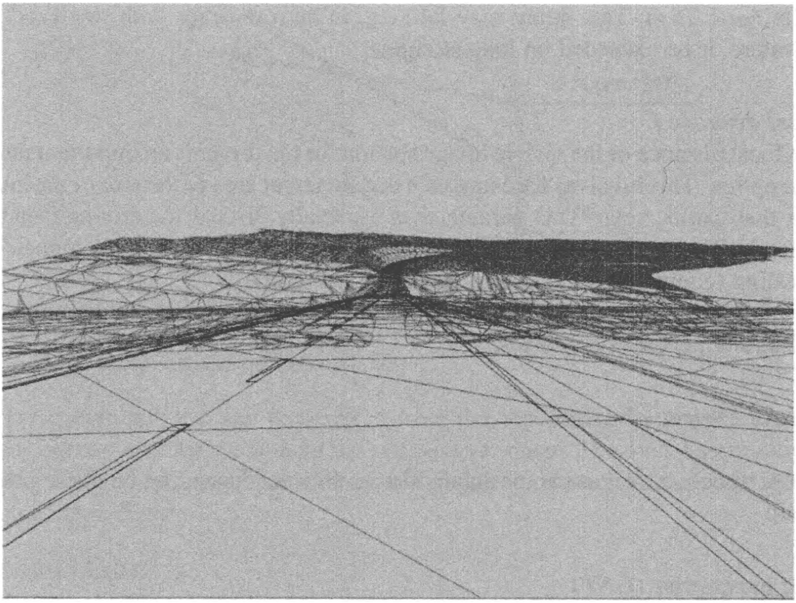


Figure 11.4: Grid perspective (GNP) (Kühn, et al., 2007)

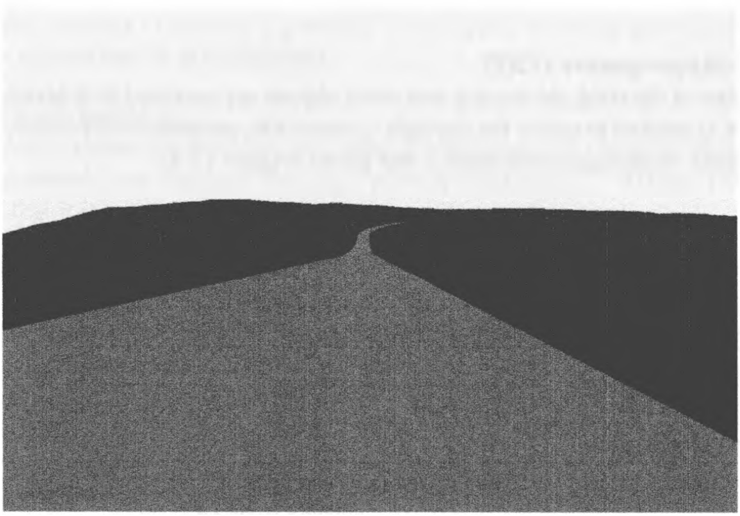


Figure 11.5: Shaded perspective (Kühn, et al., 2007)

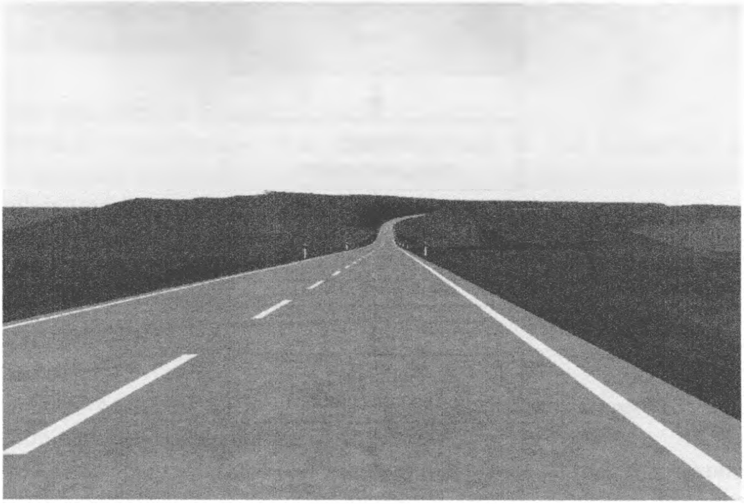


Figure 11.6: Photo-like perspective (Kühn, et al., 2007)

11.3 Computer Visualization

11.3.1 Process

By using a computer, data, structures, and interrelations can be represented in a graphical form. The process of generating images with different information is generally known as visualization (Figure 11.7). Various steps are necessary in order to generate images from data.

The raw data is smoothed out and any existing errors are corrected (filtering). In the ensuing mapping stage, non-geometrical data is converted into geometrical data. At the end of the process, image creation (rendering) is carried out, i.e. the geometrical data is converted into image data.

The following criteria can be used to make a fundamental distinction between types of visualization (Kühn, et al., 2007):

(1) Dimensions of the area being displayed:
- Two-dimensional.
- Three-dimensional

(2) Type of display dependent on time:
- static display;
- dynamic display.

(3) Completeness of the images:
- abstract display;
- realistic display.

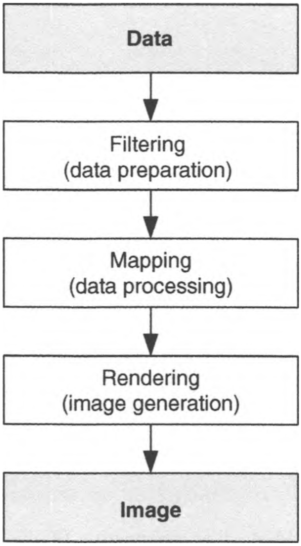


Figure 11.7: Steps in the visualization process (Kühn, et al., 2007)

11.3.2 Kinds of images

2D and p3D images

Two-dimensional images are constructed on the basis of 2D data and elements. When outputting these images, it is not necessary to carry out projections or take visibility into consideration because no objects are concealed by the missing third dimension.

Normally 3D scenes are displayed on a plane surface (paper, monitor, or screen) in the central perspective or axonometrically. This is why Buchroithner (2001) speaks of p3D images. This may lead to distortions in perspective or concealment.

Interactive and navigation technologies need to be provided in order to use the third dimension, as the input and output devices are normally only designed to be 2D.

As long as no mention is specifically made of the type of display, the terms "3D" and "three-dimensional" always relate to the basic data.

Static and dynamic images

The difference between static and dynamic images is an important criterion for classifying visualization methods. Static presentations do not change over time, i.e. changes to the image can only be caused by interaction. However, dynamic images change in time without any interaction. If the time changes are continuous, we speak of an animation.

Complete and abstract images

The degree of completeness of an image depends on how completely a given amount of data is displayed in an image. Complete images are called realistic and incomplete images are abstract.

Depending on the case in point, a decision must be made about the degree of completeness. The degree of detail and the information content are important qualitative criteria for describing the degree of completeness.

Animation

In principle, Stüttgen, et al. (1995) makes a distinction between character animation and camera animation.

In character animation, the individual image can be animated or displayed by moving or visualizing the object. By means of camera animation, it is possible to simulate driving and movement sequences along a camera path and the individual object is shown moving in the project environment.

If speed, the observation location, and driving line are fixed – generally in the direction of the road axis – and cannot be changed, a static animation is created when computing the image sequences. In every dynamic animation, the criteria mentioned above can be freely selected in every perspective image. The movement mechanism is guided by means of navigation (e.g. a joystick).

The most important disadvantage of the individual image is the so-called snapshot, which is the complete opposite of the sequence of images that the driver perceives when driving along a road. Because he moves on the road and also perceives moving images, a camera animation should be used to simulate the driving area. A static animation simulates the journey of an undisturbed single vehicle with idealized marginal conditions.

The following model assumptions should be followed:

(1) Frame rate: 25 pictures per second.
(2) Sequence duration: illustration of critical area (30 to 100 seconds).
(3) Speed: speed v_{85}.

Simulation

Simulation is generally understood to be the virtual reproduction, design, and change of an object, process, or system using computers. The results obtained by simulation should largely tally with reality (Kühn, 2002a). This reproduces real circumstances and the effects of different factors on reality are increasingly checked (Figure 11.8). Visualization is advisable to make the simulation process and its results visible and appraisable.

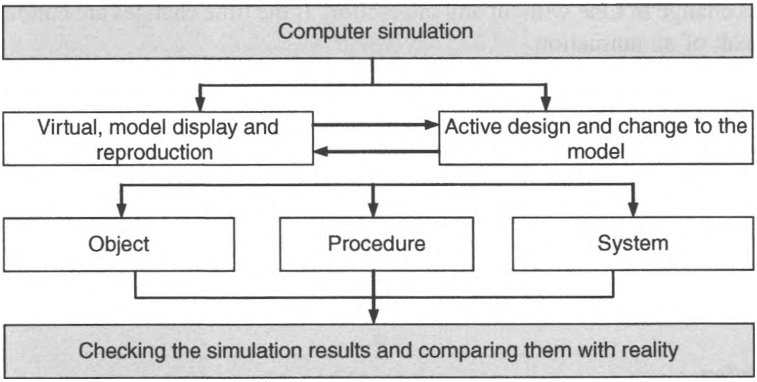


Figure 11.8: Fundamental idea in computer simulation (Kühn, 2002a)

According to Kühn (2002a), simulation can be used in the design process for the following main tasks (Figure 11.9):

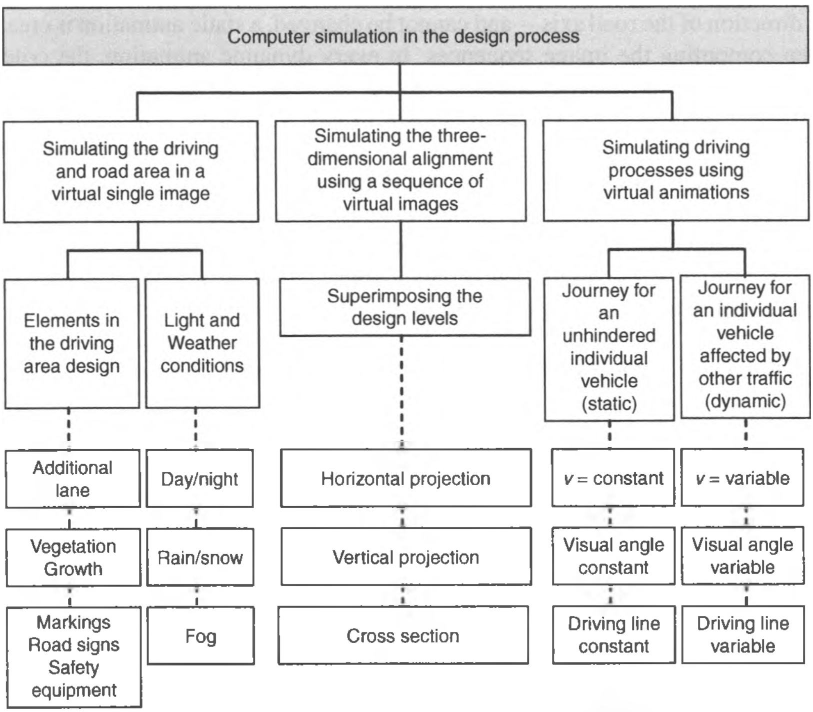


Figure 11.9: Simulation processes in road design (Kühn, 2002a)

(1) Designing the driving and road area
Simulation with an individual image is required so as to be able to check the effects of different individual features on the design and perceptibility and comprehensibility of the driving and road area.
(2) 3D alignment
A sequence of images is required to check the 3D alignment for any shortcomings at critical sections of the route.
(3) Driving procedures
Animations are required to check driving and movement sequences or to assess the expected speed.

11.4 Visualization Techniques

11.4.1 Introduction

Depending on the type of representation and the digital geometric data that are available, it is possible to make a fundamental distinction between visualization techniques (Table 11.2).

While the assignment of the basis of the data is clear, because only 2D or 3D data is available, the type of display is defined differently. It is possible not only to have 2D and 3D images, but also to have a p3D image using perspective images.

Manual techniques (marking out the site, physical models, etc.) are only seldom used nowadays for expense reasons and because they only provide a snapshot image that is unique. Montages on maps or aerial photographs and 2D animations are commonplace as visualization techniques because of the availability of 2D data. Perspective images based on 3D data are used for

Table 11.2: Classification of visualization techniques (HViSt, 2008)

	Kind of image		
Basis of data	2D	p3D	3D
Two-dimensional (x, y coordinates)	Montage on maps and aerial photos	Montage on illustrated maps and aerial photos with perspective	
	2D animations		
Three-dimensional (x, y, coordinates)		Perspective images and perspective image animations	Marking out on site
		Montage of perspective images in existing photos	Physical models
			Stereoscopic images (individual images, animation)

the normal p3D visualization techniques. p3D animations are increasingly being used in addition to perspective images and montages of perspective images (Bailey, et al., 2002).

Three-dimensional images as a freeze frame, a sequence of images, or 3D animations can be created using stereoscopic visualization techniques.

The conditions and applications for the individual visualization techniques are explained in more detail using examples in Section 11.6.

11.4.2 Amount of data

Existing data on the surrounding area and planning data on the traffic facility are required if different visualization techniques are going to be used. While planning data generally has to be created for a specific object in the design process, state surveying offices generally have existing data for the surrounding area, which can be inspected by anybody.

Digital topographical maps

Digital topographical maps are normally available from the state surveying offices on an $M = 1{:}10{,}000$ scale for digital processing as part of the design process.

Digital aerial photos

Digital aerial photos are images of the earth's surface to scale in georeferenced form. They have a ground resolution of about 30 cm and are normally provided in TIFF (Tagged Image File Format) format.

Digital terrain models (DGMs) and digital surface models (DOMs)

The terrain models available in ASCII format consist of regularly arranged grid points, which are georeferenced in position and height and normally have a grid pattern width of 10 m. The location is specified in the country's Gauss–Krüger coordinates system and the height as altitude above sea level. The accuracy of the altitude measurement depends on the method used. The accuracy for altitudes is normally ± 5 dm for photogrammetric analyses. The accuracy generally improves to ± 3 dm for laser scanning procedures. In wooded areas, industrial and residential areas, or artificially remodeled areas, the accuracy may also differ depending on the measurement method.

Digital surface data (DOM) is a similar kind of data that is generated by the laser-scanning technology. This describes not only the natural terrain, but also the surface of all objects on the earth's surface by irregularly arranged points that are georeferenced by their location and height. The location is specified in the national system of Gauss–Krüger coordinates and the height as altitude above sea level. The accuracy in altitude is normally ± 3 dm. DOM data is supplied in ASCII format.

Automated land register map (ALK), house outlines
House outlines are georeferenced polygons of the outline of buildings from the Automated Land Register Map (ALK). House outlines are described by building corner point coordinates, which are usually given in the Gauss–Krüger coordinate system with connecting information between the corner marks.

The quantity and quality of the data obtainable from the surveying offices are constantly being developed. The latest information should be obtained from surveying offices in order to guarantee the most profitable set of data for a visualization task.

11.5 Development Stages

11.5.1 Freehand sketches

Cox (2005) and Landphair and Larsen (1996) analyzed the general development stages in visualization and evaluated them.

Freehand sketches, for example a freehand line, are used as part of the graphic design process to provide a rough illustration of the line of the road on the horizontal projection. Summary vertical projections serve to evaluate the terrain conditions along the road section. Due to the gradual introduction of the tangent, base circle and flexible rod methods and the use of circular and clothoid templates, the visual quality of the preliminary graphical draft has improved considerably. Only the actual route calculation on the horizontal and vertical projections and the cross section are handled manually or using computer-aided calculation methods based on well-known computing approaches.

11.5.2 Physical models

Slater and Rahmann (1966) investigated the opportunities for using various physical models as aids in road design. The following kinds of models were used:

- wire models to illustrate the alignment
- road models with marks on steel needles to illustrate the course of the gradient
- balsa wood models as a sequence of cross-section illustrations
- layer models for 3D illustrations of terrain and the course of the road.

These models could only be provided for selected, complex route planning tasks because of the enormous expenditure involved.

Besides providing a visual check on the overall 3D impression by using physical models, special periscope cameras have been used to simulate the driver's perspective. Using this methodology, the course of the route with its terrain is created as a scale model. The special camera can take pictures from different positions from the driver's point of view and they are used to check the comprehensibility of the driving area. This very complex technology is used for the following applications:

- checking the alignment;
- checking the visibility of intersections as well as entrance and exit ramps;
- defining the correct position of traffic islands.

11.5.3 Manually generated perspective images

At the beginning of the 1970s, perspective images were created by hand to check the 3D classification of highways in topographically difficult terrain. What was called a graph method was used. Using a graph, real data was converted into image area data. The final editing was carried out manually using artistic skills, because décor details and color had to be added by hand (Landphair and Larsen, 1996).

Manually created perspective images were too complex and required a lot of experience when selecting the suitable location and required artistic skills and good 3D powers of imagination.

11.5.4 Computer-generated perspective images

Computer-generated perspective images were used in road design for the first time at the end of the 1970s. They were produced on the basis of an integrated graphics system using special coordinate geometry programs. These perspective images were created from cross sections of the road axis, which were connected to each other by lines. “Hidden” lines could not yet be shown in concealed form on grid perspectives (hidden line suppression). Without shoulders and the adjacent terrain, these perspective images were only used to visualize sharp bends and short crests.

Calculation systems were introduced at the beginning of the 1980s where perspective images could be produced and concealed lines could also be shown. It was possible to calculate sight distances because of the arrangement of the shoulders.

By introducing surfaces and DGMs, it was finally possible to compute perspective images with comprehensive information on any location. So it was possible to generate realistic photomontages on the basis of existing photos and virtual perspective images. Naumann (1976) developed a program to compute simple sequences of perspective images.

11.5.5 Animations

As even more powerful computers were developed and computer networks were introduced, the first simple animations were generated at the beginning of the 1990s.

Initially, the animations could only be generated on the fly and were viewed immediately. But as storage systems became available and the degree of detail improved, animations were increasingly used in road design processes.

11.5.6 Real-time visualization

Because of the dramatic improvement in the features, power and, especially, the calculation speed of computers and the software programs that were available, image creation (rendering) was also possible in real time. Users can also move in the modeled virtual area at will.

Wenzel (2002) developed a methodology for the 3D, interactive visualization of geometrical data in 2002. This allowed 3D data to be generated on a real-time basis on standard personal computers.

Janikula and Garrick (2002) uses a system to illustrate the 3D alignment, which films an existing road and records route planning data. Afterwards the video image is overlaid with the relevant bend and gradient data on the road using the 3D visualization toolbox, so that conclusions can be drawn on the causes of undesirable combinations of elements (e.g. discontinuous alignment or shortcomings). However, routes can only be checked after the building work has been completed.

Bella (2005) checked the coordination of the horizontal and vertical projections in an interactive driving simulator. He detected the problems that originate from processing the design levels separately in the simulator and demonstrated the limits of checking the 3D alignment for deficits by computer-generated perspective images and animations.

Han, et al. (2005) investigated virtual reality systems to support the design of rural roads. He developed a visualization system that is low-cost, simple to use, and flexible. He believed this could be used to support the planning process:

- visualizing the road;
- providing real-time illustrations of route changes;
- offering better communications with the authority having the road built.

11.6 Fields of Application

11.6.1 Review

Because of the availability of suitable hardware and software systems, computer visualization is becoming more and more important for designing roads, intersections, and engineering work from a practical point of view. Design engineers now have a new and modern tool, which they can use at an early stage to design, display, and assess the traffic construction work in a manner that reflects the real situation. It is clear that the use of computer visualization is increasingly moving from being available for selected complex individual projects to being there for daily routine tasks. The inhibition threshold for using these new technologies is also decreasing as greater success is achieved in the planning process. In the long term, the technical opportunities provided by computer visualization will gradually change design methodology.

The most important applications of visualization are as follows:

- project presentation;
- approval procedures;
- design checks.

11.6.2 Project presentation

The design of traffic facilities using traditional design elements has now reached a technically mature standard. The design engineer draws up the necessary planning documents on the horizontal and vertical projections and cross section by using tried and tested CAD (Computer Aided Design) program systems and summarizes the results numerically in tables and graphically in the form of 2D plans for expert and public assessment. In spite of good, complete planning documents that are normally free of any errors for the three design levels, the presentation of the planned building project is often unconvincing and unsatisfactory for lay people, given the technical possibilities that are now available.

It is now possible to provide an optimal presentation of the building project for a traffic facility by using perspective images, sequences of perspective images, or computer animations, which take into account the surrounding area in a 3D way.

11.6.3 Approval procedures

Overview

Planning, designing, and constructing a traffic facility often trigger a conflict between private and public interests. So by using a gradual iteration process and taking into account private and public interests, traffic engineers have to search for a route that will most probably promote joint agreement. The very sensitive process of searching for options later determines whether a building project will be approved or not and therefore affects the quality of the planning and preparation work.

Hughes (2004) showed in a study that suitable visualization technologies could accelerate negotiations, reduce implementation costs, uncover design problems at an earlier stage, and speed up approval procedures for traffic projects.

A better presentation and illustration of the building project is necessary at the rough line design stage, the preliminary draft, and the final planning approval procedure so as to be able to fully justify the solution that has been professionally developed.

Perspective images from various standpoints are increasingly aids for illustrating the planning project – they can, for example, show the arrangement of the building project in the landscape and therefore support people's 3D imagination. The planned building project is also clearer, more specific, and more comprehensible for lay people. By providing computer animations, additional key support can be made available for the decision-making process made by the bodies responsible for issues of public concern.

Visualization techniques

Map montages

The planning process produces technical planning documents for a road traffic facility within the individual design stages; these are separate for the individual design levels – the horizontal and vertical projections and the cross section. As lay people normally cannot understand these kinds of planning documents and therefore cannot evaluate them, digital montages are increasingly being prepared based on maps and aerial photos and photos.

Axis lines conveying various types of information can be transferred on to maps, e.g. topographical maps, if the digital data is available, or after scanning.

Aerial photomontages

Aerial photos of different types of quality can be used to present or illustrate different route options. The line of the road and illustration of any shoulders are entered on the aerial photo digitally using colors and textures. Other information can also be included in the montage digitally (Figure 11.10).

Comparing options with perspective image montages

The different effects of the structural solution, its integration in the environment, and individual concerns can be better illustrated. Figure 11.11 shows different options for an intersection where much of the traffic flows at different levels and its effect on the urban environment.

11.6.4 Design checks

Principles

The sequence of images of the driving area, which the driver perceives when driving along a road, is the only objective test criterion for checking the driving area and the 3D alignment. For this reason perspective images of the driving

Figure 11.10: Showing options in the aerial view (Kuhn, et al., 2007)

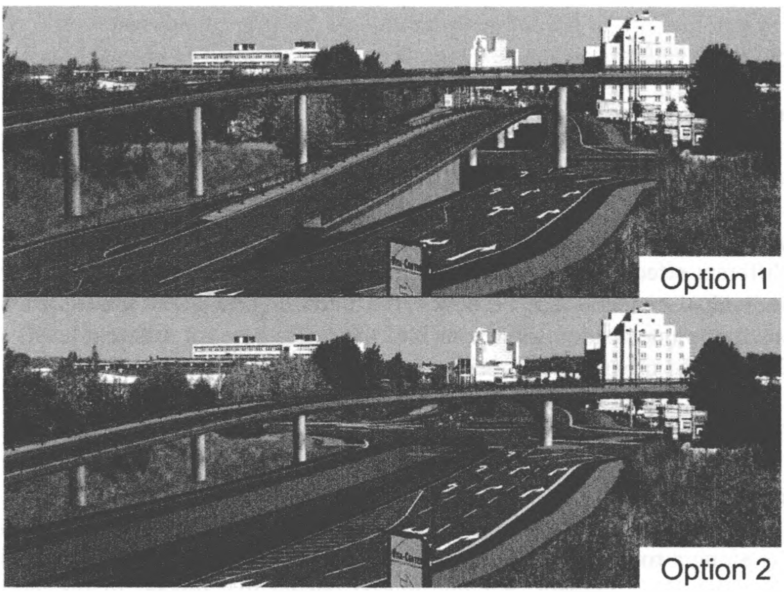


Figure 11.11: Options for an intersection where most traffic flows at different levels (Kühn, et al., 2007)

area and sequences of perspective images of the driving area make sense for quality checks in the design process. Parameters and methods for the quantitative assessment of sequences of perspective images have to be developed as well as the purely qualitative evaluation of the perspective images of the driving area and a possible comparison between virtual and real perspectives.

The most important criteria for comprehensive quality checks in the design process include (Kühn, 2002a):

- the design of the driving area for rural roads;
- road design for urban roads;
- 3D alignment;
- checking the sight distances;
- comprehensibility;
- integration in the landscape.

Design of the driving area for rural roads

The driver perceives the line of the roadway and the surroundings as the driving area when traveling along a rural road. The actual calculation of the route is very important, but the design of the driving area, which guarantees that the driver can comprehend it safely, is equally important in the design process. The driving area has the following individual elements on rural roads:

(1) the road line (alignment on the horizontal and vertical projections and cross section);
(2) the course of the terrain including shoulders (embankments and cuttings);
(3) road markings and traffic signs;
(4) safety equipment (horizontal and vertical);
(5) plants and vegetation.

The driver's ability to comprehend the driving area depends on the lighting and weather conditions to a crucial degree as well as the individual elements already mentioned. A driving area that is easy to understand and has been well and safely designed normally causes no difficulties for a driver, even in unfavorable lighting and weather conditions, if he adapts his speed and his driving style to the changes in the conditions. An unclearly designed driving area can cause sudden false reactions and accidents, even though drivers may have adapted their driving style appropriately.

Photo-like perspective images or montages of perspective images can be used for the visualization process, which should show all the elements that affect the driving area (Figure 11.12).

Road design for urban roads

The individual elements that characterize the driving area on inner city roads are the road markings, additional lanes, and any buildings alongside it. A harmonious

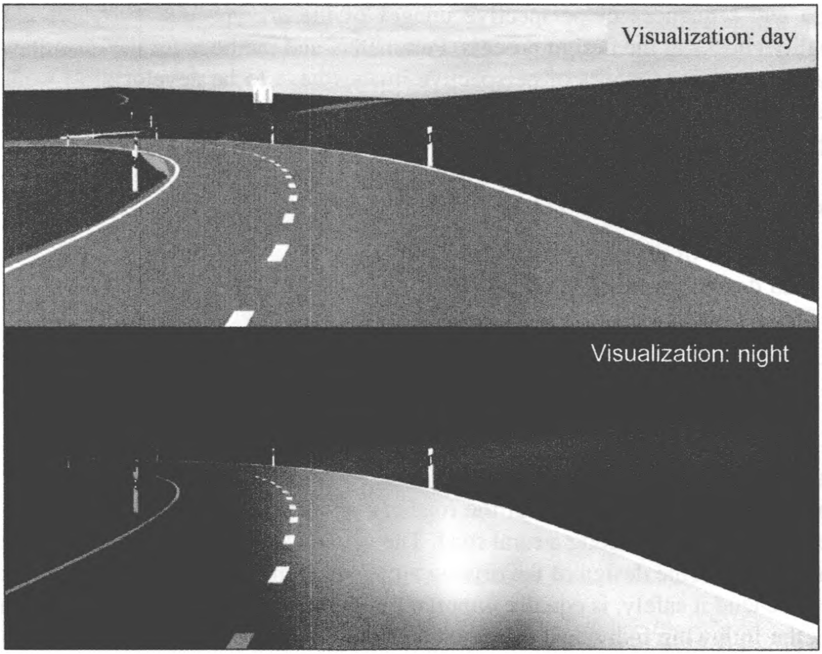


Figure 11.12: Design of the driving area for rural roads (Kühn, et al., 2007)

overall picture for the road can be obtained by consciously and sensibly designing the individual elements in the design phase. This will pay equal attention to architectural and psychological issues from the driver's point of view.

By using a target/actual comparison, it is possible to clearly illustrate and explain the planned project. The current state and the target state are contrasted in this comparison. Upgrading the cross section at a local location with all the individual elements that characterize the road normally entails considerable changes to the current situation. By using life-like textures, the design effects can be better illustrated and this increases local residents' acceptance levels (Figure 11.13).

Noise protection barriers are normally built as part of the new building or upgrading work if the permissible sound levels cannot be met for buildings or the surrounding area. They do reduce noise levels, but often create restrictions as a result of limited visibility or shadows. A noise protection barrier built subsequently is shown in Figure 11.14.

Three-dimensional alignment

A comprehensive objective check on the 3D alignment is particularly important on rural roads in addition to the design and checks that the driving area can be safely perceived and understood. The quality of the perspective images may differ

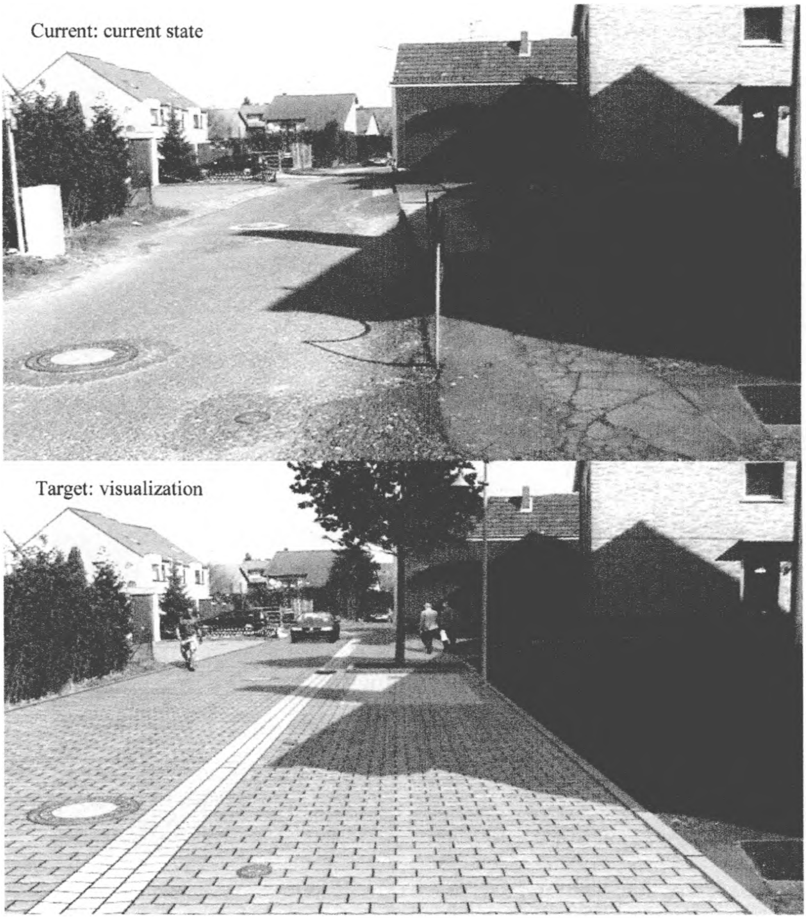


Figure 11.13: Upgrading an urban road (Kühn, et al., 2007)
Source: (NRW State road authority)

considerably from the design and assessment of the driving area for cost reasons, i.e. the 3D course of the road with its terrain is often enough for evaluation purposes.

Using a sequence of perspective images, it is possible to qualitatively check the 3D alignment of a road traffic facility at critical sections and prevent any recognizable shortcomings by changing the axis design. Figures 11.15 and 11.16 show the visualization of the 3D alignment of a rural road near a bridge building before and after the gradient has been corrected.

Checking the sight distances

When planning new road building work or upgrading a road, the standards for sight distances in the current valid rules must be met so as to provide passing

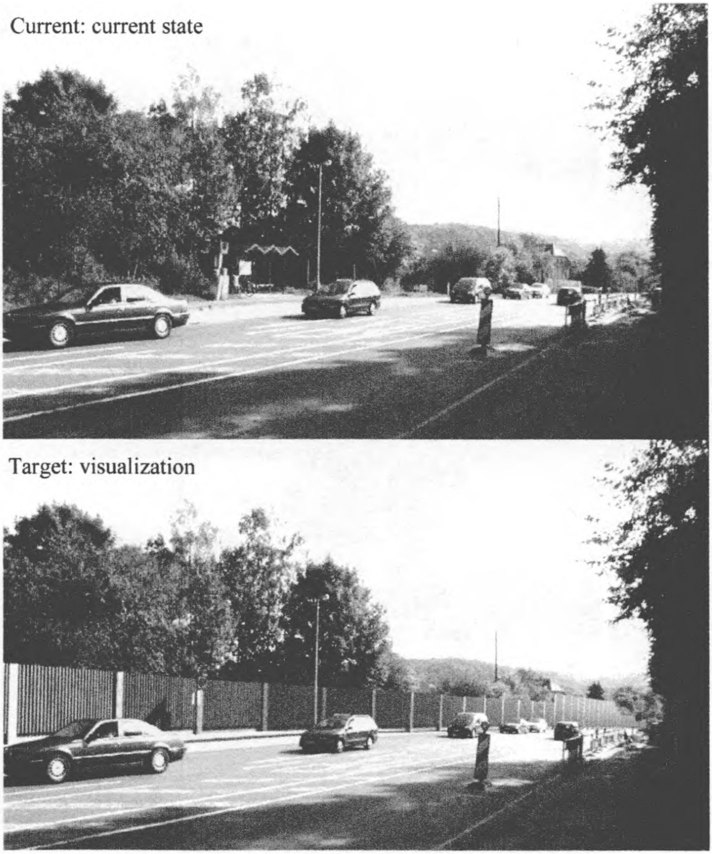


Figure 11.14: Installing a noise protection barrier (Kühn, et al., 2007)
Source: (NRW State road authority)

opportunities for drivers in the interests of maintaining the flow of traffic and efficiency. Because of the separate planning of horizontal and vertical projections and the cross section, sight distances may have been miscalculated, i.e. the desired sight distances are not achieved during a subsequent check on the finished road.

Perspective images of the driving area provide a better basis for analyzing sight distances. The sight distance can be determined pretty accurately with this new method using the piercing points of the visual ray in the individual perspectives. If the required sight distance is not met along individual route sections, the field of vision has to be changed by planning measures (removing obstacles, change the edges of shoulders, creating greater visibility, etc.).

Figure 11.15: "Flat board" bridge design (constant longitudinal gradient in the vertical projection), (Kühn, et al., 2007)

Figure 11.16: Correcting the gradient (depression in the vertical projection), (Kühn, et al., 2007)

Comprehensibility

Perception in good time and an extensive and clear apprehension of the traffic facilities by the driver is a basic condition for adequate road safety on a section of road and therefore for accident prevention in general.

The following criteria guarantee good comprehensibility, in addition to a well-balanced 3D alignment:

- clear and meaningful roadway markings;
- additional elements for designing the driving area (distance markers, safety barriers, traffic islands, vegetation, etc.);
- clear and comprehensible traffic signs and signposts.

Road users must receive comprehensive information about the road traffic facility through adequate and deliberate instructions.

Integration in the landscape

The harmonious integration of a traffic facility in the surroundings and landscape is an essential quality criterion for any planning task. The external design of the engineering work should be adapted to the landscape and its appearance should take into consideration typical landscape features.

Using computer visualization, it is possible to check the integration in the landscape at an early design stage by using virtual perspective images. This means that it is possible to recognize design shortcomings at an early stage so as to be able to correct them during the planning work.

A target/actual comparison can be used for visualization purposes. The effects on the environment should be checked during the planning phase, especially with complicated engineering construction work, i.e. the harmonious integration of a building in the landscape is an important quality criterion in the design checks. Figure 11.17 illustrates how a tunnel is integrated in the landscape.

Using a target/actual comparison, it is possible to produce a before/after comparison after the traffic building work has been completed. Figure 11.18 illustrates a before/after comparison for a bridge construction in three stages.

11.6.5 Results

Various approaches are possible for the individual criteria when introducing quality checks into the design process.

It is only possible to carry out a qualitative check to assess the driving area and road design and integration in the landscape by using perspective images or animations. Engineers must also take into account the fact that integration in the landscape should be assessed from selected positions in the surrounding area or positions above the traffic facility. But the driver's view must be selected for checking the design of the driving area and road. Quantitative checks are increasingly being used for this.

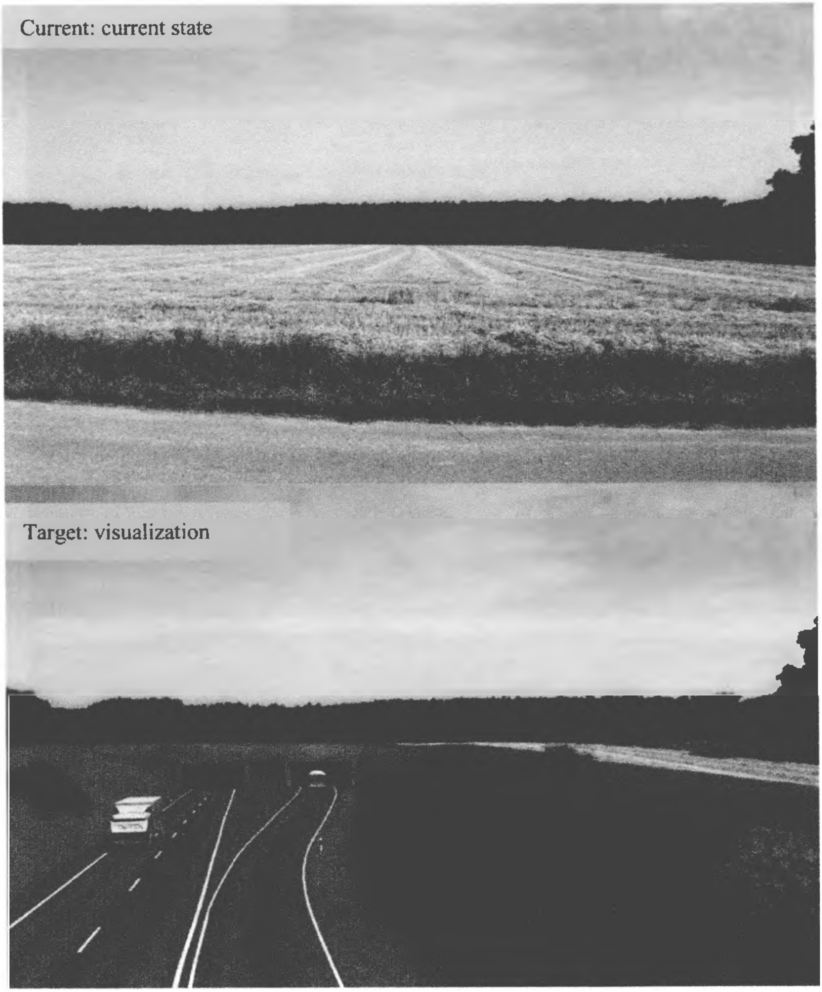


Figure 11.17: Integrating a tunnel
Source: (NRW state road authority)

11.7 Immersive Images

In traditional visualization procedures, perspective images, or animations are generated three dimensionally based on geometrical data and are shown pseudo-three-dimensionally on a 2D display. Immersive systems (stereoscopic vision) have to be used to be able to objectively simulate the driver's 3D vision.

According to Opriessnig (2000), it is generally necessary to generate a separate perspective image for both eyes for 3D vision and observation. Then the

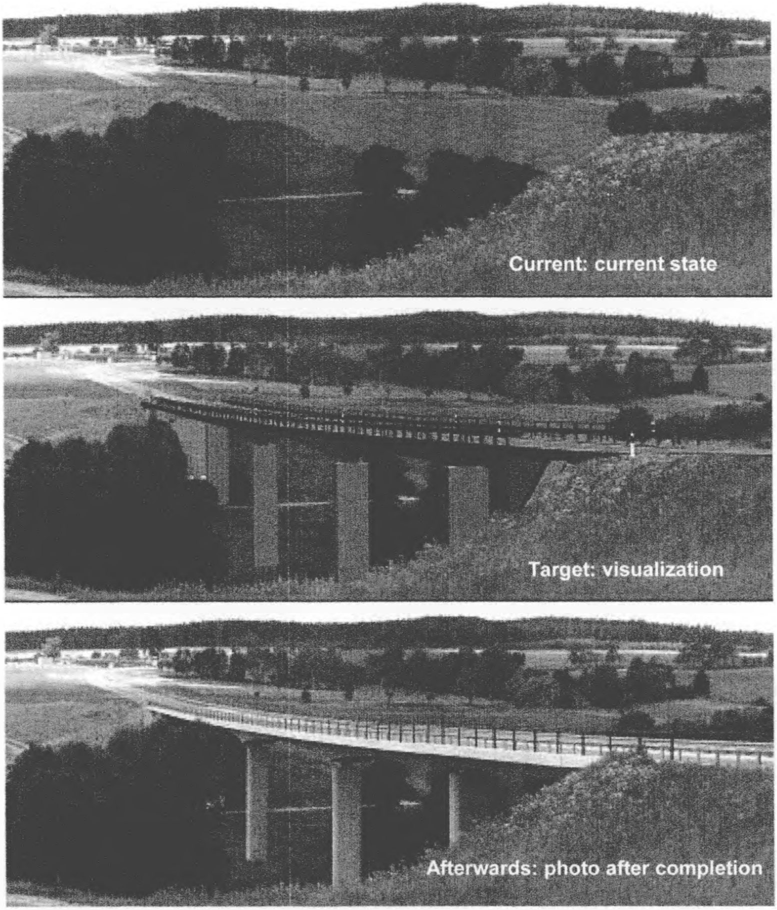


Figure 11.18: Integrating a bridge in the environment (Kühn, et al., 2007)

human brain assembles a 3D image from both perspective images (for the right and left eyes). The information on depth is obtained from the differences in the individual images as in the case of real sight.

Stereoscopic systems are required for the independent generation of perspective images and channel separation (Figure 11.19).

Stereoscopic images can be obtained using the shutter technique (active stereoscopy) with traditional CRT monitors and shutter glasses (Figure 11.20).

In virtual reality rooms, stereoscopic images are produced with projections on screens by using polarizing and interference filters (Figure 11.21).

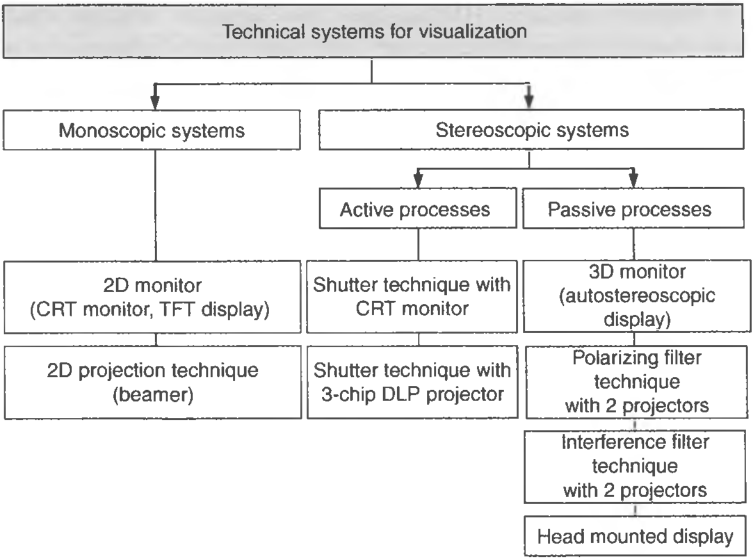


Figure 11.19: Technical systems for visualization (Kühn, 2003)

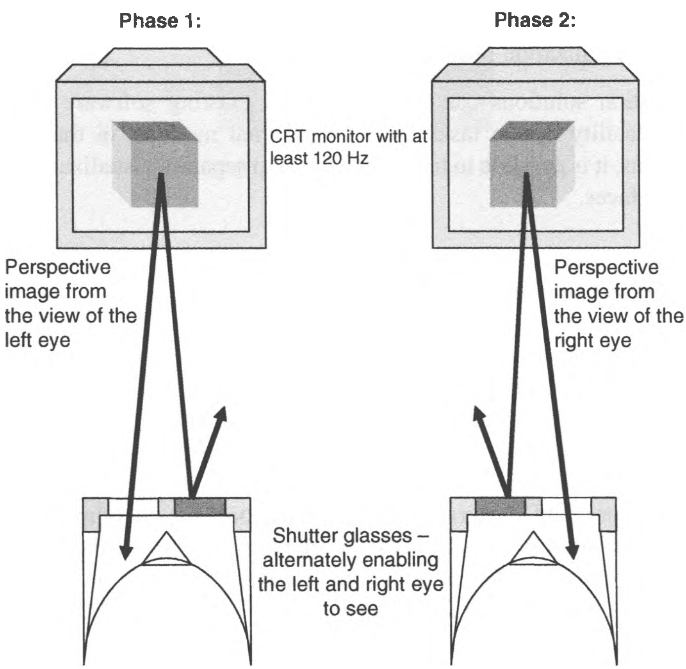


Figure 11.20: Active stereoscopic methods

Head mounted displays (HMDs) have two separate displays which are positioned directly in front of the viewer's eyes using special glasses or a helmet. Stereoscopic images can also be generated like this.

A stereoscopic image is generated directly by a screen with 3D monitors (active stereoscopy), i.e. the viewer needs no auxiliary aids (Figure 11.22).

A very high degree of immersion is achieved if so-called CAVE systems are used. Here images are projected on different sides of a room where the viewer is located (Bowman, Kruijff, et al., 2005).

11.8 Software Systems

11.8.1 Review

Engineers have access to different software programs for the planning work on traffic facilities. There are differences in their design and scope and in their program structure. In general software programs that are internationally available for visualization can be divided into three main groups (Figure 11.23):

(1) add-ons to standard CAD programs;
(2) road design programs with a visualization module;
(3) separate visualization programs.

Visualization solutions can be provided in existing software solutions for road traffic facility design tasks by using internal modules in the program. In some solutions, it is possible to transfer 3D data to separate visualization programs through interfaces.

11.8.2 Add-ons to standard CAD programs

These CAD programs have been developed and conceived for generating and documenting designs, plans, detailed drawings, and design drawings in 2D and 3D. These are mostly basic program systems without any clear reference to particular business sectors. They can be obtained through the special application add-ons.

Some standard CAD programs contain basic tools for planning and designing traffic facilities. Add-ons for various standard platforms are available for planners for specific applications. The applications use standard CAD programs directly, i.e. they use the basic function and expand this for special applications.

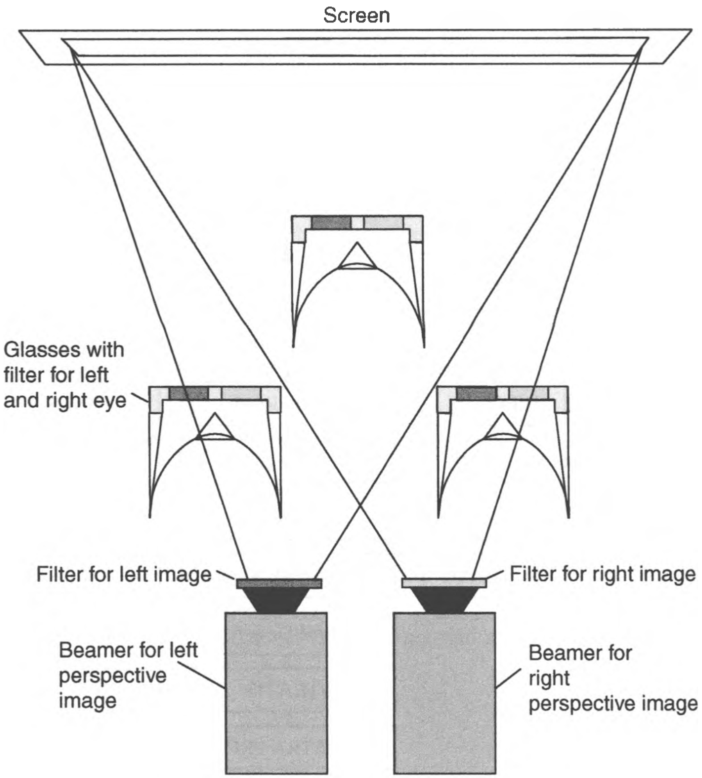


Figure 11.21: Passive stereoscopy methods

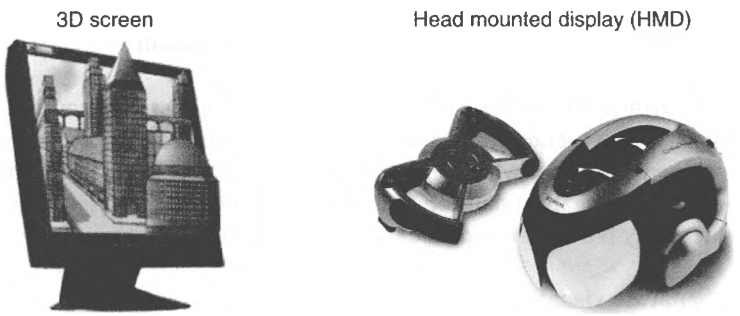


Figure 11.22: 3D screen and HMD systems

(1) Add-ons to standard CAD programs
(2) Road design programs with a visualization module
(3) Separate visualization programs

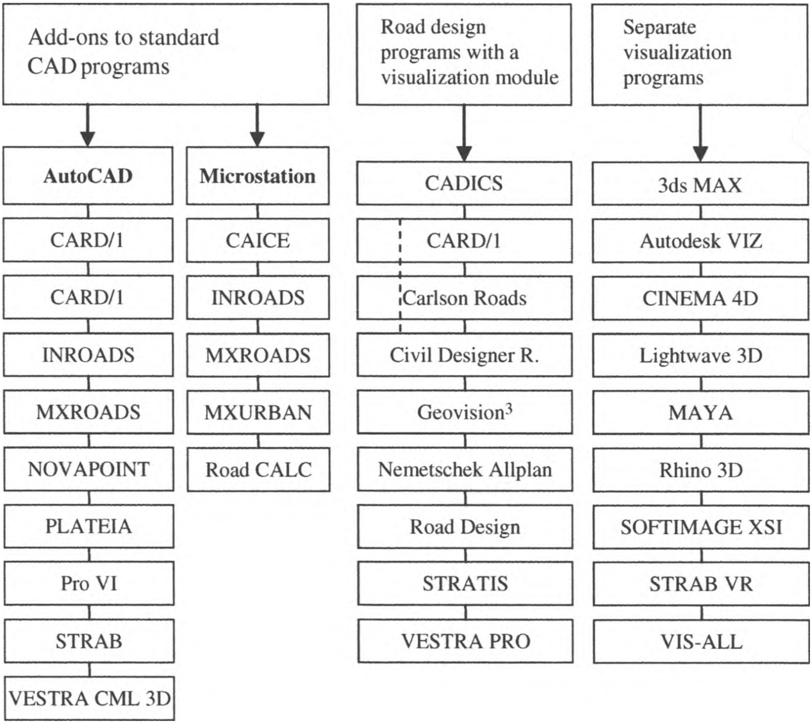


Figure 11.23: Summary of software programs (Kühn, et al., 2007)

11.8.3 Road design programs with a visualization module

The second main group consists of separate software solutions. In contrast to the applications, these programs are not add-ons to standard CAD programs, but they are autonomous systems, which mostly consist of several proprietary modules (e.g. including a visualization module). As these programs are closed systems, which often only provide restricted export and exchange facilities.

11.8.4 Separate visualization programs

As the main application for this software is providing animations for computer games, the film industry, advertisements, production technology, architecture and mechanics, and data transfer between the visualization program and design software is normally difficult. For example, some programs cannot process clothoids. These have to be broken down into polygons or splines, which differ from the original in certain tolerances.

It is not normally possible to transfer technical design data from the horizontal and vertical projections or cross section, i.e. lines, circles, triangles, or other geometrical shapes from the design are transferred to the visualization program, but not the whole.

Most visualization programs have an import function for the widely used DWG/DXF format, so that 3D data can be transferred.

11.9 Questions

(1) Explain the term "visualization."

(2) What do you understand by a perspective image and which kind of perspective image display is used in road design?

(3) Name the essential model assumptions for calculating perspective images from a driver's point of view.

(4) Explain the different stages in the visualization process.

(5) Name and explain the fundamental types of images for visualization.

(6) What is computer animation?

(7) Name the essential visualization techniques and allocate them to the various types of images.

Chapter 12

New Types of Model Projections

12.1 Preliminary Remarks

The classical methods of proceeding with road design and the calculations required for this are based on the traditional elements of straights, clothoids and circular arcs. The design work is carried out in separate stages – the horizontal projection, the vertical projection, and the cross section. The three-dimensional alignment of a road is created by superimposing the three design levels on each other.

As this classical design process is often not flexible enough, particularly if there are a variety of constraints along the route, experts have been considering for some time the idea of developing new kinds of models, which allow them to calculate a route continuously and, if necessary, use three-dimensional planning methods. This chapter will summarize new approaches to developing such model projections.

12.2 The Mathematical Model of the Flexible Rod Line

12.2.1 Problems

The high level of conformity between the flexible rod line and the traditional road planning elements on the one hand and the manual effort required to reduce the flexible rod line into the design elements (straights, clothoids, and circular arcs) on the other hand were the reasons why efforts were made at an early stage to find suitable mathematical functions to simulate the flexible rod line.

The flexible rod line provides a preliminary graphical line. What is needed is a suitable mathematical model, which links the set characteristic points (fixed points) with a constant, smooth line, while also taking into account important factors from a design point of view. This alternative has to exactly pass through all the fixed points, match the flexible rod line as closely as possible, and still be smooth and constant.

This problem is one that concerns the interpolation of a set of fixed points in a mathematical sense. The problem and the solution can be formulated in the following way mathematically:

At horizontal projection level, there are n fixed points along the flexible rod line at their abscissas x_k and ordinates y_k, $(k = 1, 2, \ldots, n)$ in a Cartesian system of coordinates. An attempt must be made to interpolate between the fixed point set using a suitable substitute function $g(x)$. In this process the function $g(x)$ should have consistent derivatives up to the mth order with $1 \leq m \leq n$ and also be the "smoothest" function, i.e.

$$\chi = \int_a^b \left[g^m(x)\right]^2 dx \qquad (12.1)$$

it must be minimal (Ralston, Wilf, 1962).

- There is no clear solution for a situation where m is $> n$, as there are an infinite number of polynomials in the $m - 1$ degree, which adhere to the set fixed points and for which $\aleph = 0$.
- In the case of $m = n$, the only solution is the clearly defined interpolation polynomial $P_n(x)$, the degree of which is $m - 1$.
- In the case where m is $< n$, there is once again a clear solution, which is represented piecewise at each interval $[x_k, x_{k+1}]$ by a polynomial, the degree of which is smaller than 2 m. The $g(x)$ function created in this manner is one of the classes known as spline functions.

12.2.2 Polynomial interpolation

Mathematical approaches

At first glance it seemed obvious to define the $n + 1$ fixed points along an axis needing to be calculated by using a polynomial of the nth degree. The following well-known approach can be selected to determine the interpolation polynomial $P_n(x)$ for $n + 1$ pairs of interpolation values (x_k, y_k) with different abscissas for each pair.

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n. \qquad (12.2)$$

Although the approach and solution are relatively simple, the final determination of the interpolation polynomial $P_n(x)$ requires a considerable amount of calculation work, particularly if there are a fairly large number of fixed points. So for practical experiments, the approximate interpolation formulae of Lagrange, Newton, and Aitken–Neville are used.

(1) Interpolation polynomial of Lagrange (Sauer, Szabo, 1968):

$$P_n(x) = \sum_{i=0}^{n} f(x_i) \prod_{\substack{k=0 \\ k \neq i}}^{n} \frac{x - x_k}{x_i - x_k}. \tag{12.3}$$

(2) Interpolation polynomial of Newton (Sauer, Szabo, 1968):

$$\begin{aligned} P_n(x) &= f(x_0) + [x_0 x_1](x - x_0) + [x_0 x_1 x_2](x - x_0)(x - x_1) + \cdots \\ &\quad + [x_0 x_1 \ldots x_n](x - x_0)(x - x_1)\ldots(x - x_{n-1}). \end{aligned} \tag{12.4}$$

(3) Multiple linear interpolation according to Aitken–Neville (Carnahan, Brice, 1969):

$$y_1^k(x) = \frac{y_k(x - x_{k+1}) - y_{k+1}(x - x_k)}{x_k - x_{k+1}} \quad (k = 0, 1, \ldots, n - 1). \tag{12.5}$$

$$y_m^k(x) = \frac{y_{m-1}^k(x)(x - x_{k+m}) - y_{m-1}^{k+1}(x)(x - x_k)}{x_k - x_{k+m}}, \tag{12.6}$$

$$(m = 2, 3, \ldots, n), \ (k = 0, 1, \ldots, n - m). \tag{12.7}$$

Results of experiments:

(1) The interpolation procedures that have been examined merely start with various approaches to the interpolation polynomial $P_n(x)$, which has to be clearly defined, i.e. the interpolation polynomial itself does not depend on which interpolation formula is selected. The degree of precision required is very different in the individual processes so that rounding errors may have an effect to a varying degree. Despite this, the same interpolation results emerged in the examples that were examined.

(2) The interpolation formulae, however, do differ with regard to their numeric features. The approaches of Lagrange and Aitken–Neville can be used to great advantage, if only a few points need to be interpolated. If there are a large number of interpolation points, however, it is better to use Newton's approach, as the coefficients have to be calculated only once in this system. In addition, the interpolation points do not have to be ordered and engineers can add as many interpolation points as they wish without having to repeat the whole calculation procedure.

(3) As the degree of the interpolation polynomial rises, its smoothness deteriorates, i.e. undulations and oscillation increase rapidly, particularly in the edge areas (Figure 12.1). If the number of fixed points is fairly large, the equidistant distribution of the fixed points has a negative effect on the

smoothness at the interval edges. However, if the fixed points are compressed in the edge areas in a manner similar to the distribution of the break points in the Chebyshev polynomial (Björk and Dahlquist, 1972), the smoothness can be improved.

(4) In classical polynomial interpolation, the geometrical shape of the interpolation curve and therefore the interpolation results largely depend on the state of the system of coordinates that has been selected. It is also not possible to calculate any retrogressive or closed curves. For this reason, the axes of the coordinates system were exchanged in this example.

Regardless of the number and constellation of the fixed points, it was not possible to achieve any ideal match with the flexible rod line using interpolation

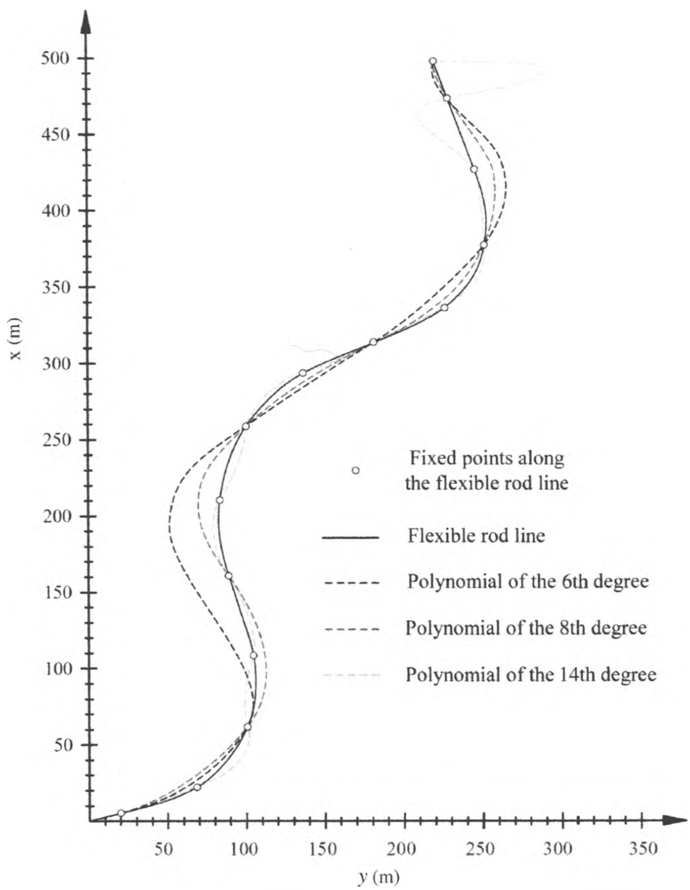


Figure 12.1: Interpolation polynomial with various degrees (Kühn, 1982b)

polynomials of the *n*th degree. They are generally unsuitable for simulating the flexible rod line and the road planning work on the horizontal projection.

12.2.3 Spline interpolation

Introduction

The interpolation polynomial has one great advantage – it can be differentiated as often as the user likes – but it has a disadvantage: It increasingly oscillates as the number of fixed points increases (Figure 12.2). If the number of points is fairly large, it is more practical to bring together low degree polynomials, which fluctuate very little, to provide a function, which can be differentiated as often as possible in the whole interval $[a, b]$ (Figure 12.3).

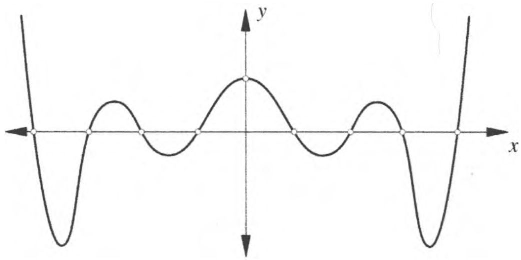


Figure 12.2: Interpolating polynomial (Sauer and Szabo, 1968)

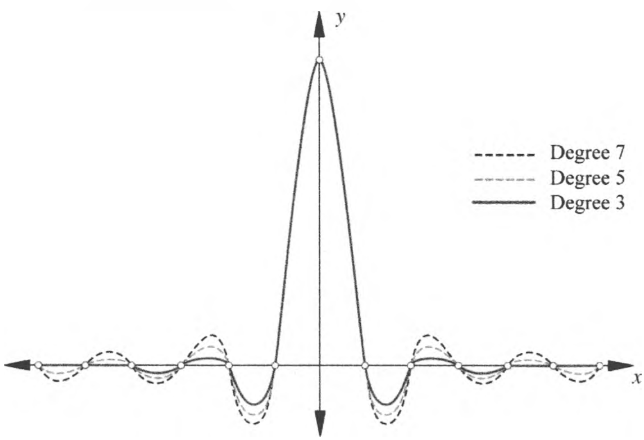


Figure 12.3: Interpolating spline function with various degrees (Sauer and Szabo, 1968)

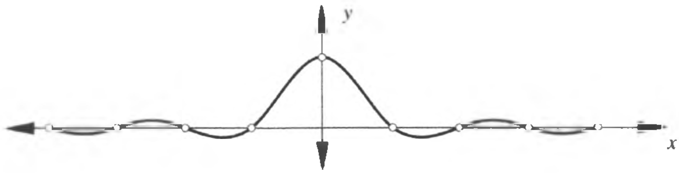


Figure 12.4: Interpolating cubic spline function (third degree) (Sauer and Szabo, 1968)

"If $n - 1$ polynomials each defined in $[x_k, x_{k+1}]$, $(k = 1, \ldots, n - 1)$ of degree $2m + 1$ are placed at the fixed points $x_2, \ldots, x_{n-1}$ so that they can be consistently differentiated by a factor of $2m$, the function s defined in $[a, b]$ is called a spline function of degree $2m + 1$" (Späth, 1973).

Research work (Kühn, 1981) has demonstrated that the cubic spline functions display positive features for design work, e.g. a low level of undulation, a high degree of smoothness, a repetition of symmetries between the fixed points, etc. (Figure 12.4), and the behavior of flexible rods can be described mathematically to an approximate degree.

Cubic spline functions

Mathematical approach

If the flexible rod is laid through the fixed points (x_k, y_k), $(k = 1, \ldots, n)$ on the horizontal projection, the flexible rod has a function $s(x)$, which, if $[s'(x)]^2$ can be disregarded in relation to the value of one, has the following features (Sauer and Szabo, 1968):

- $s(x)$ produces a second consistent derivation in $[a, b]$
- $s(x_k) = y_k$ ($2n$ interpolation conditions)
- $s'_k(x_k) = s'_{k-1}(x_k)$, $s''_k(x_k) = s''_{k-1}(x_k)$ ($2n - 2$ continuity conditions)
- Among all the functions that produce a second consistent derivation $\varphi(x)$, where $\varphi(x_k) = y_k$, $s(x)$ can be described as

$$\int_a^b [s''(x)]^2\, dx \leq \int_a^b [\phi''(x)]^2\, dx. \tag{12.8}$$

This leads to the following:

- $s(x)$ is a polynomial of the third degree at each interval $[x_k, x_{k+1}]$
- $s'''(x)$ is discontinuous at the fixed points $x_2, \ldots, x_{n-1}$.

There is no assumption that there is any equidistance in x_k, but strict monotony, i.e. the following applies:

$$a = x_1 < x_2 < \cdots < x_n = b. \tag{12.9}$$

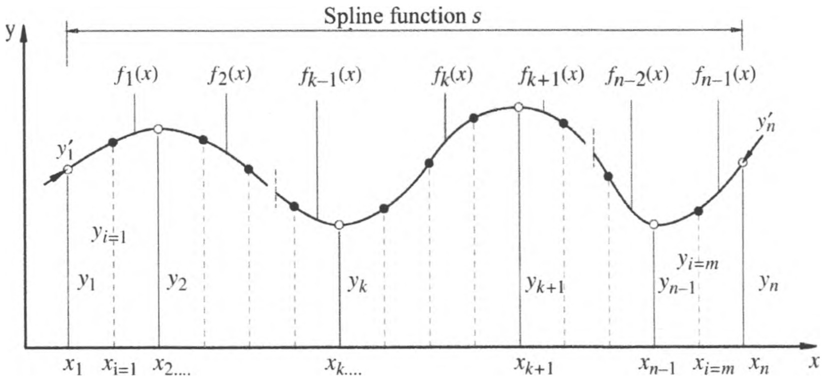


Figure 12.5: Mathematical model of a cubic spline function s (Späth, 1973)

The cubic spline functions fulfill the specified conditions. Taking into account all the functions φ, from which a second consistent derivation can be obtained, they are the ones for which the value of the integral (12.8) is smallest.

An interpolating cubic spline s consists of $n - 1$ polynomials of the third degree f_k, each of which can be defined in $[x_k, x_{k+1}]$, which are placed alongside each other in x_k, $(k = 2, \ldots, n - 1)$ and from which a second derivation can be consistently obtained (Figure 12.5):

$$s(x) = \begin{cases} f_1(x), \; x_1 \leq x \leq x_2 \\ f_k(x), \; x_k \leq x \leq x_{k+1} \\ \vdots \; f_{n-1}(x), \; x_{n-1} \leq x \leq x_n \end{cases} \tag{12.10}$$

Additional figures for the first derivation y'_1 and y'_n or second derivation y''_1 and y''_n must be set as the peripheral conditions so that the cubic spline is clearly defined.

If the following approach is selected for the polynomial of the third degree f_k:

$$f_k(x) = a_k(x - x_k)^3 + b_k(x - x_k)^2 + c_k(x - x_k) + d_K, \tag{12.11}$$

the $4(n - 1)$ coefficients a_k, b_k, c_k, d_k must be defined so that the following applies:

$$f_k(x_k) = f_{k-1}(x_k), f'_k(x_k) = f'_{k-1}(x_k), f''_k(x_k) = f''_{k-1}(x_k) \tag{12.12}$$

$$f_k(x_k) = y_k$$

$$\int_a^b [s''(x)]^2 dx = \sum_{k=1}^{n-1} \int_{x_k}^{x_{k+1}} [f_k''(x)]^2 dx = \min imum. \qquad (12.13)$$

with the derivatives

$$f_k'(x) = 3a_k(x - x_k)^2 + 2b_k(x - x_k) + c_k \qquad (12.14)$$

and

$$f_k''(x) = 6a_k(x - x_k) + 2b_k. \qquad (12.15)$$

The requirements of consistency for the inclinations and curvature – provided that these conditions prevail – result in the following correlations at the corner points of a fixed interval $[x_k, x_{k+1}]$:

$$\begin{aligned} f_k(x_k) &= y_k, & f_k(x_{k+1}) &= y_{k+1} \\ f_k'(x_k) &= y_k', & f_k'(x_{k+1}) &= y_{k+1}' \\ \Delta x_k &= x_{k+1} - x_k, & \Delta y_k &= y_{k+1} - y_k, \end{aligned} \qquad (12.16)$$

that is:

$$y_k = f_k(x_k) = d_k \qquad (12.17)$$

$$y_{k+1} = f_k(x_{k+1}) = a_k*\Delta x_k^3 + b_k*\Delta x_k^2 + c_k*\Delta x_k + d_k \qquad (12.18)$$

$$y_k' = f_k'(x_k) = c_k \qquad (12.19)$$

$$y_{k+1}' = f_k'(x_{k+1}) = 3a_k*\Delta x_k^2 + 2b_k*\Delta x_k + c_k \qquad (12.20)$$

$$y_k'' = f_k''(x_k) = 2b_k \qquad (12.21)$$

$$y_{k+1}'' = f_k''(x_{k+1}) = 6a_k*\Delta x_k + 2b_k. \qquad (12.22)$$

These correlations can be used to determine the coefficients a_k, b_k, c_k and d_k, $(k = 1, \ldots, n - 1)$, depending on the values of x_k and y_k $(k = 1, \ldots, n)$, and the second or first derivatives y_k'' or y_k', $(k = 1, \ldots, n)$.

Determining the coefficients in dependence on the second derivatives

From (12.21) and (12.22):

$$a_k = \frac{1}{6\Delta x_k}(y_{k+1}'' - y_k'') \qquad (12.23)$$

from (12.21):

$$b_k = \frac{1}{2} y_k'' \quad (k = 1, \ldots, n - 1) \qquad (12.24)$$

from (12.18), (12.16), (12.23), (12.24) and (12.26):

$$c_k = \frac{\Delta y_k}{\Delta x_k} - \frac{1}{6}(y''_{k+1} + 2y''_k)\Delta x_k \tag{12.25}$$

from (12.17):

$$d_k = y_k \tag{12.26}$$

The following is obtained as a spin-off from (12.19) and (12.20):

$$y'_k = \frac{\Delta y_k}{\Delta x_k} - \frac{1}{6}(y''_{k+1} + 2y''_k)\Delta x_k \quad (k = 1, \ldots n-1) \tag{12.27}$$

and

$$y'_n = \frac{\Delta y_{n-1}}{\Delta x_{n-1}} + \frac{1}{6}\Delta x_{n-1}(2y''_n + y''_{n-1}). \tag{12.28}$$

the need for consistency in the first derivative means that there is a need to combine

$$3a_{k-1}\Delta x^2_{k-1} + 2b_{k-1}\Delta x_{k-1} + c_{k-1} = c_k \tag{12.29}$$

or ultimately

$$\Delta x_{k-1}y''_{k-1} + 2(\Delta x_{k-1} + \Delta x_k)''y''_k + \Delta x_k y''_{k+1} = 6\left(\frac{\Delta y_k}{\Delta x_k} - \frac{\Delta y_{k-1}}{\Delta x_{k-1}}\right) \tag{12.30}$$

$(k = 2, \ldots, n-1)$

If y''_1 and y''_n are specified, a linear equation system with $n-2$ equations of the following type is obtained for the unknowns $y''_2, \ldots, y''_{n-1}$ in a matrix scheme

$$A * y'' = B. \tag{12.31}$$

$$A = \begin{bmatrix} 2(\Delta x_1 + \Delta x_2) & \Delta x_2 & \cdot & \cdot & \cdot & \cdot \\ \Delta x_2 & 2(\Delta x_2 + \Delta x_3) & \Delta x_3 & \cdot & \cdot & \cdot \\ \cdot & \Delta x_3 & 2(\Delta x_3 + \Delta x_4) & \Delta x_4 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \Delta x_{n-2} & 2(\Delta x_{n-2} + \Delta x_{n-1}) \end{bmatrix} \tag{12.32}$$

$$y_4'' = \begin{bmatrix} y_4'' \\ y_4'' \\ y_4'' \\ \vdots \\ y_4'' \end{bmatrix} \tag{12.33}$$

$$B = \begin{bmatrix} 6\left(\frac{\Delta y_2}{\Delta x_2} - \frac{\Delta y_1}{\Delta x_1}\right) - \Delta x_1 y_1'' \\ 6\left(\frac{\Delta y_3}{\Delta x_3} - \frac{\Delta y_2}{\Delta x_2}\right) \\ 6\left(\frac{\Delta y_4}{\Delta x} - \frac{\Delta y_3}{\Delta x_3}\right) \\ \vdots \\ 6\left(\frac{\Delta y_{n-1}}{\Delta x_{x-1}} \frac{\Delta y_n}{\Delta x_n}\right) - \Delta x_n y_n'' \end{bmatrix} \tag{12.34}$$

The coefficient matrix (12.29) is tridiagonal, symmetrical, diagonally dominant and has positive diagonal elements. This means that the existence and uniqueness of the cubic spline is guaranteed if y_1'' und y_n'' are specified. The GAUSS elimination process (Spăth, 1973) can be used to solve the equation system. The second derivatives therefore result in the following

$$y_k'' = F_k - G_k * y_{k+1}'', \quad (k = 2, \ldots, n-1) \tag{12.35}$$

if the value of $F_1 = 0$, $G_1 = 0$,

$$F_k = \frac{6\left(\frac{\Delta y_k}{\Delta x_k} - \frac{\Delta y_{k-1}}{\Delta x_{k-1}}\right) - \Delta x_{k-1} * F_{k-1}}{2(\Delta x_{k-1} + \Delta x_k) - \Delta x_{k-1} * G_k} \tag{12.36}$$

and

$$G_k = \frac{\Delta x_k}{2(\Delta x_{k-1} + \Delta x_k) - \Delta x_{k-1} * G_{k-1}}. \tag{12.37}$$

If the values for the second derivative y_k'' are known, the calculation of the polynomial coefficients a_k, b_k, c_k and d_k is made for each interval $[x_k, x_{k+1}]$. Then the related function values y_i $(i = 1, \ldots, m)$ can be interpolated between the fixed points for as many abscissa values x_i, $(i = 1, \ldots, m)$ as are needed using formula (12.11). The following applies if the peripheral conditions are different:

$$y_1'' = u * y_2'' \quad \textit{and} \quad y_n'' = v * y_{n-1}''. \tag{12.38}$$

$u = v = 1$ is often selected in order to obtain the same value in x_1 or x_n for the second derivative as in x_2 or in x_{n-1}. It is normal to stipulate $u = v = 0.5$. If we use (12.38) in (12.31), modified approaches result if $k = 2$ and $k = n - 1$:

$$((2+u)\Delta x_1 + 2\Delta x_2)y_2'' + \Delta x_2 * y_3'' = 6\left(\frac{\Delta y_2}{\Delta x_2} - \frac{\Delta y_1}{\Delta x_1}\right) \quad (12.39)$$

and

$$\Delta x_{n-2}y_{n-2}'' + (2\Delta x_{n-2} + (2+v'')\Delta x_{n-1})y_{n-1}) = 6\left(\frac{\Delta y_{n-1}}{\Delta x_{n-1}} - \frac{\Delta y_{n-2}}{\Delta x_{n-2}}\right). \quad (12.40)$$

The other equations remain unchanged. The existence and uniqueness of the cubic splines with the peripheral conditions (Kühn, 1982b) are guaranteed if the positive definability of the matrix remains intact. An effective sub-program is specified in Späth (1973) in order to solve this modified equation system.

If the values y_1' and y_n' are set as the peripheral condition for the first derivatives x_1 and x_n, the equation system (12.28) is expanded by using the formulae (12.25) and (12.26) for the two equations

$$2\Delta x_1 y_1'' + \Delta x_1 y_2'' = 6\left(\frac{\Delta y_1}{\Delta x_1} - y_1'\right) \quad (12.41)$$

and

$$\Delta x_{n-1}y_{n-1}'' + 2\Delta x_{n-1}y_n'' = 6\left(y_1' - \frac{\Delta y_{n-1}}{\Delta x_{n-1}}\right). \quad (12.42)$$

The resulting equation system therefore has n equations for the unknowns that need to be determined: $y_1'', \ldots, y_n''$. The existence and uniqueness of the cubic spline when setting $y_1'', \ldots, y_n''$ are also guaranteed.

Determining the coefficients in dependence on the first derivatives

In the statements made so far, a cubic spline with various peripheral conditions has always been characterized by triple figures (x_k, y_k, y_k''), $(k = 1, \ldots, n)$. But a cubic spline can also be characterized by n triples (x_k, yk, y_k'), $(k = 1, \ldots, n)$. But first the coefficients have to be eliminated from the formulae (12.16) and (12.18). The results obtained are:

$$a_k = \frac{1}{\Delta x_k^2}\left(-2\frac{\Delta y_k}{\Delta x} + y_k' + y_{k+1}'\right) \quad (12.43)$$

$$b_k = \frac{1}{\Delta x_k}\left(3\frac{\Delta y_k}{\Delta x} - 2y_k' - y_{k+1}'\right), \quad (k = 1, \ldots, n-1) \quad (12.44)$$

$$c_k = y_k' \quad (12.45)$$

$$d_k = y_k. \tag{12.46}$$

This also applies:

$$y''_k = \frac{2}{\Delta x_k}\left(3\frac{\Delta y_k}{\Delta x} - 2y'_k - y'_{k+1}\right) \tag{12.47}$$

and

$$y''_n = \frac{2}{\Delta x_{n-1}}\left(-3\frac{\Delta y'_{n-1}}{\Delta x_{n-1}} + y_{n-1} - 2y'_n\right). \tag{12.48}$$

The need for consistency in the second derivative means that there is a need to combine

$$6a_{k-1}\Delta x_{k-1} + 2b_{k-1} = 2b_k \tag{12.49}$$

or ultimately

$$\frac{1}{\Delta x_{k-1}}y_{k-1} + 2\left(\frac{1}{\Delta x_{k-1}} - \frac{1}{\Delta x'_k}\right)y'_k + \frac{1}{\Delta x_k}y_{k-1}$$
$$= \frac{3}{\Delta x_{k-1}} * \frac{\Delta y_{k-1}}{\Delta x_{k-1}} + \frac{3}{\Delta x_k} * \frac{\Delta y_k}{\Delta x_k}, \ (k = 2, \ldots, n-1) \tag{12.50}$$

If y'_1 and y'_n are specified, these $n - 2$ equations can be solved in order to determine the unknowns $y'_2, \ldots, y'_{n-1}$ using the GAUSS elimination procedure, as the coefficient matrix is defined positively once again. This also guarantees the existence and uniqueness of the cubic spline. So the first derivatives provide these results

$$y'_k = F_k - G_k * y'_{k+1}, \quad (k = 2, \ldots, n-1) \tag{12.51}$$

if the value of $F_1 = 0$, $G_1 = 0$,

$$F_k = \frac{3\left(\frac{1}{\Delta x_{k-1}}\right)^2 \Delta y_{k-1} + 3\left(\frac{1}{\Delta x_k}\right)^2 \Delta y_k - \frac{F_{k-1}}{\Delta x_{k-1}}}{2\left(\frac{1}{\Delta x_{k-1}} + \frac{1}{\Delta x_k}\right) - \frac{G_k}{\Delta x_{k-1}}} \tag{12.52}$$

$$G_k = \frac{1}{\Delta x_k\left[2\left(\frac{1}{\Delta x_{k-1}} - \frac{1}{\Delta x_k}\right) - \frac{G_{k-1}}{\Delta x_{k-1}}\right]} \tag{12.53}$$

Using the first derivative, we can then calculate the coefficients a_k, b_k, c_k, and d_k and then interpolate between the fixed points using formula (12.11).

Parametric representation

When calculating the axis, the strict monotony of the abscissa values can also not always be maintained by selecting a suitable system of coordinates. If we wish to use cubic spline interpolation in the case of regression curves, a parameter t must be introduced, which grows monotonically with the course of the curve:

$$t_1 < t_2 < \cdots < t_n. \tag{12.54}$$

The cubic spline function s is then represented parametrically by the two functions $x = x(t)$ and $y = y(t)$. The ideal values for the parameter t_k, $(k = 1, \ldots, n)$ would be the accumulated length of the arc of the curve, which still has to be calculated, however, Theoretical and practical experiments have demonstrated that the most appropriate representation of t is the straight line between fixed points (chord length) (Figure 12.6). The parameters t_k can be determined recursively:

$$\begin{aligned} t_1 &= t_a \\ t_k &= t_{k-1} + \sqrt{\Delta x_{k-1}^2 + \Delta y_{k-1}^2}, \quad (k = 2, \ldots, n). \end{aligned} \tag{12.55}$$

Ultimately there are two equations for the cubic polynomials in parameter form:

$$\begin{aligned} x_i &= ax_k(t_i - t_k)^3 + bx_k(t_i - t_k)^2 + cx_k(t_i - t_k) + dx_k \\ y_i &= ay_k(t_i - t_k)^3 + by_k(t_i - t_k)^2 + cy_k(t_i - t_k) + dy_k. \end{aligned} \tag{12.56}$$

In contrast to cubic spline interpolation in explicit form, a separate calculation of the determining elements of the two cubic splines is made with the points (t_k, x_k) or (t_k, y_k).

Curvature

An important criterion for assessing the driving characteristics of a road is its curvature. When calculating the axis with the aid of the traditional road planning elements of straights, clothoids, and circular arcs, the familiar curvature images of a combined curve, an apex clothoid, an inflection line, and oval lines are created. When adhering to the parameter correlations for the road planning elements, curvature images emerge, which should lead to homogeneous speeds and a balanced driving style.

As a result of extensive experimental tests (Kühn, 1982b), evidence was provided that by using cubic spline functions in parameter form and when specifying characteristic fixed points (weight points and selected additional points), mathematical sequences of elements can be calculated and their curvature matches the results of using traditional design elements.

The most important experimental results obtained using the example of a simple curve can be summarized as follows:

(1) The set starting and finishing point and the summit of the curve (three fixed points) approximately provide the individual curvature of an apex clothoid (Figure 12.7).
(2) If, however, the starting and finishing point of the assumed circular arc are specified in addition to the starting and finishing point (four fixed points), this approximately matches the curvature of a combined curve (Figure 12.8).

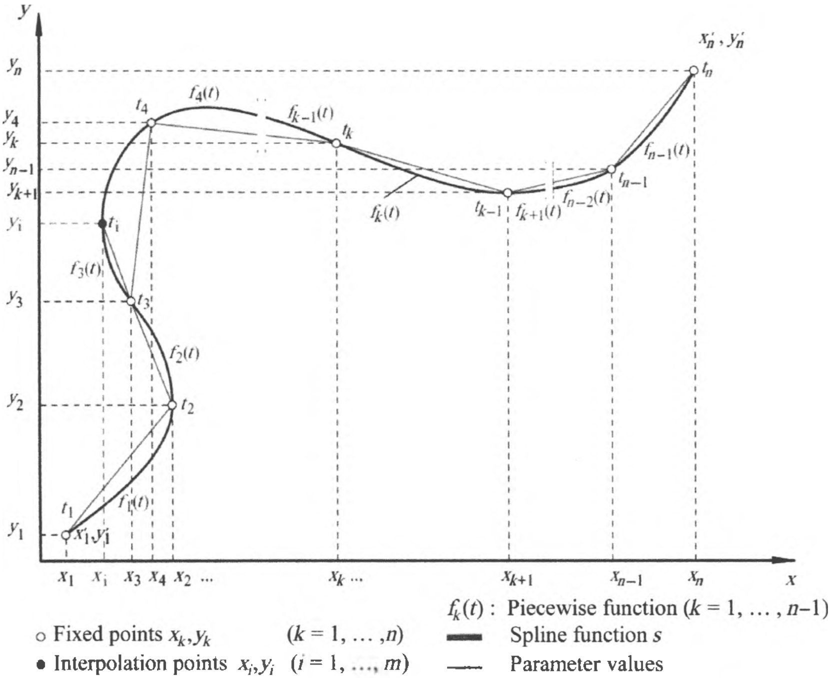


Figure 12.6: Parametric representation of a cubic spline function s (Kühn, 2003)

(3) By specifying the inclination as a constraint at the starting and finishing point of the curve, discrepancies in the constraints can be kept to a minimum (Figure 12.9).

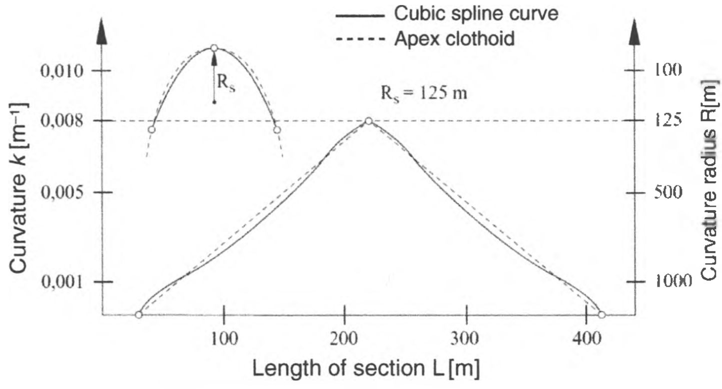


Figure 12.7: Curvature of a cubic spline curve when specifying three fixed points (simulating an apex clothoid) (Kühn, 1985)

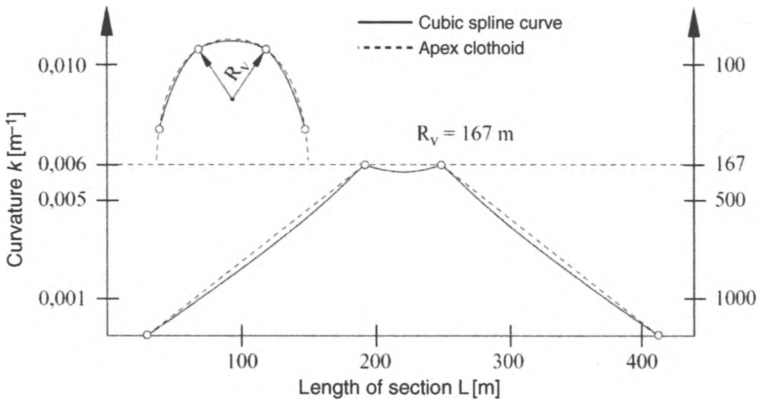


Figure 12.8: Curvature of a cubic spline curve when specifying four fixed points (simulating a combined curve) (Kühn, 1985)

Results of experiments

(1) The course of the interpolation curve can be affected by the peripheral conditions and the constellation of fixed points in a manner similar to the flexible rod planning method.

(2) The different peripheral conditions only have a slight effect on the course of the spline curve in the edge intervals (Figure 12.9). But they do play a major role in determining the smoothness achieved at the crossover between the spline curve and the straight. From a technical design point of view, it is best to specify the inclination (first derivative) at the starting and finishing point as peripheral conditions, as the direction of the tangent line is normally known at the start and end of the building work.

(3) In the case of the cubic spline interpolation in explicit form, there are often larger differences between the flexible rod line and the spline curve. A good

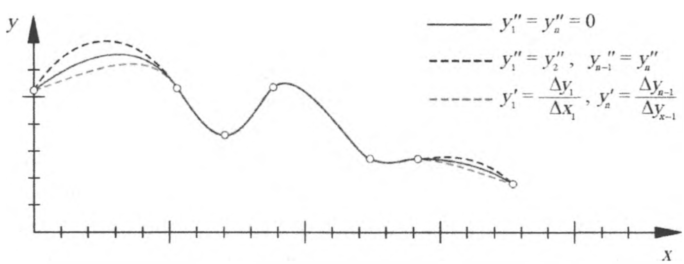


Figure 12.9: Effect of different peripheral conditions (Späth, 1973)

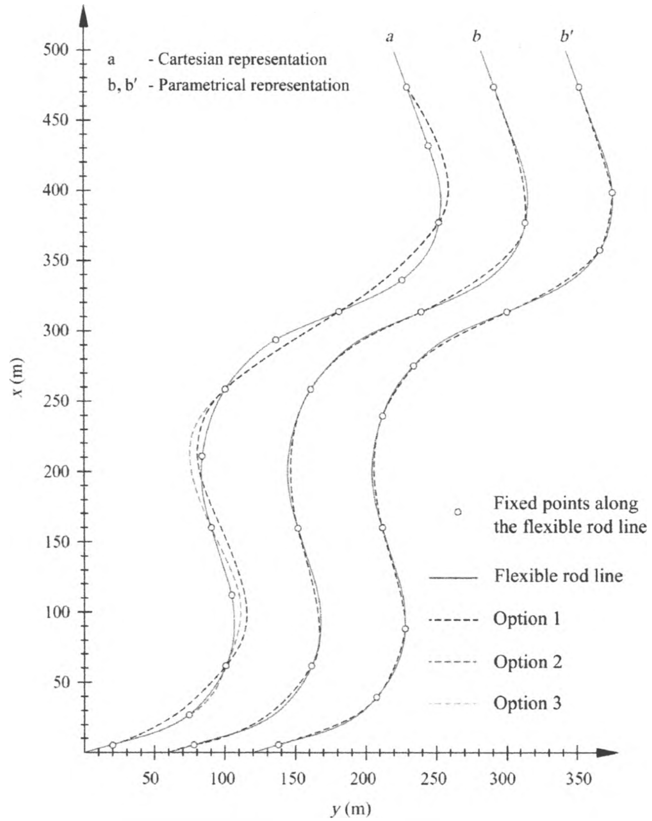


Figure 12.10: Cubic spline interpolation with different fixed point constellations (Kühn, 1985)

level of approximation can only be achieved if the number of fixed points is increased and the distance between the points is equal (Figure 12.10a). The discrepancies, some of which are major, emerge because the geometric shape of the curve depends on the state of the system of coordinates in the case of a cubic spline interpolation in explicit form. The axes of the coordinates system were therefore interchanged in this example.

(4) In the case of the cubic spline interpolation in parametric form, however, there are only slight differences between the flexible rod line and the spline curve. In order to minimize discrepancies and still obtain curvature that is similar to that achieved by a traditional sequence of elements, the starting and finishing point of the supposed arc should be specified in addition to the supposed inflection point (Figure 12.10b,b′).

(5) In the case of cubic splines in parameter form, the invariance of the interpolation curves is clearly shown in contrast with a rotation of the coordinates system (Figure 12.11).

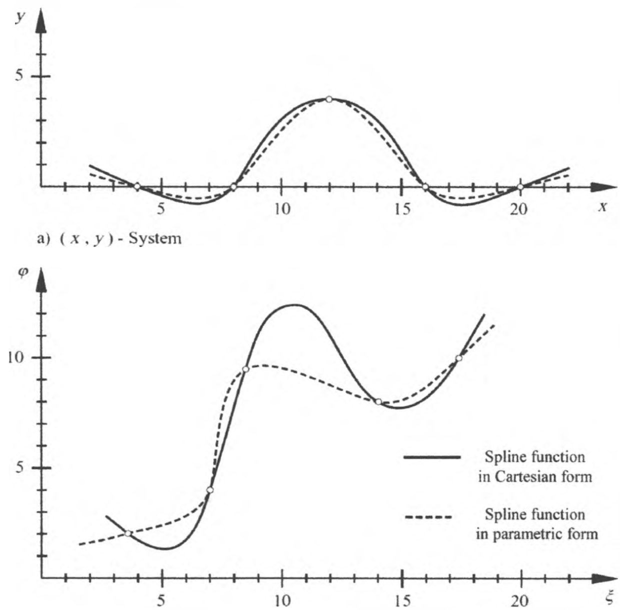


Figure 12.11: How the system of coordinates affects the geometric design of the interpolation curve (Bönsch, 1973)

Summary

Cubic spline functions in a parametric form are particularly suitable for simulating the flexible rod line mathematically.

As is the case with the flexible rod, they roughly minimize the bending energy of the interpolation curve running through the fixed points (x_k, y_k), $(k = 1, \ldots, n)$.

By specifying a characteristic constellation of fixed points, there are only slight discrepancies between the flexible rod line and the cubic spline curve.

The curvature of a cubic spline curve almost completely matches the curvature of a traditional sequence of elements if a suitable sequence of fixed points is set.

The cubic spline functions display good design features like smoothness, consistency, and a repetition of symmetries on account of their mathematical design.

Cubic spline functions in parametric form do not depend on the state of the system of coordinates, i.e. regression and closed curves can be calculated without any problems.

12.2.4 Generalized spline functions

General information

Experiments so far have demonstrated that cubic splines in parametric form are very suitable for simulating the flexible rod line and generally for calculating the axis on a horizontal projection (Kühn, 1981). The major drawback of the cubic spline functions is that the course of the interpolation curve can only be affected by the constellation of fixed points. If, for example, a diagnosis of constraints for any specified point is negative and the mathematical axis that has been calculated matches the design engineer's ideas, the axis calculation has to be started again from scratch with a different constellation of points. But using what are known as generalized spline functions, however, the selected parameters only have to be changed at the relevant interval.

Mathematical model

In contrast to cubic splines, a generalized cubic spline function f is composed of $n - 1$ part elements f_k in this form:

$$f_k(x) = a_k g_1(x) + b_k g_2(x) + c_k g_3(x) + d_k g_4(x). \tag{12.57}$$

The g_i may include as many functions as needed, but it must be possible to obtain a consistent second derivation from them. Their design may be very different and may even depend on the interval $[x_k, x_{k+1}]$, which is not clearly expressed in the formula (12.57). In order to have smooth interpolation curves and restrict their mathematical complexity, certain requirements must be made on g_i. The abscissa values must be strictly monotonical even in the case of the generalized spline interpolation. So a parametric representation is absolutely necessary to calculate regression curves. The first derivatives according to t $[x(t), y(t)]$ at the starting and finishing point must be also be specified as peripheral conditions. The most important feature of the generalized spline function is that the g_i functions depend on parameters that can be selected. Normally one or two parameters are used. The course of the interpolation curve can be controlled in each interval by varying the parameters until the desired curvature is obtained (Kühn, 1983a). As a result, the curve will look more or less like a polygon line or a cubic spline at various intervals. But quite different curves may emerge. It is ideal to have interactive communications with a computer via a screen for these purposes.

Exponential spline functions with a variable parameter p_k

If we use the following at each interval $[x_k, x_{k+1}]$ for the non-polynominal function f_k

$$f_k(x) = a_k u + b_k v + c_k \varphi_k(u) + d_k \varphi_k(v) \tag{12.58}$$

$$\text{with } v = \frac{x - x_k}{\Delta x_k}, \quad u = 1 - v \tag{12.59}$$

$$j_k(v) = \frac{\sinh(p_k * v) - v * \sinh p_k}{\sinh p_k - p_k} \tag{12.60}$$

and

$$\varphi_k(u) = \frac{\sinh(p_k * u) - u * \sinh p_k}{\sinh p_k - p_k} \tag{12.61}$$

an exponential spline function is created (Späth, 1969). The function φ now carries an index k as a sign that its design may depend on the interval $[x_k, x_{k+1}]$. The p_k are variable parameters with

$$0 \leq p_k < \infty, \quad (k = 1, \ldots, n-1), \tag{12.62}$$

which may of course coincide.

Under these conditions:

$$\begin{aligned} f(x_k) &= y'_k, & f(x_{k+1}) &= y_{k+1} \\ f'(x_k) &= y'_k, & f'(x_{k+1}) &= y'_{k+1} \\ \Delta x_k &= x_{k+1} - x_k, & \Delta y_k &= y_{k+1} - y_k, \end{aligned} \tag{12.63}$$

the coefficients a_k, b_k, c_k, d_k can be specified from the corner point correlations of a fixed interval $[x_k, x_{k+1}]$.

Results of experiments

(1) In the case of an exponential spline interpolation, the parameter p_k in each interval can be varied in such a way that a curve occurs between a third-degree polynomial and the straight. Theoretical and practical tests have demonstrated that a cubic spline is obtained if $p_k = 0$. If $p_k \rightarrow \infty$, an interpolating polygon line is obtained, i.e. a first-degree spline (Figure 12.12).
(2) A high degree of approximation is achieved between the flexible rod line and the spline curve if the p_k values are close to zero – this generates what is roughly a cubic spline.

Rational spline functions with a variable parameter p_k

A further form of spline interpolation with variable parameters is the rational spline interpolation system, which can be used with an interactive dialogue system for interpolation between set fixed points to create smooth curves.

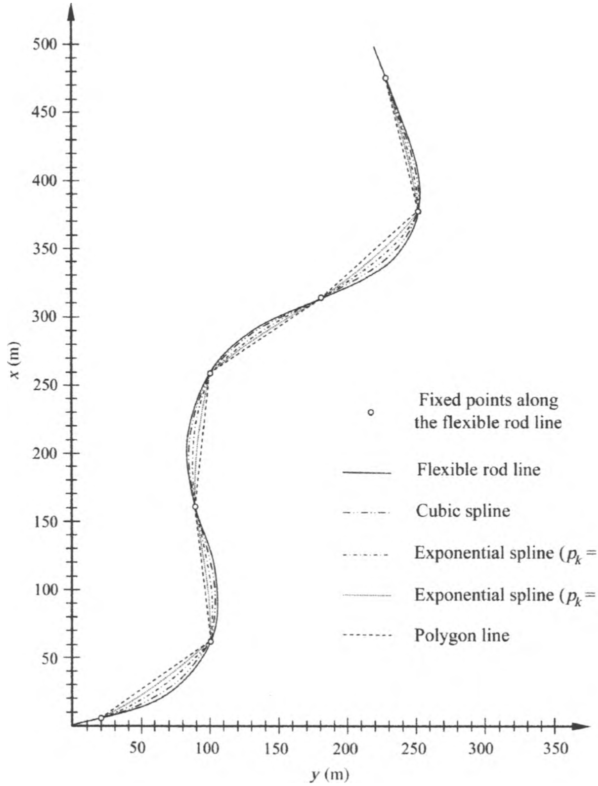


Figure 12.12: Approximation of the flexible rod line using exponential splines (Kühn, 2002a)

In contrast to formula (12.58), the rational spline function f is composed of $n - 1$ part elements f_k of the shape (Späth, 1971):

$$f_k(x) = a_k + b_k(x - x_k) + c_k(x - x_k)^2 + \frac{d_k}{x - x_k + p_k} \tag{12.64}$$

$$p_k \in [-\infty, -\Delta x_k] \, or \, p_k \in [0, \infty], \quad (k = 1, \ldots, n - 1) \tag{12.65}$$

This means that there is a parabola in each interval $[x_k, x_{k+1}]$, which is superimposed by a hyperbola, the pole of which lies outside the interval $[x_k, x_{k+1}]$ in each case. The parameters p_k determine the status of the poles. But formula (12.64) is unstable numerically, as differences involving numbers of almost the same size occur. This negative factor does not occur if the equivalent approach is used with the same demands on p_k:

$$f_k(x) = a_k(x - x_k) + b_k(x_{k1} - x_k) + \frac{p_k(x - x_k)(x_{k+1} - x)}{x - x_k + p_k}[c_k(x - x_k) + d_k(x_{k+1} - x)] \quad (12.66)$$

Based on the familiar conditions (12.63), the equations for the coefficients a_k, b_k, c_k, d_k depending on the values x_k, y_k and y'_k can be determined from formula (12.66).

Results of experiments

(1) In the case of rational spline interpolation using approach (12.66), the parameter p_k can be varied in each interval $[x_k, x_{k+1}]$ according to formula (12.65). Theoretical and practical work have demonstrated that a cubic spline results if $p_k \rightarrow \pm\infty$. If however the engineer aims for $p_k \rightarrow 0$, different curvature is achieved than if cubic spline interpolation is used. The curves often have less sharp bends at the fixed points and do not "shorten the route" at all. Figure 12.13 demonstrates the results described here.
(2) The rational spline function responds in a most sensitive manner if negative values for p_k are selected near $-\Delta x_k$. However, the reaction is weak in the intervals where the cubic spline function has already established an elongated course to the curve (Figure 12.14).
(3) Undesirable bulges in the cubic splines can also be avoided by selecting suitable p_k values (Figure 12.14).

Rational spline functions with variable parameters for p_k and q_k
In the case of rational spline interpolation, the spline function f can also consist of $n - 1$ part items f_k where (Späth, 1973):

$$f_k(x) = a_k u + b_k v + c_k \frac{u^3}{p_k v + 1} + d_k \frac{v^3}{q_k u + 1} \quad (12.67)$$

$$\varphi_k(v) = \varphi(q_k, v) = \frac{v^3}{q_k u + 1} \quad (12.68)$$

$$\varphi_k(u) = \varphi(p_k, u) = \frac{u^3}{p_k v + 1} \quad (12.69)$$

With regard to u and v, the same approaches apply as in Section 12.2.4. In contrast to (12.60), (12.61), and (12.62), f_k in each interval $[x_k, x_{k+1}]$ depends on two variable parameters p_k and q_k

$$-1 < p_k, q_k < \infty, \quad (k = 1, \ldots, n - 1) \quad (12.70)$$

which may of course coincide. Different values for p_k and q_k characterize different gaps for the poles for $q_k(u)$ from zero to the left and for $q_k(v)$ from one to the right. By analogy to the other approaches using generalized spline functions, the

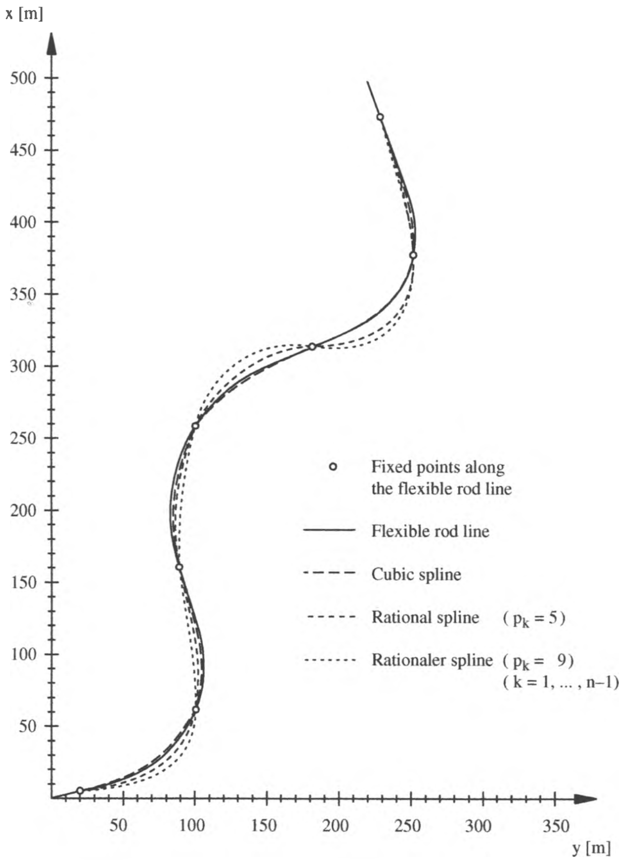


Figure 12.13: Approximate matching of flexible rod line using rational splines with a variable parameter p_k (Kühn, 2002a)

calculation of the coefficients a_k, b_k, c_k, d_k depends on the values x_k, y_k and the first derivatives y'_k.

Results of experiments

(1) If rational spline interpolation with two variable parameters p_k and q_k is used, the course of the curve can more or less match a cubic spline or an interpolating polygon line. In a similar way to exponential spline interpolation, a cubic spline is obtained if $p_k = q_k = 0$ and a spline of the first degree if $p_k, q_k \rightarrow \infty$ (Figure 12.15).
(2) If rational spline interpolation is used, similar curve courses or interpolation results are achieved to exponential spline interpolation, as the formulae (12.58) and (12.67) also react in a similar way when parameters are varied.
(3) Normally it is enough to use the same value for p_k und q_k in an interval $[x_k, x_{k+1}]$.

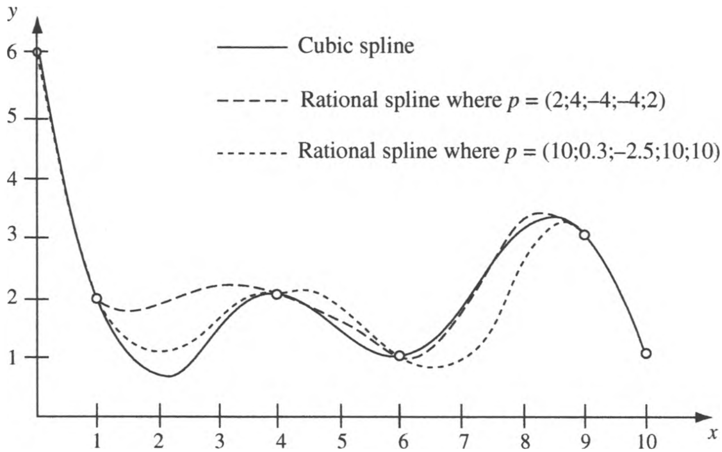


Figure 12.14: Eliminating undesirable undulations by using rational spline functions (Späth, 1971)

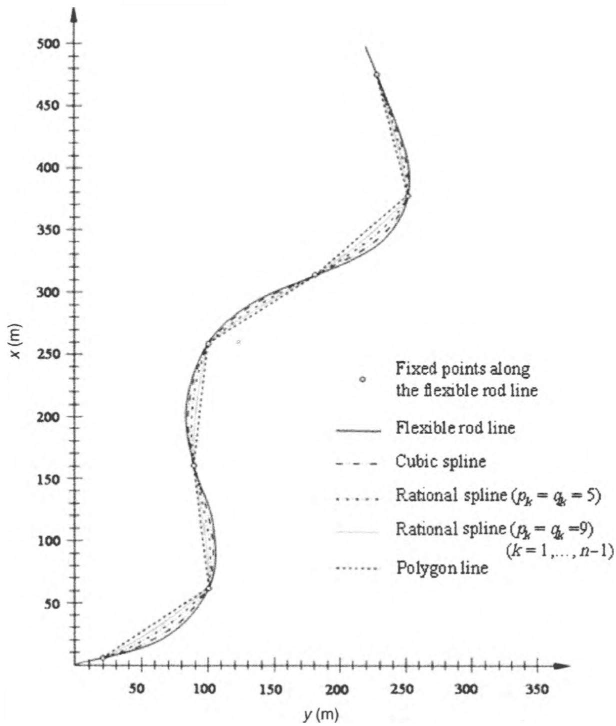


Figure 12.15: Resemblance with the flexible rod line using rational splines with variable parameters p_k and q_k (Kühn, 2002a)

Curvature graph

When using generalized spline functions for calculating axes, the curvature plot of the spline curve also depends on the design of the piecewise combined mathematical function and the freely selected parameter *ab* – in addition to the constellation of fixed points – in contrast to the cubic spline function. Because of the special design of the generalized spline functions, the mathematical approach and thereby the design of the interpolation curve can be reduced to a cubic spline function when setting the basic parameters ($p_k = 0$ for exponential spline, $p_k \rightarrow \pm\infty$ for one parameter-rational spline, p_k, $q_k = 0$ for two parameters-rational spline). In this case the curvature plot of the traditional road planning elements approximately matches the statements made in Section 12.2.3. But if the freely variable parameters are altered, curvature plots ranging between a cubic spline function and a polygon line will emerge, depending on the design of the function – or completely different curvature plots will be created.

Example (Figure 12.16)

A section of road, consisting of three curves following each other, was designed using a flexible rod. The axis calculation was based on the constellation of fixed points using an exponential spline function with a variable parameter p_k. As part of setting the basic parameters, p_k was immediately set at zero in all the intervals at the beginning of the calculation process and therefore the exponential spline function was reduced to a cubic spline function. The curvature plot that emerged matches that of a sequence of apex clothoids very well. However, if p_k is changed from $p_k = 0$ towards infinity, curves are created, which have a sharper degree of curvature at the fixed points and also have the effect of shortening the route. If $p_k = 7$, a curve is created, which has a high degree of curvature at the fixed points. Consequently, the curvature plot has marked curvature apexes at the fixed points.

The most important results of experiments can be summarized as follows:

(1) When using exponential or rational spline functions with one or two variable parameters (p_k or p_k, q_k), the course of the interpolation curve may vary between a cubic spline function and an interpolating polygon line. If the variable parameters are changed from zero towards infinity, curves are created, which have a high degree of curvature at their fixed points, i.e. curvature apexes occur. In order to be able to provide curvature plots that are similar to those achieved when using traditional road planning elements as part of any practical application, it only makes sense to only make minor changes to the parameters close to zero.

(2) If rational spline functions of a special kind are used with just one variable parameter for the axis calculation process, curves can be calculated, which are very smooth and have positive curvature despite the variation of the parameter.

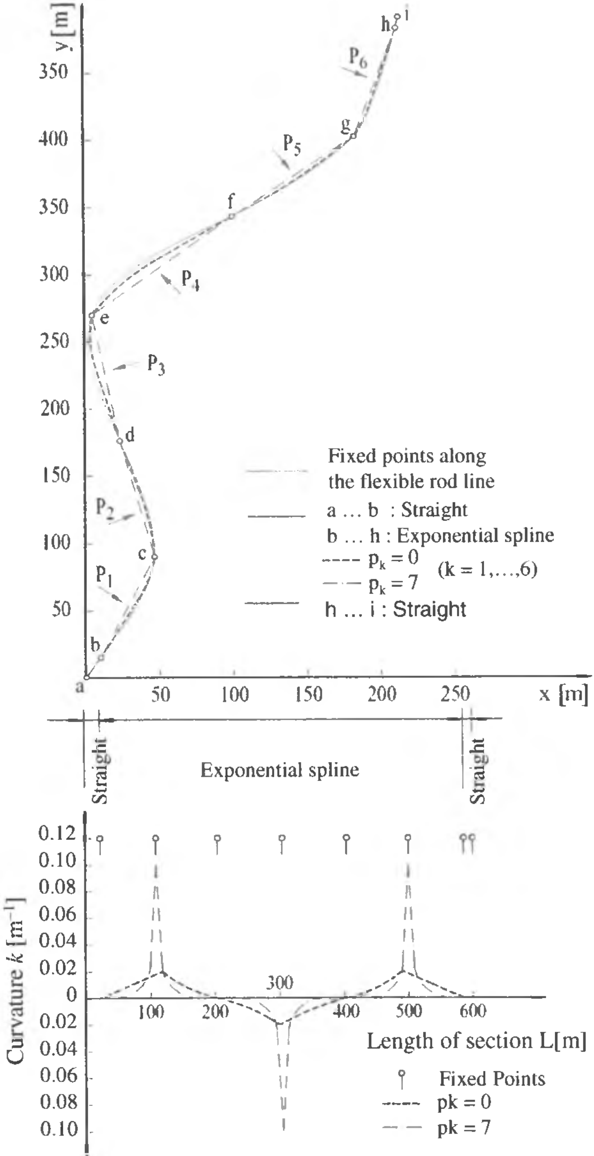


Figure 12.16: How varying the parameters affects the design of the interpolation curve and the curvature graph (Kühn, 1989b)

(3) Even if generalized spline functions are used, additional fixed points in the area of the supposed circular arc on the flexible rod line should be specified in order to achieve workable curvature plots.

Summary

Generalized spline functions are only partially suitable for the mathematical model projection of the flexible rod line, depending on their mathematical type.

Exponential spline functions with a variable parameter p_k and rational spline functions with two variable parameters p_k and q_k can only be used if slight corrections to the axis are required, i.e. if the parameters only slightly differ from zero. If the parameter variations are larger, curves that shorten the route will be created, which generally have unsuitable curvature.

If, however, rational spline functions with one variable parameter p_k are used, interpolation curves can be created, the course and curvature of which is ideally suitable from a design point of view. The curves are often smoother than when using cubic spline interpolation.

All the approaches and curvature plots of the generalized spline functions can be reduced to cubic spline functions by setting the basic parameters. By varying the parameters, axis alterations can be made in certain sections by altering the constellation of fixed points.

The use of mathematical approaches for generalized spline functions with more than one variable parameter should be avoided, as these approaches are normally hard to manage in practice. However, it makes sense to specify various parameters in the individual intervals in conjunction with any alterations to the axis that are planned. Data specifying standard values and thresholds are needed to improve the way that parameters are varied in practice.

12.2.5 Conclusions

In the procedure used so far, a spline function s – regardless of the type of piecewise functions in the individual intervals – was laid on all the fixed points along the flexible rod line and the axis calculation work was then carried out by interpolation.

When planning upgrading work, especially in urban locations with a line of buildings along the road, it is often necessary to determine independent straights at selected points along the route. Experiments have demonstrated that no exact straight can be calculated using familiar spline approaches if two or more fixed points are specified. Because of the familiar way that spline functions link specified fixed points with straight lines, a more or less oscillating curve will emerge depending on the distance between the interpolation points, and this is unusable from a design point of view. So it is essential when calculating the axis to define independent straights and also calculate them accurately.

Cubic spline functions may be used without any problems along sections of the route that can be designed relatively freely. However, if a number of constraints exist, rational spline functions should be used for calculation purposes as the course of the road can then be corrected in various sections using the variable parameters without altering the constellation of fixed points.

When drawing up a design methodology for implementing and calculating the flexible rod line, it makes sense to define individual elements depending on the constellation of fixed points and their mathematical type, and carry out the actual axis calculation work with CAD programs based on selected combinations of elements.

As a parametric representation is generally selected for the mathematical approach to the individual elements, the design engineer has the option of defining the element in a three-dimensional way at the same time.

12.3 Parameters for the route characteristic

12.3.1 Preliminary remarks

The major difference when formulating design factors for a mathematical sequence of elements is that they should not consist of the geometrical individual elements of straights, clothoids, and circular arcs, but special mathematical functions. For this reason, parameters that are common for particular types of roads have to be modified for traditional road planning elements.

All the mathematical approaches for parameters for familiar types of roads are shown in Table 12.1 and will be explained in the following text.

12.3.2 Curvature, bendiness and parameters derived from this

The curvature K_i for each geometric individual element can be calculated without difficulty from the radius of curvature. By specifying a characteristic constellation of fixed points for each curve, similar values for the curvature or the radius of curvature are obtained, even with the mathematical sequence of elements.

The same calculation approaches apply to the mean curvature and the parameters derived from this ∂_k, KÄ, and ν_k, regardless of which mathematical model is used. The only difference is found when drawing up the curvature K_i. While the K_i values are calculated for a traditional sequence of elements from the course of the curvature using the familiar formula, the determination process with a mathematical road section takes place using the curvature formula of any two-dimensional curve. The functions $x(t)$ and $y(t)$ are arbitrary mathematical functions. As the selected distance for interpolation points is low, any formation of mean values can be waived and $K_i = \chi_i$ applies.

A low figure for the mean curvature normally means that the route has been elongated or has a high ratio of straights. However, it is possible that similar curvature values will emerge for existing main roads, which were designed with long straights and small radii, as for upgraded sections, which have an alignment with many similar curve radii. As a result, the standard discrepancy ∂_k is used to

Table 12.1: Mathematical approaches for typical parameters for types of routes using mathematical models (Kühn, 1983a, 1999b)

Parameter description	Traditional sequence of elements	Mathematical sequence of elements
1. Curvature $K_i[\mathrm{m}^{-1}]$	Straight: $K_i = 0$	$K_i = \chi_i = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}$
	Cirular arc: $K_i = \frac{1}{R_i}$	
	Clothoid: $K_i = \frac{K_p + K_{p+1}}{2}$	
	$K_p = \frac{1}{A^2} * L_p$	
2. Mean curvature $\bar{K}[\mathrm{m}^{-1}]$	$\overline{K} = \frac{1}{q}\sum_{i=1}^{q} *\lvert K_i \rvert$	
3. Empirical standard discrepancy $\delta_K[\mathrm{km}^{-1}]$	$\delta_K = \sqrt{\frac{1}{q}\sum_{i=1}^{q}(K_i - \overline{K})^2}$	
4. Mean change of curvature per km $K\ddot{A}[\mathrm{km}^{-2}]$	$K\ddot{A} = \frac{1}{L}\sum_{i=1}^{q-1}\lvert K_{i+1} - K_i \rvert$	
5. Bendiness $KU[\mathrm{gon*km}^{-1}]$	$KU = \frac{1}{L}\sum_{i=1}^{m}\lvert \tau_i \rvert$	$KU' = \frac{1}{L}\sum_{i=1}^{n}\lvert \beta_i \rvert$
6. Mean quadratic deviation $Q[\mathrm{gon*km}^{-1}]$	$Q = \sqrt{\frac{1}{L}\left[\sum_{i=1}^{m}\frac{\tau_i^2}{L_i} - \frac{\left(\sum_{i=1}^{m}\lvert \tau_i \rvert\right)^2}{\sum_{i=1}^{m} L_i}\right]}$	$Q' = \sqrt{\frac{1}{L}\left[\sum_{i=1}^{n}\frac{\beta_i^2}{L_i'} - \frac{\left(\sum_{i=1}^{n}\lvert \beta_i \rvert\right)^2}{\sum_{i=1}^{n} L_i'}\right]}$
7. Inhomogeneity criteria $v_K, v_{KU}[-]$	$v_K = \frac{\delta_K}{K}, v_{KU} = \frac{Q}{KU}$	
8. Parameters for specific route types $SP[\mathrm{gon*km}^{-1}]$ $SP'[\mathrm{gon*km}^{-3}]$	$SP = \frac{\sum_{i=1}^{m}\left\lvert \frac{L_i}{R_i} \right\rvert *63,7*K_i + \sum_{i=1}^{m}\left\lvert \frac{L_i}{R_i} \right\rvert *63,7*K_j}{L}$	$SP' = \frac{\sum_{i=1}^{n}\frac{\lvert \beta_i \rvert}{L_i} *\lvert \max K_i \rvert}{L}$
9. Fluidity criterion $FL[\mathrm{km}^{-2}]$	$FL = \sum_{i=1}^{n}\lvert \max K_i \rvert * \frac{1}{L_i'}$	

supplement any assessment of road sections with the same or similar mean curvature. The ∂_k figure is normally small if the sequence of elements is well graded and harmonious.

The mean change of curvature KÄ can also be drawn on to check the features of a route. But as a weighted mean value, it does not provide any information on whether the change of curvature is abrupt or gradual or constant. So it makes more sense to calculate the change of curvature between two fixed points using a mathematical sequence of elements in order to be able to check and restrict them in the area of the supposed circular arc.

By way of analogy to the inhomogeneity criterion of bendiness, that of flection v_k as a quotient is formed from a standard variation and average individual curvature. The smaller v_k is, the greater the homogeneity of the road section.

12.3.3 Bendiness and parameters derived from this

Comprehensive research work carried out by Köppel and Bock (1970), Trapp (1974), Lamm (1972, 1973) and Leutner (1974) has demonstrated that bendiness (KU, KU′) is a crucial parameter for describing the characteristics of a route, because it significantly affects speed, driving behavior, and therefore accidents (Figure 12.17). Roads in irregular territory (foothills of the central uplands) have a high degree of curvature and their alignment on the horizontal projection is characterized by a large number of major changes of direction with small curve radii. Low values for bendiness are, however, an expression of generous and elongated road alignment. The calculation of bendiness (Table 12.1) is carried out with the mathematical sequence of elements from the direction angles β_i of the individual curves. The best way to determine β_i values is to use the scalar product of the vectors enclosed. But the two approaches to calculation are in principle the same.

In a manner similar to individual curvature, the mean quadratic deviation (Q) is derived for general curvature too (Schlichter, 1977) in order to be able to separately assess road sections that have the same bendiness, but use a different sequence of elements. A modified approach must be selected for the mathematical sequence of elements, which starts with the individual element "curve" and therefore provides generally lower values for Q:

$$\sum_{i=1}^{n} \frac{\beta_i^2}{L_i'} < \sum_{i=1}^{m} \frac{\tau_i^2}{L_i}. \qquad (12.71)$$

It is generally possible to draw on the mean quadratic deviation of bendiness to assess the characteristics of the route. Depending on its size, conclusions can be drawn about their "balance" or "homogeneity."

Schlichter (1977) derived the inhomogeneity criterion for bendiness v_{KU} (Table 12.1) as a further parameter for specific route types from bendiness KU and its mean quadratic deviation Q on the basis of variation coefficients in mathematical statistics. It is also used to assess the homogeneity of a sequence of elements. The smaller v_{KU} is, the more harmonious the alignment will be on the horizontal projection, i.e. the road section consists of well-graded individual geometric elements.

12.3.4 Specific route parameters (SP)

Lamm (1972) derived the specific route parameter SP from the bendiness (Table 12.1) in order to assess the characteristics of a section of a road in the lower radius region. He takes into account the fact that not only bendiness, but also curves with radii of $R_i \leq 350$ m have a major influence on driving behavior. By weighting the bendiness values with the appropriate curvature factors, more attention is paid to the individual element as well as the sequence of elements. These curvature factors have been drawn up on the basis of driving experiments and are listed in tabular form. They represent the speed restrictions, which are needed for small radii related to a radius of 500 m.

12.3.5 Fluidity criterion (FL)

Based on ideas presented by Schek (1973), what is known as a fluidity criterion was defined (Table 12.1). If there is a fixed number of curves n, the section will be all the more fluid on the horizontal projection, the smaller the maximum curvature of each individual curve is. A reduction in FL therefore normally means that small and short curves are avoided.

12.3.6 Curves with similar radii

The parameters discussed so far in relation to specific roads are generally mean values, i.e. they can only be drawn on to assess the "mean" characteristics of a route. But they do not provide any details about the true size and correlation of individual elements located next to each other. These parameters do not recognize individual discontinuity in the course of a road (Figure 12.17).

Köppel and Bock (1970) discovered that as the size of the radius increased, the distribution of driving speeds increases accordingly and if the radius is $R = 300$–700 m, it is possible to drive at 85 km h^{-1} in the curves. Taking into account this fact, the sequence of elements becomes more important when assessing the road characteristics.

Köppel and Bock (1970) recognized in the 1970s that the correlation between radii next to each other or following each other has a major effect on driving behavior, driving speeds, and therefore accidents. So the evenness of curves is a major criterion in ensuring that the characteristics of a road are balanced.

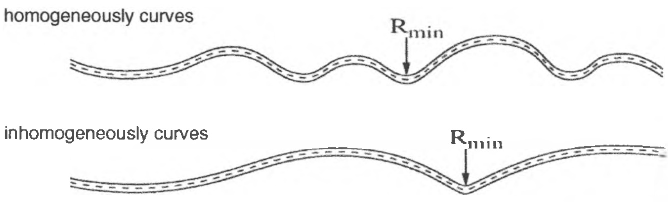


Figure 12.17: Road with curves with similar and dissimilar radii (Köppel and Bock, 1970)

12.3.7 Results of experiments

In the case of the traditional and mathematical sequence of elements, it was possible to achieve a high degree of conformity with the "road length" parameter. But the individual lengths of straights are very different because the straights are specified exactly for the axis calculation for the mathematical sequence of elements, while they are usually random when breaking down the flexible rod line into the traditional road planning elements.

Despite different mathematical models, a high degree of conformity was established between the bendiness and mean curvature parameters for specific routes, i.e. these parameters are comparable to each other.

On account of the modified approach to the mean quadratic deviation of overall curvature, smaller values emerged with the mathematical sequence of elements than from the traditional road section. In order to obtain the same or similar figures, the modified approach to the mathematical road section should be used in the traditional sequence of elements as well.

The parameters derived from the curvature ∂_k, ν_k, KÄ, FL could not be used for comparative tests, as the calculation figures fluctuate too much.

The results of the investigations demonstrated that it is often adequate to calculate the general curvature or mean curvature and use this information to assess a road section, as both parameters depend on each other in a linear fashion. The parameters derived from these two figures help with the supplementary assessment, especially if route sections with the same curves or general curvature are to be analyzed. An attempt has been made with the SP specific route parameter to capture the effect of overall curvature and curves on driving behavior as a result of one parameter.

12.4 "DITRA" Dialogue Routing Program System

12.4.1 Preliminary remarks

When calculating the axis using the GRUTRA (Kühn, 1987a, 1987b) program system, the contours on the flexible rod line are relatively stiff on account of the fixed points that have been specified when using the mathematical model. The course of the road is clearly determined by the constellation of fixed points and there are only minimum differences between the flexible rod line and the mathematical axis.

But if undesirable inflection points occur during the axis calculation process or if the diagnosis of constraints is negative in some areas, the preliminary graphical draft has to be corrected and the axis calculation repeated. This process is often very wearisome and time-consuming, particularly as the final course of the road normally only differs slightly from what is supposed to be a faulty course of the axis. A tiny correction to the route in the sections concerned is usually all that is necessary to meet the demands for continuity and smoothness and also comply with the diagnosis of the constraints.

By developing, defining, and introducing what are known as mathematical design elements, the conditions for a new kind of dialogue-oriented design methodology for roads are created in difficult situations, where a smooth and continuous outline is being sought and has to be calculated.

The design methodology that has been developed in principle opens up the opportunity of bringing together a wide variety of different thematic elements by defining the same peripheral and transitional conditions as axis lines on the horizontal projection.

This allows a possible three-dimensional approach when formulating suitable mathematical elements and sequences of elements.

12.4.2 Model projection

Introduction

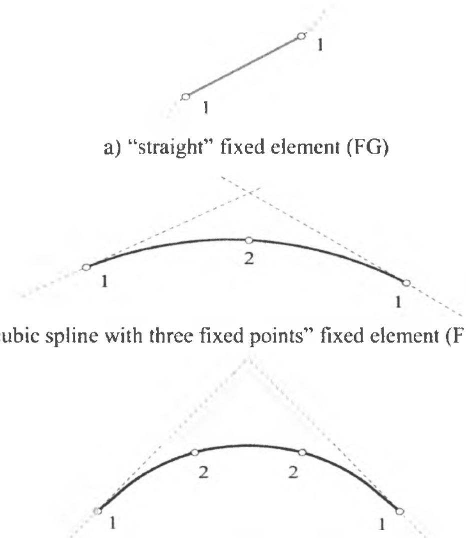
The mathematical model projection provided by DITRA integrates the mathematical principles of GRUTRA. Taking into account the particular conditions when tracing the route in tight situations and the necessary dialogue work, the following design elements were defined mathematically and introduced so that the axis can be calculated in DITRA on the horizontal projection (Kühn, 1987, 1988, 1989a, 1999a):

- fixed elements,
- dialogue elements,
- coupling elements.

When breaking down a flexible rod line into a sequence of mathematical elements, the fixed elements are determined first by the constraints that exist and the constellation of fixed points. Depending on the case in point, dialogue elements can be incorporated between the fixed elements using a coupling element. The mathematical design and a characteristic constellation of fixed points are clearly set for the individual elements.

Fixed elements

The following fixed elements were developed for DITRA based on the mathematical model in GRUTRA (Figure 12.18):



a) "straight" fixed element (FG)

b) "cubic spline with three fixed points" fixed element (FKS 3)

c) "cubic spline with four fixed elements" fixed element (FKS 4)

Figure 12.18: Fixed elements (Kühn, 2002a)

- "straight" fixed element (FG),
- "cubic spline with three fixed points" fixed element (FKS 3),
- "cubic spline with four fixed elements" fixed element (FKS 4).

The fixed elements are clearly defined by a characteristic constellation of fixed points, the mathematical approach in each case, and the constant peripheral conditions and they do not allow for any variations. Any change to the course of the road can only be introduced by moving individual local fixed points. But to obtain a homogeneous and constant sequence of elements, the preliminary graphical draft must be corrected in the areas concerned and the axis calculation has to be repeated.

Figure 12.19 illustrates the mathematical model for a sequence of fixed elements. The beginning and end straight is not a fixed element, but simply clarifies the peripheral conditions at the start or end of the building work (consistency of the gradient). Depending on the change in direction, three fixed points (simulating an apex clothoid) are specified for curve 1 and four fixed points for the cubic spline (simulating a combined curve) in curve 2. There is an independent straight included as a fixed element between these two fixed elements.

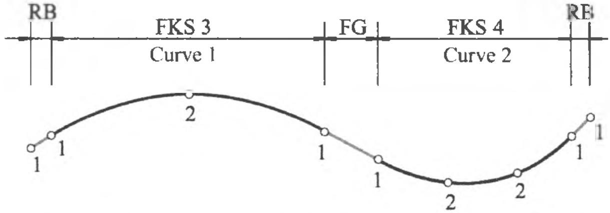


Figure 12.19: Sequence of fixed elements (Kühn, 2002a)

Dialogue elements

Dialogue elements are defined and introduced in order to also be able to manage the course of the road in individual sections without having to pay regard to the fixed points. The dialogue elements provide a certain amount of room to maneuver when shaping the flexible rod line. Depending on which mathematical approach is used, the parameters for dialogue elements can be freely selected. By varying the parameters, the course of the road can be altered in various sections in dialogue with the computer.

Generalized spline functions can be used as dialogue elements and they differ from each other in the following areas:

(1) Number and location of the fixed points,
(2) Mathematical design (exponential, rational splines),
(3) Number of free parameters (p_k or p_k, q_k).

In general, the following dialogue elements can be defined depending on the constellation of fixed points (Figure 12.20):

- Simple dialogue element: (DVS 2),
- Free dialogue element: (DVS 3),
- Bound dialogue element: (DVS 4),
- Support dialogue element: (DVS 6).

The number of dialogue sections and the room to maneuver overall are determined by the individual dialogue elements as a result of the constellation of fixed points. As the number of fixed points increases, the number of areas for possible variation rises too, but, in the end, the room to apply corrections to the route decreases. If the number of fixed points is too large, the constraints on the course of the axis are too great and extremely elaborate dialogue work must be carried out as a result of the many dialogue areas within the dialogue element. Normally dialogue elements should be selected with a constellation of fixed points that fits the case at hand.

The mathematical design of the dialogue element has a crucial effect on the characteristics and the geometrical course of the interpolation curve as well as the constellation of fixed points.

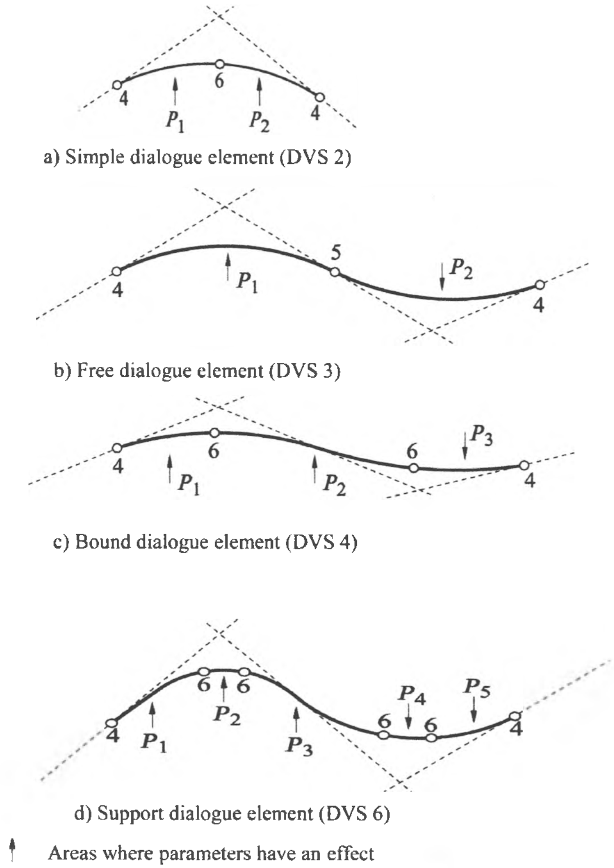


Figure 12.20: Dialogue elements (Kühn, 2002a)

Comprehensive research work has demonstrated that the mathematical approach best suited to the dialogue elements in each interval $[x_k,\ x_{k+1}]$ is a piecewise function where:

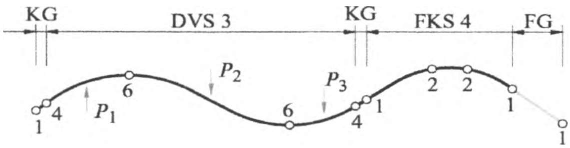
$$f_k(x) = a_k(x - x_k) + b_k\left(x_{k+1} - x_k\right)$$

$$+\frac{p_k(x - x_k)\left(x_{k+1} - x\right)}{x - x_k + p_k}\left[c_k(x - x_k) + d_k\left(x_{k+1} - x\right)\right]. \qquad (12.72)$$

By varying the parameters, the course of the interpolation curve can be toggled between a cubic spline ($p_k \to \pm\infty$) and other smooth curves ($p_k \to 0$). It should also be said that a free parameter is adequate to cover the dialogue needs if a sensible design for the generalized spline function has been adopted.

Coupling element

A coupling element is used to link fixed and dialogue elements mathematically (Figure 12.21). The coupling element may also lie at the beginning or end of a route section and therefore be placed before or after a fixed or dialogue element. In both cases the coupling element merely guarantees the peripheral conditions at the start, transition, or end of the spline and does not act as a design element. Independent straights as fixed elements can also adopt the function of a coupling element.



KG: Straight coupling element
DVS 3: Rational spline dialogue element
FKS 4: Cubic spline fixed element
FG: Straight fixed element

Figure 12.21: Sequence of fixed, dialogue, and coupling elements (Kühn, 2003)

12.4.3 Curvature graph

While the individual curvature graph in a traditional sequence of elements can be determined directly from the parameters R and A, it depends on the following criteria for a mathematically calculated route (Kühn, 1999a):

- the constellation of fixed points
- the mathematical approach adopted
- variable parameters.

The constellation of fixed points is set during the preliminary graphical draft phase. Care must be taken to heed the constraints that exist: the straight areas, changes of direction, and design relations with adjacent curves.

As part of setting the fundamental parameters, the mathematical route is normally reduced to a sequence of fixed elements at the beginning of the calculation process, the curvature of which largely matches the classical sequence of elements (Figure 12.22). The fixed elements FKS 3 and FKS 4 adequately simulate the course of the classical apex clothoid and combined curve sequence of elements. Relatively little room for maneuver is available for any corrections to the route because of the relatively tight specification of the sequence of elements, but the curvature graph does not alter to a great degree.

But if dialogue elements of a special kind (Figures 12.23 and 12.24) are used to break down the flexible rod line and these dialogue elements can be attributed

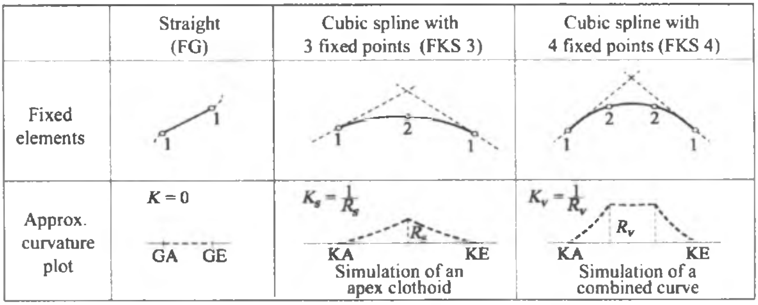


Figure 12.22: Curvature graph for fixed elements (Kühn, 1989b)

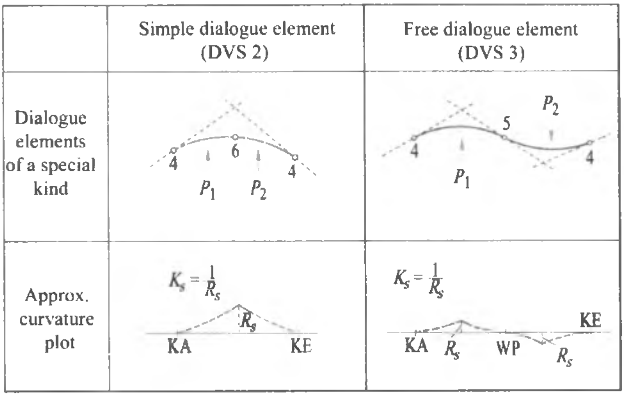


Figure 12.23: Curvature graph for dialogue elements DVS 2 and DVS 3 when setting basic parameters (Kühn, 1989b)

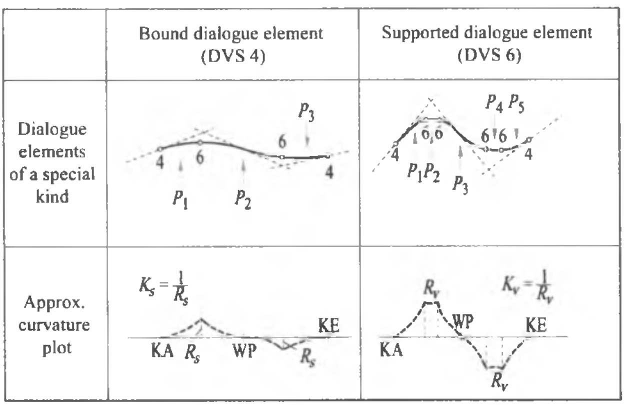


Figure 12.24: Curvature graph for dialogue elements DVS 4 and DVS 6 when setting basic parameters (Kühn, 1989b)

to the familiar fixed elements because of the constellation of fixed points that has been selected, a dialogue route is created, which allows a great deal of room for maneuver when making corrections to the route.

As the route increasingly deviates from the flexible rod line, the design features of the dialogue route also alter in the sections concerned.

If the corrections to the route as a result of varying the parameters are only slight, there is normally only a phase displacement of the individual curvature graph within the relevant dialogue section and the curvature factors are retained (Figure 12.25):

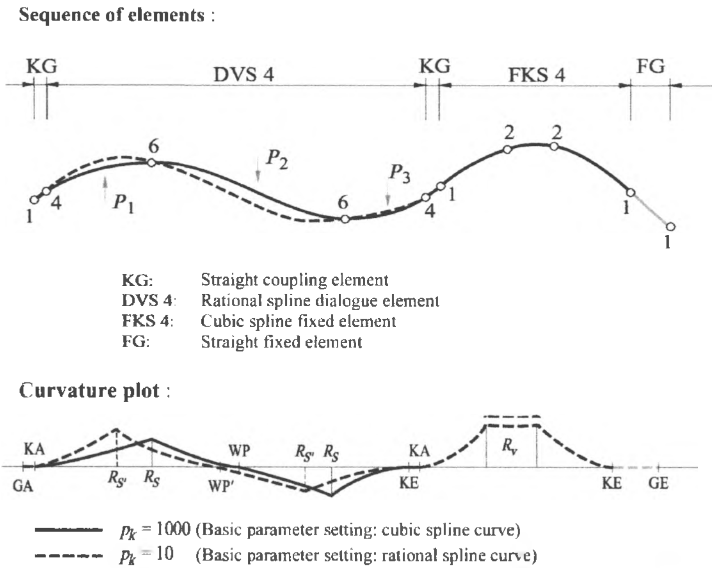


Figure 12.25: Effects of corrections to the route on the curvature graph (Kühn, 1989b)

12.4.4 Explanations of the design methodology

Introduction

The technical procedures when using DITRA for working out the design at points of constraint can in principle be divided into the following phases (Kühn, 1999a; Figure 12.26):

- searching for a route with a flexible rod
- determining the sequence of elements
- digitalizing the point coordinates and determining other input data
- calculating the axis for different sections taking into account individual constraints

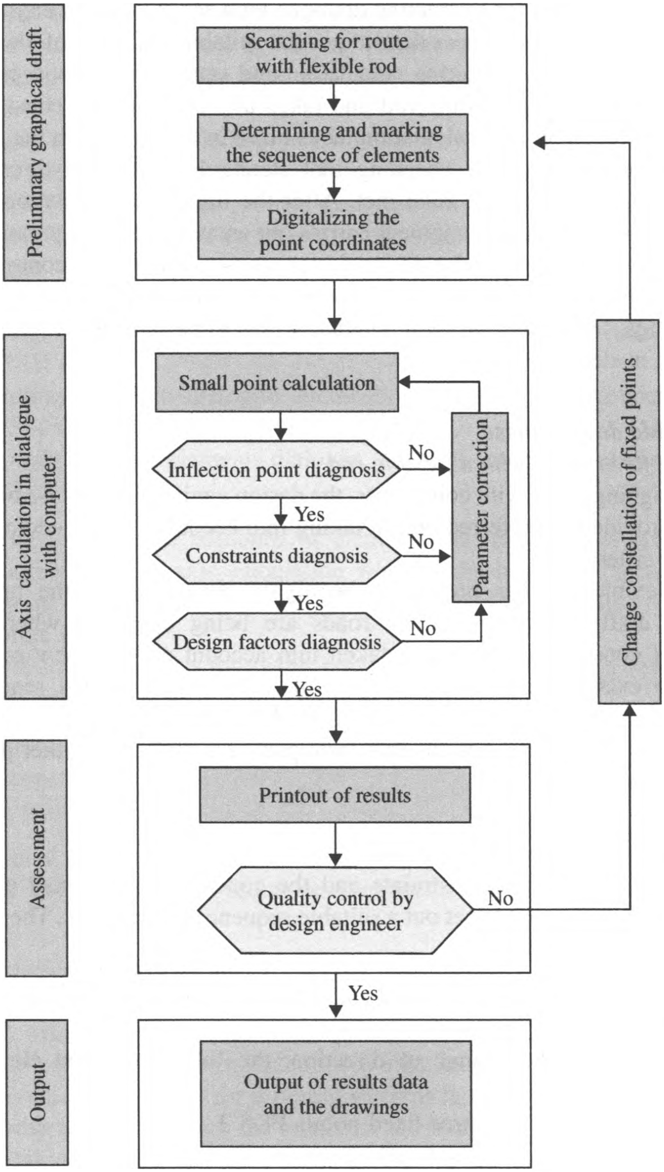


Figure 12.26: Design methodology using DITRA (Kühn, 1999a)

- calculating the staking out figures and design factors for the whole route
- quality checks and assessment
- preparing the drawings.

DITRA was designed for interactive dialogue between the design engineer and a computer. The design engineer determines the suitable sequence of elements and the individual sections requiring calculation and starts with a good preliminary graphical draft using a flexible rod and takes into account any constraints that exist. The subsequent axis calculation takes place in dialogue with the computer. This process also involves checking that standard values and thresholds for defined design factors have been met. After the final axis calculation work for the whole route, the design engineer carries out an assessment of parameters that are specific to the route and the design factors as part of a quality control process. If the route meets the demands that have been set, the engineer starts to produce the drawings. Otherwise the axis calculation has to be repeated after alterations have been made to the input.

The graphic design phase

Searching for a route with a flexible rod

When designing roads with constraints, the design engineer has to use his creative skills to provide the preferred option taking into account design, ecological, and economic criteria.

The flexible rod has proved to be highly beneficial for the preliminary graphical draft, particularly when roads are being upgraded where a large number of constraints have to be taken into account and the new road has to match the existing road as far as possible. The result of this search is the graphical image of the flexible rod on the horizontal projection – the actual flexible rod line. Several options that form part of a study for further processing are also possible.

Determining the sequence of elements

Taking into account the constraints and the constellation of fixed points, the design engineer now has to set out a suitable sequence of elements. The following principles must be followed:

(1) The route must always start and end with a straight, which acts as a fixed or coupling element.
(2) Depending on the change of direction, the following fixed elements are chosen:
   - a cubic spline with three fixed points FKS 3
     (simulating an apex clothoid)
     or
   - a cubic spline with four fixed points FKS 4
     (simulating a combined curve).
(3) The fixed elements FKS 3 and FKS 4 can be placed alongside each other arbitrarily. In principle, a fixed element (FG) or a coupling element (KG) can be interconnected.

(4) Dialogue elements should only be planned in curves with constraints so as to be able to change the route by varying parameters, if necessary.
(5) Depending on the specific situation, suitable dialogue elements should be chosen.
(6) Dialogue elements are incorporated into a sequence of fixed elements by using a "straight" coupling element. Independent straights at the beginning, end, or intermediate points are also fixed elements and may adopt the function of a coupling element. The following transition conditions are possible in principle:
   - coupling element–dialogue element–coupling element (KG–DVS–KG),
   - "straight" fixed element–dialogue element–"straight" fixed element (FG–DVS–FG),
   - coupling element–dialogue element–"straight" fixed element (KG–DVS–FG),
   - "straight" fixed element–dialogue element–coupling element (FG–DVS–KG).

After setting the suitable sequence of elements, the fixed points are marked. This guarantees that the axis calculation work in the individual sections takes place using the correct mathematical approach (straight, cubic spline function or rational spline function).

The coordinates for the following points are determined as a result of the preliminary graphical draft on the horizontal projection:

- fixed points on the route,
- critical constraints,
- polygon points.

The individual calculation sections for the axis calculation work are also well-established, depending on the sequence of elements that has been selected.

Axis calculation work in dialogue with the computer

General issues

In contrast with procedures used in the past for the GRUTRA program system, DITRA has been conceived in such a way that the design engineer can carry out the axis calculation work for different sections in dialogue with the computer. He starts with the areas where the course of the road can be varied by dialogue elements, taking into account any constraints. If the axis that is calculated provides the necessary spacing between the constraints and also meets the checking conditions set for the relevant section, the course of the road has been set. After completing all the dialogue areas, the fixed elements are included in the course of the final, integrated axis calculation work. This not only computes all the staking out points, but also determines the parameters required for the route and the design factors on the whole route for the subsequent quality control procedures. It is also possible to convert the small points along the route to any system of coordinates.

DITRA implementation program

Figure 12.27 illustrates the general design of the DITRA implementation program. After inputting data and finding any errors in the input, the search

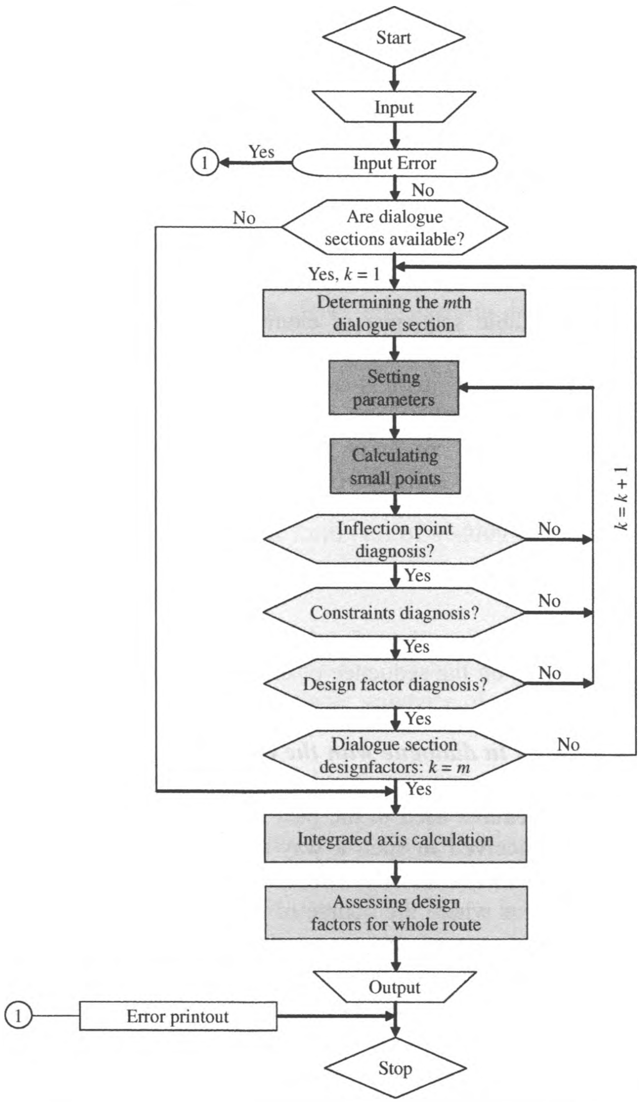


Figure 12.27: Block diagram of the DITRA program system (Kühn, 1999a)

for an initial dialogue section takes place using the same numbering system. The parameters p_k, $(k = 1, \ldots, n - 1)$ are set for each dialogue section, depending on the constraints present at each interval $(k = 1, \ldots, n - 1)$. Normally the basic parameters are set first – i.e. all the parameters are the same size $(p_1 = p_2 = \cdots = p_k,\ k = 1, \ldots, n - 1)$ and are given the basic parameter value according to the numbers $(p_k = 1000,\ k = 1, \ldots, n - 1)$. This means that the dialogue route is reduced to a fixed sequence of elements in each dialogue section at the start of the calculation work, which optimally simulates the course of the flexible rod line and also takes into account fundamental design conditions.

Depending on the constraints present, the design engineer can now alter the optional parameters for intervals in dialogue with the computer. If no undesirable inflection points occur and the necessary spacing between constraints is met and if the design factors meet the stipulations, the first dialogue section can be completed and the route is set for this section. When all the dialogue sections have been completed, the integrated axis calculations take place based on the fixed and dialogue elements.

Quality control and assessment
Using the printouts of results, the design engineer now has to carry out quality checks. As important checks for the section calculations have already been carried out in dialogue with the operator as part of the program, the quality control process is largely restricted to assessing parameters for specific stretches and design factors for the whole route.

12.4.5 Example

The dialogue oriented design methodology developed and explained using the DITRA implementation program is now explained using an example (Kühn, 1999a; Figure 12.28) and the results achieved are then analyzed.

Taking into account the existing buildings, the preliminary graphical design line is determined on the horizontal projection as the preferred option using a flexible rod. The "coupling element – dialogue element – fixed element" sequence of elements is set for the subsequent axis calculation work. As part of setting the basic parameters, the dialogue element (generalized spline) is reduced to a fixed element (cubic spline) (broken line) at the start of the calculations. As the analysis of the constraints at points z_0, z_1, and z_2, z_3 is negative, parameter p_k has to be altered at certain intervals $(p_k \rightarrow 0)$. It is possible to meet the set spacing between constraints if $p_k = 1$. At the same time there are no undesirable inflection points and the standard values for the design checks on the individual elements and sequences of elements are met (dotted line).

The coupling element at the start of the route simply determines the starting conditions and acts as a fixed element. The fixed element at the end of the route

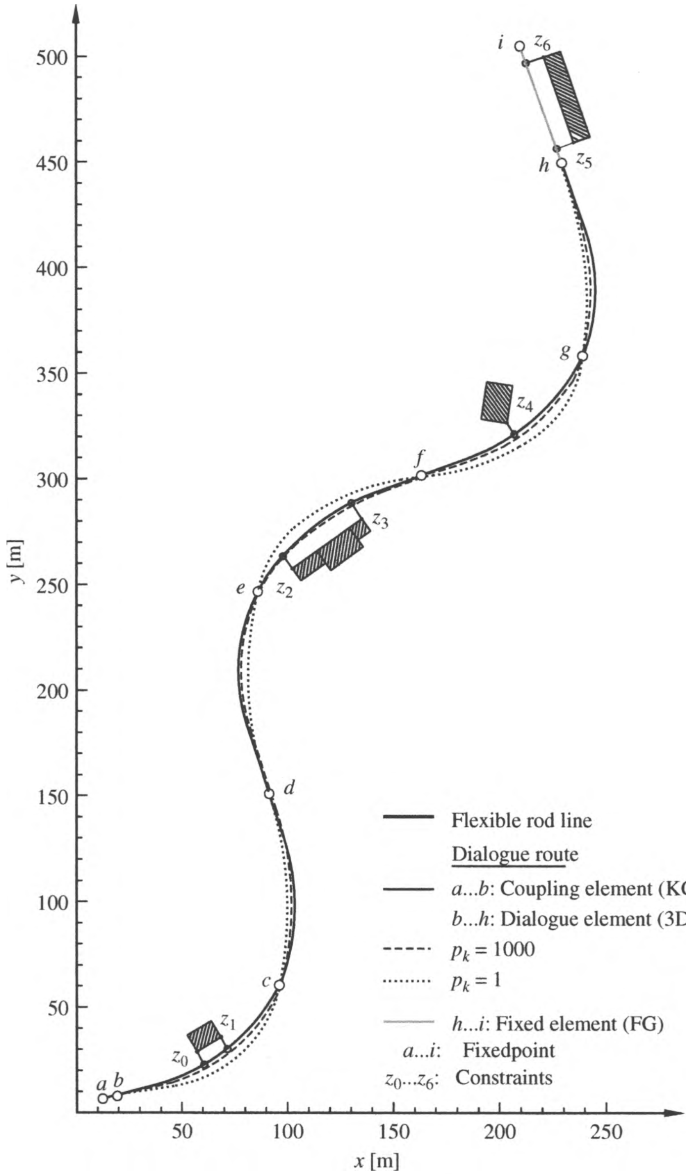


Figure 12.28: Example (Kühn, 2002a)

has to be selected because of the buildings in a line at the edge. A dialogue element is defined between the coupling element and fixed element because of the buildings along the whole route. This provides a relatively large amount of room for maneuver for possible corrections to the route, i.e. the axis can be altered in this area by varying the parameters without any problem.

The same value is attributed to the free parameter p_k in every interval of the dialogue element in this example ($p_1 = p_2 = \cdots = p_6$). Then in dialogue with the computer and after the basic parameters have been set, a suitable free parameter is selected using four iteration stages ($p_k = 100, 10, 5, 1$); this sets and calculates the dialogue route, which takes into consideration all the major checking criteria.

12.4.6 Summary and conclusions

Direct dialogue working procedures between the design engineer and the computer are absolutely essential for drawing up a route with constraints. For this reason, a modified model projection with defined mathematical design elements and permissible sequences of elements has been developed for the DITRA program system. The design engineer has access to individual elements to break down and calculate the preliminary graphical draft (flexible rod line): fixed, dialogue and coupling elements.

Depending on the constellation of constraints, the design engineer must decide on the suitable sequence of elements for calculating the axis. He will set dialogue elements at critical sections along the route and allow slight corrections to the route by varying the parameters. Fixed elements will only be used if no change to the route is needed in the area concerned.

By using dialogue elements, the design engineer can directly gain access to the axis calculation work during the work procedures without altering the constellation of fixed points. Using slight shifts to the route in selected sections, any undesirable inflection points can be eliminated without any problem and negative diagnoses on constraints can be corrected easily.

When making corrections to the route by varying parameters, the curvature plot of the route must be checked constantly to prevent any unfavorable curvature graphs. The optional parameters p_k should be set at the beginning of the calculation work in such a way that the dialogue elements can be reduced to "cubic spline" fixed elements. The course of the road can be altered iteratively by gradually varying the parameters.

Normally only slight corrections to the route should be made without altering the constellation of fixed points in order to avoid negatively affecting the continuous transitional conditions at the coupling points in the sequence of elements.

The program system that has been developed and the design methodology that has been drawn up have to be adjusted to match the practical applications and tested on selected examples. Particular attention should be given to setting the standard values for the optional parameters. The parameters and control checks for GRUTRA should also be tested.

12.5 Three-Dimensional Design Approach

12.5.1 The problem

Road design is still carried out in three separate design stages: the horizontal projection, the vertical projection, and the cross section. This does not provide a three-dimensional image of the road during the design phase so that it can be assessed and checked.

Using calculated perspectives and sequences of images depicting the driving area, it is possible to check the perceptibility of the driving area and the three-dimensional alignment of the road during the design stage, assess this and, if necessary, alter the alignment on the horizontal projection, the vertical projection, and the cross section.

At an early stage, consideration was given to developing a three-dimensional design methodology, which would use suitable mathematical approaches and models, in order to avoid design errors that occur as a result of processing the individual design levels separately and then superimposing them. Despite some attempts, a procedure that enables engineers to calculate the axis three-dimensionally still does not exist.

The fact that the search for a suitable mathematical model is still going on is not the only problem; there is still no three-dimensional land model of the earth, which can be shown on a computer screen and used for processing three-dimensional design work. Taking into account the fact that a three-dimensional survey of the earth's surface is going on, we can expect a compact, digital, three-dimensional model of the earth to be available in the medium term and this would provide another important pillar for a three-dimensional design methodology.

12.5.2 Review of developments

Brauer (1942) turned his attention to the differential geometric principles of three-dimensional curves for producing a three-dimensional route for roads for the first time in 1942. In order to solve this task mathematically, he developed what is known as a "moving trihedral," consisting of a unit vector t on the tangent line, the unit vector situated in a vertical relationship to the former and lying on the wheel axis that is parallel to the road surface and the unit normal vector η located in a vertical relationship to both (Figures 12.29 and 12.30).

Lorenz (1943) determined the axis with the help of cylinder barrels in his experiments on three-dimensional route planning. These cylinder barrels serve as supports for the three-dimensional transition curve, which is designed to be interposed between the two warped straights g_1 and g_2 running tangentially to the cylinder barrels (Figure 12.31).

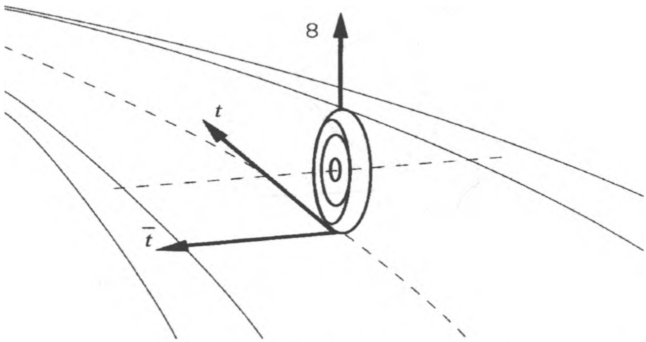


Figure 12.29: Moving trihedral on the road (Brauer, 1942)

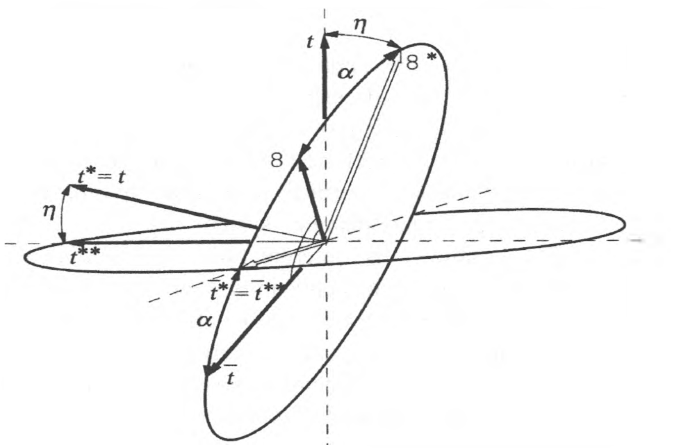


Figure 12.30: The three moving trihedrals (Brauer, 1942)

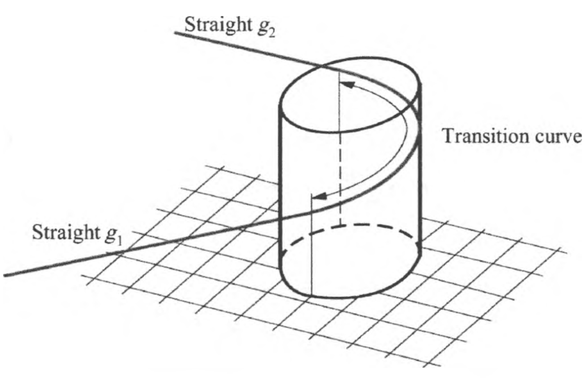


Figure 12.31: The Lorenz model (Lorenz, 1943)

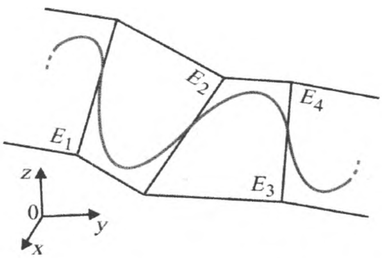


Figure 12.32: The Freising model (Freising, 1949)

Freising (1949) proposed a geometric design system using his Freising model, where the route planning axis consists of even three-dimensional curves ($\tau = 0$). A shallow curve was used as the three-dimensional element here (Figure 12.32).

Scheck (1973) examined various approaches for optimizing three-dimensional route planning. He gradually optimized the route on the horizontal and vertical projections. Normally the vertical projection was calculated and set using dynamic optimization based on a preferred option on the horizontal projection. But this iterative horizontal and vertical projection optimization does not represent a three-dimensional route planning system.

Borgmann (1976) examined the interpolation of three-dimensional fixed points with a flexible rod for a specific application where the hyperbolic transition curve was used as a three-dimensional transition curve.

Psarianos (1982) carried out extensive research into developing a model using the three-dimensional design elements of a straight, a helix, and a general apex clothoid. The general apex clothoid is used as a transition curve between the straight and the helix (Figure 12.33).

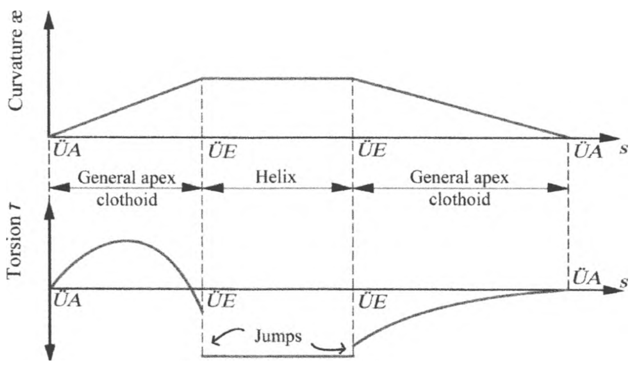


Figure 12.33: Model projection according to Psarianos (1982)

The use of interpolation and compensation methods on the basis of three-dimensional fixed points and constraints using various mathematical functions provides new opportunities for a three-dimensional approach to route planning.

These approaches, model projections, and mathematical theories on three-dimensional route planning clearly demonstrate that no practical solution has yet been found to realize this demanding task. Some of the model projections are very abstract, are mathematical calculations, and are not yet generally valid.

12.5.3 Mathematical model projections

As the traditional route planning elements, clothoids and circular arcs, are not very suitable for describing or calculating a road axis in a three-dimensional manner, it seems to be more appropriate to start with the experience gained in route planning on two levels and also look for suitable mathematical functions, which link set three-dimensional fixed points with smooth curves and at the same time take into account important design principles.

By way of analogy with axis calculation on the horizontal projection, spline functions can also be used to depict and calculate a three-dimensional road axis. By using a suitable parametric representation for the spline function, a three-dimensional road axis can be calculated using a mathematical approach by setting number triples (x_k, y_k, z_k), $(k = 1, \ldots, n)$ (Kühn, 1999b, c).

Three-dimensional approach to a spline function

If a three-dimensional approach is to be used to depict and calculate a road axis with the help of spline functions, a suitable system for representing parameters must be introduced.

If a parameter t_k, $(k = 1, \ldots, n)$, which grows monotonically with the course of the curve, is selected by way of analogy to axis calculations on two levels,

$$t_1 < t_2 < \cdots < t_n \qquad (12.73)$$

the spline function can be depicted parametrically using the following three functions

$$x = x(t), y = y(t), z = z(t) \qquad (12.74)$$

where the parameter t_k is the accumulated three-dimensional arc length (Figure 12.34).

When calculating the axis, a spline curve, which is clearly set by determining the first and second derivations according to (t), is laid through the points (t_k, x_k) on the (t, x) level, (t_k, y_k) on the (t, y) level and (t_k, z_k) on the (t, z) level. If t_k runs through the interval $[t_1, t_n]$, the triples $[x(t_i), y(t_i), z(t_i)]$ are the small points on the three-dimensional interpolation curve.

The ideal value for the t_k parameters would be the accumulated arc length of the curve, but this first has to be calculated. An iteration process is one

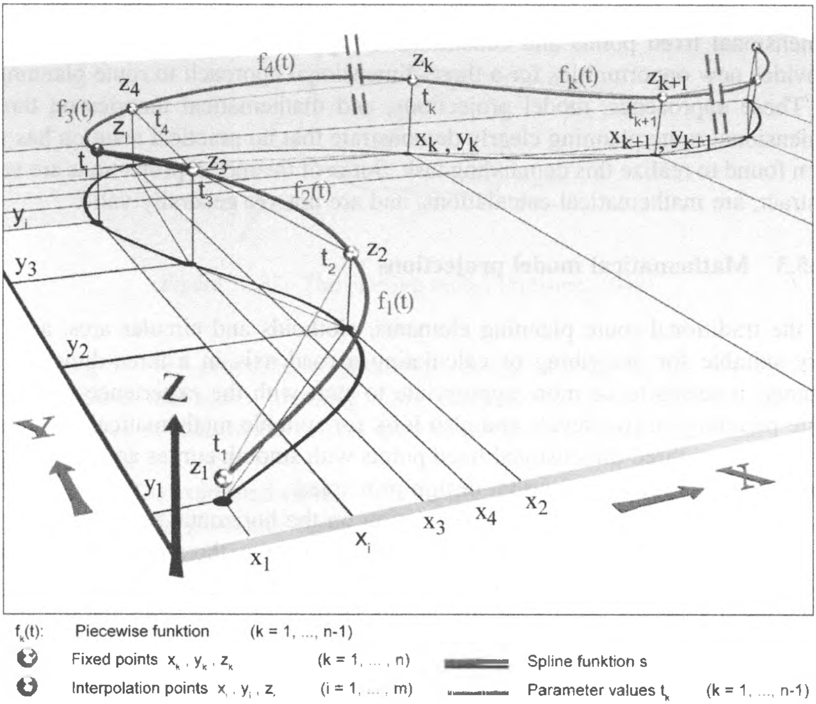


Figure 12.34: Parametric representation of a three-dimensional spline function (Kühn, 2002a)

conceivable approach. Theoretical and practical experiments have demonstrated that the most appropriate representation of t_k is a straight link between the fixed points (chord length). However, attention must be paid to the fact that the chord length does not deviate too much from the arc length.

The parameter t_k can be calculated recursively as follows:

$$
\begin{aligned}
t_1 &= 0, \\
t_k &= t_{k-1} + \sqrt{\Delta x_{k-1}^2 + \Delta y_{k-1}^2 + \Delta z_{k-1}^2}; (k = 2 \ldots n).
\end{aligned}
\tag{12.75}
$$

The derived mathematical approaches for calculating the first or second derivations in the fixed points in order to determine the coefficients and interpolate are generally valid for the three-dimensional parametric representation in a spline function too. Depending on the t_k parameters, however, a separate calculation of the determining points of the three splines takes place using the points (t_k, x_k), (t_k, y_k), (t_k, z_k).

If the coefficients are established using the second derivations according to t, the following modified correlations are the result:

$$\ddot{x}_k = Fx_k - G_k * \ddot{x}_{k+1};\ (k = 2, \ldots, n-1) \tag{12.76}$$

$$Fx_k = \frac{6\left(\frac{\Delta x_k}{\Delta t_k} - \frac{\Delta x_{k-1}}{\Delta t_{k-1}}\right) - \Delta t_{k-1} * Fx_{k-1}}{2(\Delta t_{k-1} + \Delta x_k) - \Delta t_{k-1} * G_{k-1}};\ Fx_1 = 0 \tag{12.77}$$

$$G_k = \frac{\Delta t_k}{2(\Delta t_{k-1} + \Delta t_k) - \Delta t_{k-1} * G_{k-1}};\ G_1 = 0 \tag{12.78}$$

$$ax_k = \frac{1}{6\Delta t_k}\left(\ddot{x}_{k+1} - \ddot{x}\right) \tag{12.79}$$

$$bx_k = \frac{1}{2}\ddot{x}_k \tag{12.80}$$

$$cx_k = \frac{\Delta x_k}{\Delta t_k} - \frac{1}{6}(\ddot{x}_{k-1} + \ddot{x}_k) \tag{12.81}$$

$$dx_k = x_k;\ (k = 1, \ldots, n-1). \tag{12.82}$$

If x is replaced by y and z, the relevant equations are formed for $\ddot{y}_k$, Fy_k, ay_k, by_k, cy_k, dy_k and $\ddot{z}_k$, Fz_k, az_k, bz_k, cz_k, dz_k. The formulae for the coefficients depending on the first derivations according to t can be determined in a similar fashion.

After determining the parameters t_k and the coefficients, the appropriate x_i, y_i and z_i values can be calculated for any values of t_i, $(i = 1, \ldots, m)$ using the following interpolation formulae:

$$\begin{aligned} x_i &= ax_k(t_i - t_k)^3 + bx_k(t_i - t_k)^2 + cx_k(t_i - t_k) + dx_k \\ y_i &= ay_k(t_i - t_k)^3 + by_k(t_i - t_k)^2 + cy_k(t_i - t_k) + dy_k \\ z_i &= az_k(t_i - t_k)^3 + bz_k(t_i - t_k)^2 + cz_k(t_i - t_k) + dz_k \quad (k = 1, \ldots, n-1). \end{aligned} \tag{12.83}$$

12.5.4 Axis calculation with three-dimensional elements

Three-dimensional elements of a special mathematical type, which are placed alongside each other in prescribed fixed point triples (x_k, y_k, z_k), $(k = 1, \ldots, n)$ and from which it is possible to obtain a second derivation, are the basis for calculating a three-dimensional axis. The geometrical course of the interpolating three-dimensional curve thereby directly depends on the prescribed constellation of fixed points and the type of mathematical function in the relevant interval. By using special rational spline functions, the course of the three-dimensional curve between any two interpolation points can be altered by varying optional parameters. This means that the course of the axis of the three-dimensional curve can be adjusted in dialogue with the computer without altering the constellation of fixed points.

Taking into account the planned manner of the dialogue work for the three-dimensional route planning, a modified mathematical model has been drawn up based on DITRA with the following three-dimensional elements:

- three-dimensional fixed elements,
- three-dimensional dialogue elements,
- a three-dimensional coupling element.

This model guarantees greater flexibility for calculating the axis and also allows direct intervention during the design work (Kühn, 2005).

Three-dimensional fixed elements

A fixed element is clearly defined by the constellation of fixed points. The three-dimensional curve is calculated using interpolation between the fixed point triples on the basis of a three-dimensional cubic spline function.

Alongside a three-dimensional straight, which is determined by two fixed points, it is possible to set three fixed points (simulating the geometrical course of a three-dimensional apex clothoid) or four fixed points (simulating the geometrical course of a three-dimensional combined curve) depending on the intended change of direction.

The following three-dimensional fixed elements are possible:

(a) Three-dimensional independent straight (dFG):

Using the approach for the parameter t_k from Müller (1975), the three-dimensional independent straight can be depicted using the three functions $x = x(t)$, $y = y(t)$, $z = z(t)$ (Figure 12.35).

The coordinates (x_i, y_i, z_i) of the interpolation points on the straights can be calculated as follows:

$$
\begin{aligned}
x_i &= mx_k(t_i - t_k) + bx_k \\
y_i &= my_k(t_i - t_k) + by_k \\
z_i &= mz_k(t_i - t_k) + bz_k \quad (k = 1, \ldots, n), \ (i = 1, \ldots, m).
\end{aligned}
\tag{12.84}
$$

(b) Three-dimensional cubic spline with three fixed points (dFKS 3) and

(c) Three-dimensional cubic spline with four fixed points (dFKS 4):

The following approaches are possible for calculating the interpolation points on a three-dimensional cubic spline and using this parametric representation:

$$
\begin{aligned}
x_i &= ax_k(t_i - t_k)^3 + bx_k(t_i - t_k)^2 + cx_k(t_i - t_k) + dx_k \\
y_i &= ay_k(t_i - t_k)^3 + by_k(t_i - t_k)^2 + cy_k(t_i - t_k) + dy_k \\
z_i &= az_k(t_i - t_k)^3 + bz_k(t_i - t_k)^2 + cz_k(t_i - t_k) + dz_k.
\end{aligned}
\tag{12.85}
$$

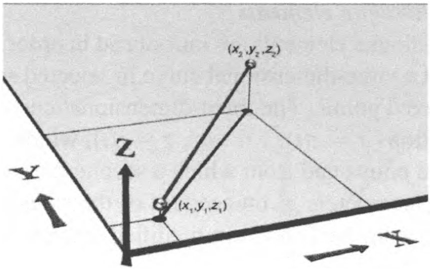


Figure 12.35: Fixed element: three-dimensional independent straight (Kühn, 2002a)

Figures 12.36 and 12.37 illustrate the mathematical model for a three-dimensional fixed element with three and four fixed points.

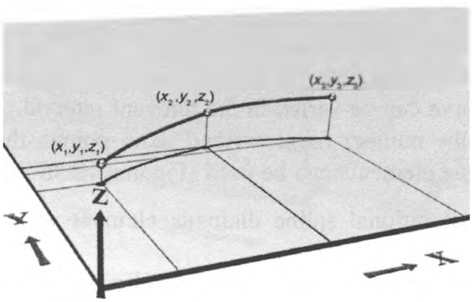


Figure 12.36: Three-dimensional cubic spline fixed element with three fixed points (Kühn, 2002a)

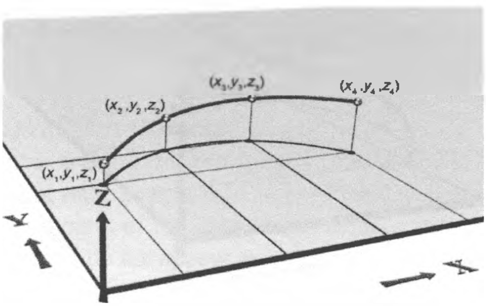


Figure 12.37: Three-dimensional cubic spline fixed element with four fixed points (Kühn, 2002a)

Three-dimensional dialogue elements

Three-dimensional dialogue elements are introduced in order to be able to vary the geometrical course of a three-dimensional curve in selected sections, regardless of the constellation of fixed points. The three-dimensional curve then consists of the three piecewise functions $x = x(t)$, $y = y(t)$, $z = z(t)$, which are placed alongside each other at the fixed points and from which a second derivation can be obtained and which have free parameters p_k on account of their design. The interpolation points are calculated using the following modified approaches:

$$x_i = ax_k(t_i - t_k) + bx_k(t_{k+1} - t_k) + \frac{p_k(t_i - t_k)(t_{k+1} - t_i)}{t_i - t_k + p_k}\left[cx_k(t_i - t_k) + dx_k(t_{k+1} - t_i)\right]$$

$$y_i = ay_k(t_i - t_k) + by_k(t_{k+1} - t_k) + \frac{p_k(t_i - t_k)(t_{k+1} - t_i)}{t_i - t_k + p_k}\left[cy_k(t_i - t_k) + dy_k(t_{k+1} - t_i)\right]$$

$$z_i = az_k(t_i - t_k) + bz_k(t_{k+1} - t_k) + \frac{p_k(t_i - t_k)(t_{k+1} - t_i)}{t_i - t_k + p_k}\left[cz_k(t_i - t_k) + dz_k(t_{k+1} - t_i)\right]. \tag{12.86}$$

By varying the p_k parameter between zero and infinity, the geometrical course of the interpolation curve can be varied in the relevant interval.

Depending on the number of prescribed fixed points, the following three-dimensional dialogue elements can be used (Figures 12.38–12.40):

- Three-dimensional rational spline dialogue element with three fixed points (dDRS 3),
- Three-dimensional rational spline dialogue element with four fixed points (dDRS 4),
- Three-dimensional rational spline dialogue element with five fixed points (dDRS 5).

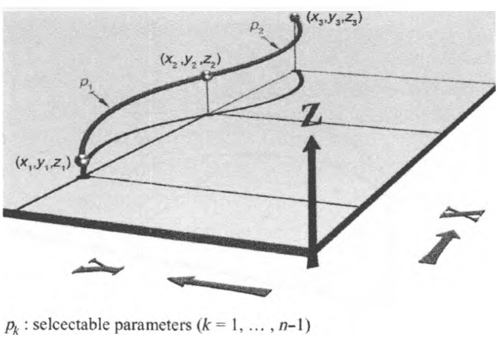


Figure 12.38: Three-dimensional rational spline dialogue element with three fixed points (Kühn, 2002a)

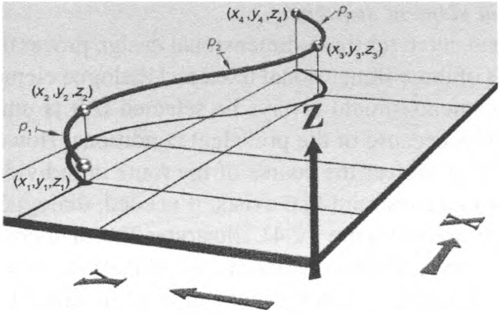


Figure 12.39: Three-dimensional rational spline dialogue element with four fixed points (Kühn, 2002a)

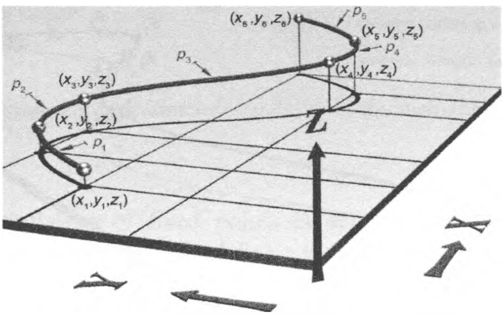


Figure 12.40: Three-dimensional rational spline dialogue element with six fixed points (Kühn, 2002a)

The fewer fixed points are stipulated for a dialogue element, the greater freedom there is, i.e. the course of the interpolation curve can be altered in the individual sections if the number of fixed points is low and if the distance between the fixed points is fairly large.

Three-dimensional coupling element

A coupling element is used to mathematically link fixed and dialogue elements. The coupling element only guarantees the peripheral conditions at the starting, transitional and final points of the three-dimensional spline. Coupling elements are short straights, which do not appear as independent straights on account of the short distance between the fixed points, but are simply necessary as a coupling to determine the first derivations (inclination). Independent straights as a fixed element can also assume the function of a coupling element between fixed and dialogue elements.

Three dimensional element sequence

A three-dimensional curve for three-dimensional design processing can be defined from various kinds of three-dimensional fixed and dialogue elements and coupling elements. Fixed elements should always be selected if it is unnecessary to vary the course of the axis because of the prevalent conditions. However, if the aim is to have more freedom to steer the course of the route in individual sections so as to be able to adapt to a constraint that exists, if needed, dialogue elements should be defined in these areas. Figure 12.41 illustrates the mathematical model of a three-dimensional curve, which consists of a combination of fixed, dialogue, and coupling elements. Initially a coupling element is set to define the spline between the fixed points S2... S5. The rational spline dialogue element is also linked to the cubic spline fixed element using a coupling element. A three-dimensional straight is defined as a fixed element at the end of the three-dimensional curve, which again guarantees the peripheral conditions of the spline.

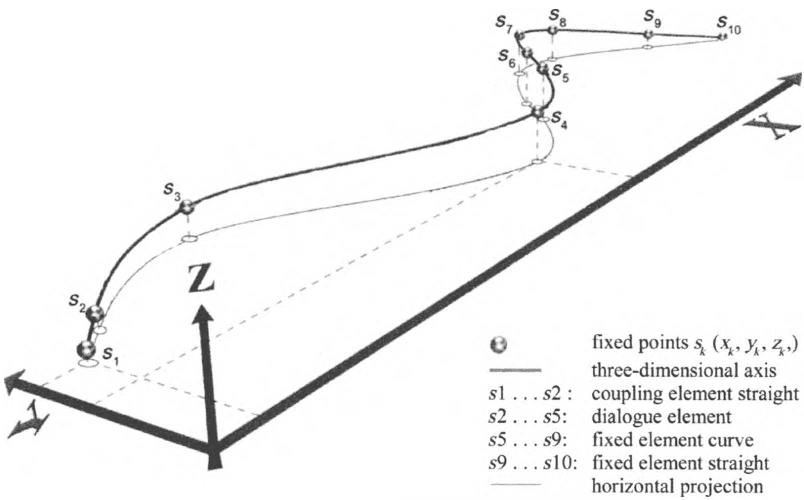


Figure 12.41: Three-dimensional curve consisting of fixed, dialogue, and coupling elements (Kühn, 2002a)

Taking into account the sequence of three-dimensional elements that have been selected, it is only possible to vary the course of the route in the area of the dialogue element between the fixed points S2 and S5 (Figure 12.42). The links between the constellation of fixed points determines the rest of the route.

12.5.5 Summary and conclusions

The research that has been carried out has demonstrated that cubic spline functions and rational spline functions in parameter form are highly suitable for use as mathematical functions for modeling three-dimensional elements.

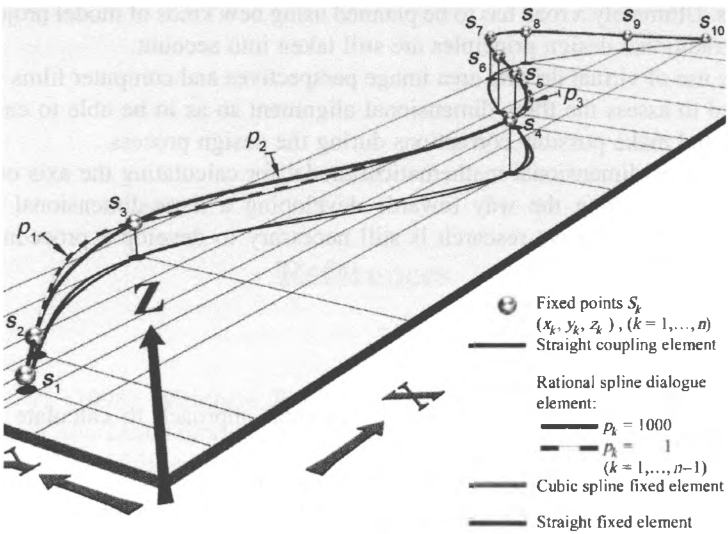


Figure 12.42: Altering the three-dimensional curve in the dialogue element area (Kühn, 2002a)

If a set constellation of fixed points is defined, three-dimensional fixed, dialogue and coupling elements can define a three-dimensional route and the axis can be calculated by interpolating between the fixed points.

While it is possible to use three-dimensional fixed elements to link up set fixed points smoothly and consistently through three-dimensional curves – by only following the constellation of fixed points – three-dimensional dialogue elements allow variations to a route even when the constellation of fixed points is not changed. By altering optional parameters along individual sections of the route, the course of the three-dimensional interpolation curve can be varied without having to correct the constellation of fixed points and it can therefore be adapted to meet any constraints that exist.

The calculation of the axis using three-dimensional elements opens up new possibilities for further developing previous design technology for road design. The three-dimensional axis small points are calculated in one working stage on the basis of the three-dimensional fixed points – i.e. in principle, it is no longer necessary to superimpose the horizontal and vertical projections.

The mathematical model projection described here is only one possible approach along the road towards developing a three-dimensional design methodology. Its practical application still has to be tested on everyday examples.

The development of suitable monitoring procedures to assess the three-dimensional mathematical axis from a design point of view is still in its infancy and this field in particular is extremely important for practical work in the design process. Ultimately a road has to be planned using new kinds of model projections while traditional design principles are still taken into account.

The use of virtual driving area image perspectives and computer films will be required to assess the three-dimensional alignment so as to be able to carry out checks and make possible corrections during the design process.

This three-dimensional mathematical model for calculating the axis of roads is a first step along the way towards developing a three-dimensional design methodology. Extensive research is still necessary to develop a procedure that fully functions in practice.

12.6 Questions

(1) Why is it not possible to use a polynomial approach to calculate a road section?
(2) Explain the mathematical principles of a spline function.
(3) Which general conditions should be used for a spline function from a design point of view?
(4) Why is the parametric representation of a spline function necessary for calculating axes?
(5) Explain the differences between a cubic, exponential, and rational spline function.
(6) How is it possible to adapt the curvature of a spline function to the classical design elements on the horizontal projection?
(7) Name and explain the essential design control checks when analyzing spline functions.
(8) What do you understand by the "fluidity criterion"?
(9) Name and explain the three-dimensional elements for three-dimensional route design work.

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Intelligent Road Design

***M.K. JIA**, Morgan State University, USA; **P.M. SCHONFELD**, University of Maryland, USA; **J.-C. JONG**, Sinotech Engineering Consultants, Inc., Taiwan and **E. KIM**, University of Incheon, South Korea*

Traditional textbooks on roadway design focus on fundamental alignment elements and design criteria. When applying these in practice, engineers must try various design combinations, check whether the resulting alignments satisfy requirements, and evaluate their relative effectiveness.

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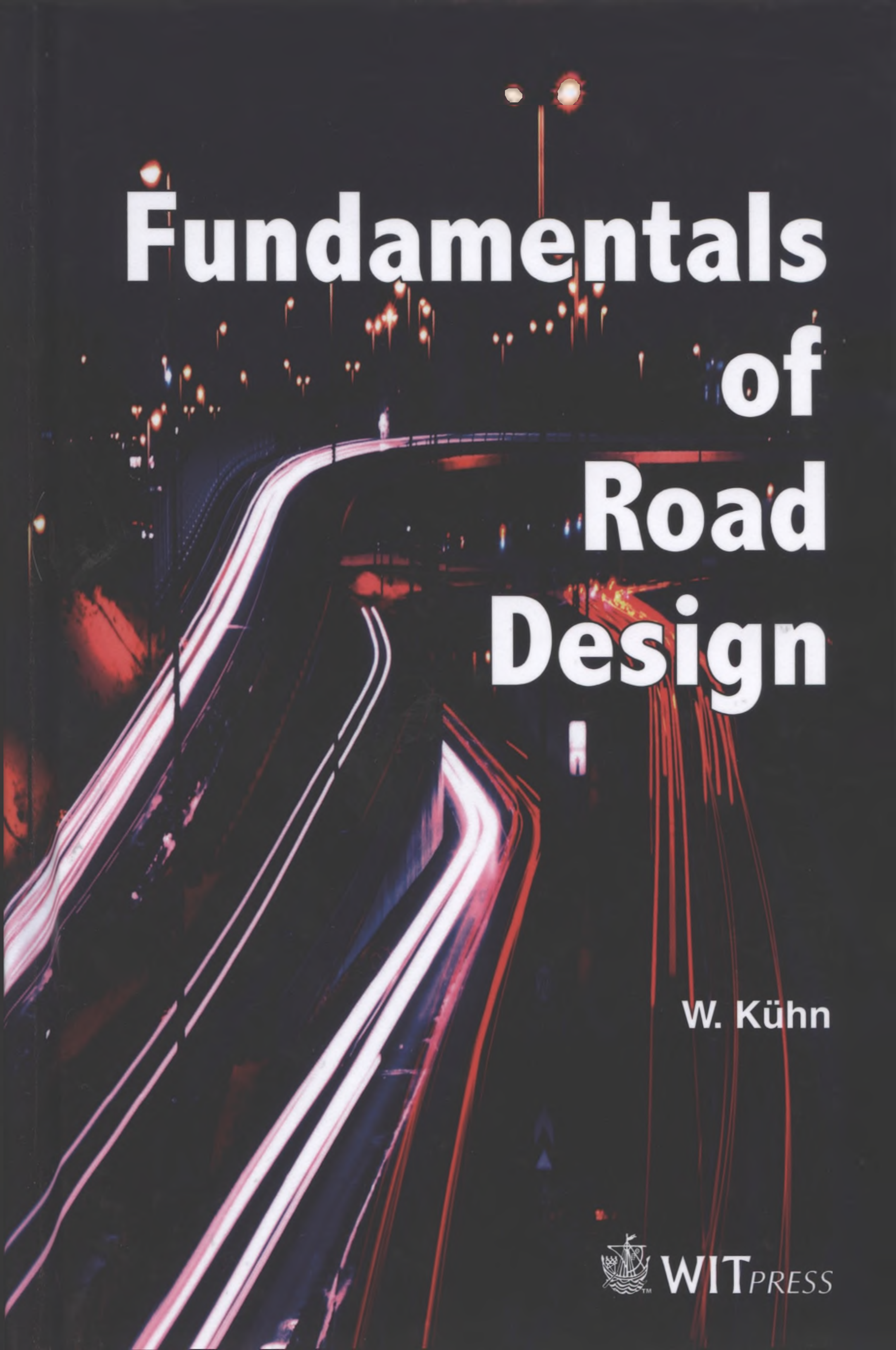
Fundamentals of Road Design



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Fundamentals of Road Design

This reference book communicates the basic theoretical knowledge, the practical requirements and experience for designing, mapping, calculating and checking roads and the planning process overall – and at the same time reveals important development trends.

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The terms, definitions, abbreviations and formula symbols are based on German usage, but can be transferred to an international framework without any difficulties.

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